Automorphisms of Hyperkähler varieties

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On Hyperkähler varieties

HYPERKÄHLER VARIETIES are simply connected complex analitic varieties endowed with a symplectic holomorphic 2-form generating $H^{2,0}$ or, equivalently, are Riemannian manifolds of real dimension $4m$ such that their holonomy group is $Sp(m)$.
The simplest example of such manifolds are K3 surfaces and it turns out that higher dimensional examples behave very much like K3's (see [8]): The second integral cohomology forms a lattice which determines much of the variety itself due to the weaker global Torelli theorem of M.Verbitsky (see [5]).
However there are few examples of such varieties, mainly 2 families in each even complex dimension due to Beauville and two isolated examples in dimension 6 and 10 due to O'Grady. We are mainly interested with the two families of Beauville, which consist of Hilbert schemes of points (or Douady space in the analytic case) of a K3 surface and its deformations (in the following we will call them varieties of $K3^{[n]}$-type) and Generalized Kummer varieties and their deformations (they are a subvariety of the Hilbert scheme of points of a complex torus).
We remark that projective Hyperkähler varieties are dense among any family of Hyperkähler varieties.

My research

AMONG my research interests
I am currently interested in assessing the nature of finite order automorphisms of Hyperkähler varieties of $K3^{[2]}$-type to determine which of those come from automorphisms of $K3$ surfaces and which do not. We do this by analizing the action of the automorphism on the second integral cohomology.
Some work on those automorphisms was done by Beauville [1] shortly after the discovery of higher dimensional examples of Hyperkähler varieties but yet there is no analogue of the work of Nikulin [7] or Mukai [6] in the case of K3 surfaces.
The simplest possible case is that of involutions, which has been fully analized in the case of antisymplectic involutions by Beauville and the fixed locus of symplectic involutions has been determined by Camere [2], however in an incomplete way. We refined her results by proving that a symplectic involution on a $K3^{[2]}$-type variety X has 1 fixed K3 surface and 28 isolated fixed points and moreover even the action of the involution on the lattice $H^2(X,\mathbb{Z})$ is uniquely determined: the invariant sublattice is isomorphic to $(-2)\oplus^{\perp} U^3 \oplus^{\perp} E_8(-2)$ and the anti-invariant is isomorphic to $E_8(-2)$, where U is the hyperbolic lattice and E_8 is the positive definite unimodular lattice of rank 8. The anti-invariant lattice is thus as that of an involution on a K3 surface and moreover in the moduli space of varieties of $K3^{[2]}$-type endowed with a symplectic involution the points corresponding to a variety $S^{[2]}$ with an involution $\phi^{[2]}$ where S is a K3 and ϕ a symplectic involution are dense. However not all automorphism arise in this way, there is an example (see [4]) of an automorphism of order 3 having an Abelian variety as fixed locus and such that the orthogonal to the invariant lattice has rank 18, which cannot happen for an order 3 automorphism on a K3 surface (the orthogonal to the invariant lattice has rank 12). We also proved that in order 3 all automorphisms have invariant lattices of order 5 or 11 using a formula of Donovan [3], the case of order 5 being that of [4].

References

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