Around the tropical vertex

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The tropical vertex group

THE tropical vertex group of Kontsevich and Soibelman is generated by certain formal symplectomorphisms of the 2-dimensional algebraic torus $\mathbb{C}^* \times \mathbb{C}^*$, modelled on

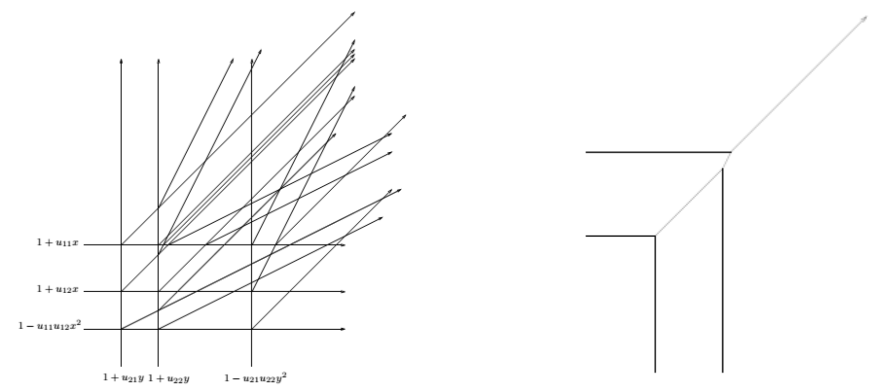
$$x \mapsto x; \quad y \mapsto y(1+tx)^{\ell_2}$$

and

$$x \mapsto x(1+ty)^{-\ell_1}; \quad y \mapsto y.$$

The group plays a role in many problems in algebraic geometry (e. g. Donaldson-Thomas theory) and mathematical physics (e. g. BPS spectra of supersymmetric gauge theories). The group itself can be understood in very different ways. There are approaches involving the enumeration of rational curves (Gross, Pandharipande and Siebert (2009)), respectively the topology of moduli spaces of representations (Reineke (2009); Gross and Pandharipande (2010)).

Figure (1(b)) displays how rational tropical curves in the plane emerge when attempting to compute the commutator of the two formal symplectomorphims described above using the method of scattering diagrams (due to Gross, Pandharipande and Siebert, [1]). The emergent tropical counts combine to yield interesting Gromov-Witten invariants.



(a) (b)

Figure 1: *Examples. (1(a)) Saturated scattering diagram to second order for the scattering diagram associated to functions $f_1 = 1 + t_1 x$ and $f_2 = 1 + t_2 y$. (1(b)) Tropical curve associated to a scattering diagram.*

These different interpretations lead to nice correspondences. For example, Reineke, Stoppa and Weist (2012) determine the dual of the remarkable Manschot-Pioline-Sen formula for quiver representations as a degeneration formula in Gromov-Witten theory. Conjecturally, these dualities fit into the general framework of Donaldson-Thomas/Gromov-Witten correspondences (Bousseau-Thomas, in progress (2014)). The refined tropical counts of Block and Göttsche emerge naturally when attempting to "quantize" this picture (Stoppa-Filippini (2012)).

The method of scattering diagrams plays a crucial role in recent work of Filippini, Garcia-Fernandez and Stoppa ([3], 2014) in which a nontrivial deformation of the Bridgeland-Toledano irregular meromorphic connections is constructed and related to tropical-Gromov/Witten invariants. This deformation is very much inspired by ideas from physics (four-dimensional tt^*-connections).

My research

IDEAS from physics (especially in the work of Gaiotto, Moore and Neitzke, [2]) also strongly suggest that an extension of the tropical vertex group should be considered, given roughly by a semidirect product of a group of matrix gauge transformations and a group of formal Poisson morphisms, acting on matrix-valued functions on the algebraic torus $\mathbb{C}^* \times \mathbb{C}^*$. This extension is very natural from the physics perspective (corresponding to the coupling of a $2d$ and a $4d$ theory), and understanding it mathematically should produce interesting new ideas.

I would like to understand this extension both from the point of view of scattering diagrams/tropical geometry, and from that of isomonodromy for irregular connections.

It is possible that new interesting tropical invariants will be encoded in an extension of the method of scattering diagrams to the matrix-valued group (much as it happened in the case of q-deformations and Block-Göttsche invariants).

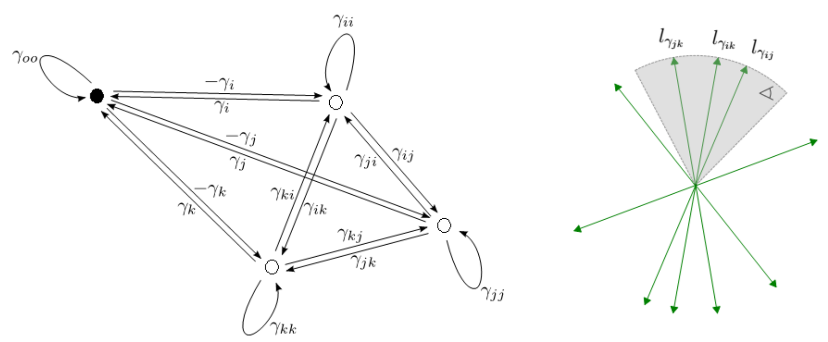


Figure 2: *Partial depiction of a pointed groupoid on the left. On the right, the definition of $A(\ast) := \mathcal{S}^{\mu_{ij}}_{\gamma_{ij}} \mathcal{S}^{\mu_{ik}}_{\gamma_{ik}} \mathcal{S}^{\mu_{jk}}_{\gamma_{jk}}$ as a product of $\mathcal{S}-$factors associated to the bps rays l_{ij}*

References

[1] M. Gross, R. Pandharipande, B. Siebert, *The Tropical Vertex*, Duke Math. J. 153, no. 2, 297-362 (2010).

[2] D. Gaiotto, G. W. Moore, A. Neitzke, *Wall-Crossing in Coupled $2d$-$4d$ Systems*, Journal of High Energy Physics, 2012.

[3] S. A. Filippini, M. Garcia-Fernandez, J. Stoppa, *Stability data, irregular connections and tropical curves*, arXiv:1403:7404v1