Groups Acting Freely on Calabi-Yau Threefolds Embedded in a Product of del Pezzo Surfaces

Filippo Francesco Favale
University of Pavia
Department of Mathematics "F. Casorati"
Advisor: Prof. Gian Pietro Pirola

My research

AMONG my research interests there is the classification of smooth Calabi-Yau threefolds, in particular the ones with "low" *height*. A Calabi-Yau threefold is a projective complex threefold Y with trivial canonical divisor and $\mathrm{h}^{1,0}(Y) = \mathrm{h}^{2,0}(Y) = 0$. The *height* $\mathrm{h}(Y)$ of a Calabi-Yau is defined as the sum $\mathrm{h}^{1,1}(Y) + \mathrm{h}^{1,2}(Y)$. Most of the known Calabi-Yau have heigth between 30 and 502. Not a single Calabi-Yau is known with $\mathrm{h} > 502$ whereas a few have height less than 30. Among those, many of them are obtained by taking the quotient of a Calabi-Yau with greater height by a finite group acting freely. Several recent works in this direction are, for example, [1], [2] and [3].

In [4], a work in collaboration with Gilberto Bini, from University of Milan, I have investigated the quotients of Calabi-Yau manifolds Y embedded in a Fano variety X which is a product of two del Pezzo surfaces with respect to a group $G \leq \mathrm{Stab}_{\mathrm{Aut}(X)}(Y)$ that acts freely on Y. A pair (Y,G) is said to be admissible if it satisfies these requests. We say that Y/G is a Calabi-Yau of minimal height if the height of Y/G is minimal among all the quotients where (Y, G) is an ammissible pair. In particular it can be interesting to determine what are the values of $\min \mathrm{h}(Y/G)$ where $Y \subseteq X$ and (Y, G) is an ammissible pair. To obtain this number we first observe that the order of G has to be as big as possible (this follows from the choices made on X). We estimate the maximum order $m(Y)$ of G such that (Y, G) is an ammissible pair with

$$M := \gcd(\chi(Y)/2, \chi(-K_X|_Y)).$$

This estimate is obtained by checking some numerical conditions and depends only on X whereas $m(Y)$ depends on Y and on the particular embedding of Y in X. There are cases for which $M = m$ (for example if $S_1 = S_2$ del Pezzo surfaces of degree 6 one has $M = m = 12$) and cases for which $M \neq m$ ($M = 16$ and $m = 1$ for $X = S_1 \times S_2$ with $S_1 = S_2$ del Pezzo surfaces of degree 8).

We obtain several quotients with minimal heigth and, among those, there are three pairs $(h^{1,1}, h^{1,2})$ of Hodge numbers never observed before and with low height.

Current research

AT the moment I am interested in studying when $\mathrm{Aut}(Y)$ is an epimorphic image of $\mathrm{Stab}_{\mathrm{Aut}(X)}(Y)$ where Y is a Calabi-Yau and X is a product of two del Pezzo surfaces. This is interesting because, if this is the case, the results about the minimality will be independent on the particular embedding chosen.

References

[1] Volker Braun, *The 24-cell and calabi-yau threefolds with hodge numbers (1,1)*, (2011).

[2] Volker Braun, Philip Candelas, and Rhys Davies, *A three-generation Calabi-Yau manifold with small Hodge numbers*, Fortschr. Phys. **58** (2010), no. 4-5, 467–502. MR 2662013

[3] Rhys Davies, *The expanding zoo of calabi-yau threefolds*, (2011).

[4] Filippo Francesco Favale and Gilberto Bini, *Groups acting freely on calabi-yau threefolds embedded in a product of del pezzo surfaces*, (2011).