The Noncommutative McKay Correspondence

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Background

THE Mckay correspondence, in its original form, relates the representation theory of two dimensional canonical singularities to their minimal resolutions. More precisely, let R be a 2-dimensional complete local ring with canonical singularities. Then $R = k[[V]]^G$ where $G \in SL(V)$. The following are isomorphic

- the McKay quiver of G
- the Auslander-Reiten quiver of R
- the intersection graph of the exceptional curves on the minimal resolution $\sigma : \tilde{Z} \to \mathrm{Spec}(R)$

The noncommutative version

IN the noncommutative version, we consider the action of a finite dimensional Hopf algebra on an Artin-Schelter regular algebra of dimension 2. These are graded noncommutative algebras which resemble commutative polynomial rings. In dimension 2, their classification is not difficult, such an algebra is isomorphic to

$$\begin{gathered} k_q[u,v] := k\langle u,v\rangle/(vu - quv) \\ \text{or} \\ k_J[u,v] := k\langle u,v\rangle/(uv - vu + u^2) \end{gathered} \tag{1}$$

The classification of graded actions of finite dimension semisimple Hopf algebra actions on (1) which are

- inner-faithful (the Hopf version of faithful action)
- with trivial determinant (the Hopf version of $G \subseteq SL(V)$)

is given below [1]. Let $V = ku \oplus kv$, assume A is noncommutative.

A	H	Action
$k_{-1}[u,v]$	$kC_n, n \geqslant 2$	diagonal
	kC_2	flip
	kD_{2n}	V faithful simple
	$(kD_{2n})^\circ$	
	$\mathcal{D}(\tilde{\Gamma}), \tilde{\Gamma} \subset SL(V)$	V innerfaithful simple
$k_q[u,v], q \neq \pm 1$	$kC_n \quad n \geqslant 2$	diagonal
$k_J[u,v]$	kC_2	diagonal

As we can see from the above, the most interesting actions occur when $A = k_{-1}[u,v]$. We will focus on the case where $\mathcal{D} := \mathcal{D}(\tilde{\Gamma})$ is a cocycle deformation of the group $\tilde{\Gamma}$. The McKay quiver of $\mathcal{D}(\tilde{\Gamma})$ with respect to any inner-faithful simple module is isomorphic to the McKay quiver of $\tilde{\Gamma}$ with respect to any faithful simple module. Moreover, the invariant ring $k_{-1}[u,v]^{\mathcal{D}}$ is a commutative canonical surface singularity of the same type as $\tilde{\Gamma}$.
This (almost) proves a graded version of the McKay correspondance in the case where $H = \mathcal{D}$; there following are isomorphic

- the McKay quiver of H
- the (graded, left) Auslander-Reiten quiver of A^H
- (in the case $H = \mathcal{D}$) the intersection graph of the exceptional curves on the minimal resolution $\sigma : \tilde{Z} \to \mathrm{Spec}(A^H)$

References

[1] K. Chan, E. Kirkman, Chelsea Walton, and James Zhang, *Quantum binary polyhedral groups and their actions on quantum planes*, J. Reine Angew. Math. **To appear**.