GAeL XXII

Schedule

	Monday	Tuesday	Wednesday	Thursday	Friday
9:00-9:30	**Registration**				
9:30-10:00	Mellit	Esnault	Mellit	O'Grady	Esnault
10:00-10:30					
10:40-11:10	Panizzut	Grandi	**GAeL XXIII**	Veniani	Kapfer
11:10-11:30	**Coffee Break**				
11:30-12:00	Esnault	O'Grady	O'Grady	Mellit	O'Grady
12:00-12:30					
12:30-14:30	**Lunch and free discussion time**				
14:30-15:00	Comparin	Mellit	**Free Afternoon**	Esnault	Zdanowicz
15:00-15:30	Thibault				Turchetti
15:30-16:00	**Coffee Break**			**Coffee Break**	
16:00-16:30	Angelini	Kaur		Dawes	Bultot
16:40-17:10	Ricolfi	Cauwbergs		Kesteloot	
18:00-20:00	**Poster session**				
20:00 onwards		**Social dinner**			

Program of the courses

- Anton Mellit *Counting stable objects*

  Working in two settings: vector bundles on algebraic curves, representations of quivers, I am going to introduce the notions of stable/semistable objects and their moduli, and explain Harder-Narasimhan theory. Later lectures will be devoted to counting semistable objects over a finite field, the main result will be the formula of Zagier, which gives the numbers of semistable objects in terms of the numbers of all objects.

- Hélène Esnault *Miscellaneous on rational points and fundamental groups*

  The guiding conjecture of the lectures is the C_1 conjecture, due to Lang-Manin-Kollár, predicting that a rationally connected variety over a C_1 field has a rational point (with some separability assumption in positive characteristic). The essential case where it is still unknown is over the maximal unramified extension of $\mathbb{Q}_p$. We will discuss various aspects of the problem, and related questions.

- Kieran Gregory O'Grady *The geometry of hyperkähler varieties*

  Definition and examples of compact hyperkähler manifolds; General results on compact hyperkähler manifolds: deformations, period map, B-B quadratic form, Kähler cone, Torelli theorems. Moduli of polarized hyperkähler varieties. Explicit examples of locally complete families of polarized hyperkähler varieties. Chow ring of hyperkähler varieties.

Abstracts of junior talks

- Elena Angelini

  Logarithmic bundles of multi-degree arrangements in $\mathbb{P}^n$

  Let $D = D_1, ..., D_l$ be a multi-degree arrangement of smooth hypersurfaces with normal crossings on the complex projective space $\mathbb{P}^n$ and let $\Omega^1_{\mathbb{P}^n}(\log D)$ be the logarithmic bundle attached to it. We show that $\Omega^1_{\mathbb{P}^n}(\log D)$ admits a resolution of length 1 which explicitly depends on the degrees and on the equations of $D_1, ..., D_l$. Then we prove a Torelli type theorem when D is made of a "sufficiently" large number of hypersurfaces: indeed, we recover the components of D as unstable smooth hypersurfaces of $\Omega^1_{\mathbb{P}^n}(\log D)$. Moreover we analyze the case of one quadric and a pair of quadrics, which yield examples of non-Torelli arrangements. In particular, through a duality argument, we prove that two pairs of quadrics have isomorphic logarithmic bundles if and only if they have the same tangent hyperplanes. Finally we give a description of the conic case and of some line-conic cases.

- Emmanuel Bultot

  The motivic Serre invariant

  Let R be a complete discrete valuation ring with fraction field K and algebraically closed residue field k. The motivic Serre invariant of a proper smooth K-variety X gives a measure of the set of rational points of X.

  Using this invariant, Esnault and Nicaise showed that any finite $\mathcal{L}$-group (with $\mathcal{L}$ invertible in k) acting on the affine space $\mathbb{A}^n_K$ admits a K-rational fixed point when K has characteristic 0, answering a question raised by Serre for this class of fields.

  This result requires to extend the invariant to non-proper K-varieties, what has been done by Nicaise using the weak factorization theorem. Unfortunately, this theorem has no counterpart in positive characteristic.

  We present a new construction of the motivic Serre invariant of a smooth K-variety avoiding the use of the weak factorization theorem. Assuming resolution of singularities, we are then able to extend the definition to arbitrary K-varieties.

- Thomas Cauwbergs

  Distinguishing elements in Grothendieck rings

  The Grothendieck ring of complex varieties is used to generalize Igusa's p-adic zeta functions and others to a motivic zeta function. While this ring has a simple definition, actually proving two elements are distinct can be challenging. I will discuss an example stemming from motivic zeta functions where two varieties are topologically the same but their classes are not. For this I use an argument of Ekedahl to prove that equality of classes implies that the Jacobians are isogenous.

- Paola Comparin

  Mirror symmetry for K3 surfaces with non-symplectic automorphism

  The Berglund-Hübsch-Chiodo-Ruan (BHCR) construction provides a method to obtain pairs of Calabi-Yau manifolds that are defined as hypersurfaces in weighted projective spaces and that are mirror in the classical sense. When one has a Calabi-Yau manifold of dimension two, i.e., a K3 surface, another type of mirror symmetry was introduced by Dolgachev: the mirror symmetry for lattice polarized K3 surfaces. In a joint work with C. Lyons, N. Priddis and R. Suggs, we consider K3 surfaces with a non-symplectic automorphism that are defined by an equation of the form $x^p + f(y, z, w)$ in a weighted projective space. We prove that for this class of surfaces the two forms of mirror symmetry agree. I will describe these surfaces and the mirror constructions and I will show how to prove the result.

- Matthew Dawes

  Irreducible symplectic manifolds, orthogonal modular varieties, and modular forms

  For each of the known families of Irreducible symplectic manifold with polarisation, one can construct a Hodge theoretic moduli space. Such moduli spaces exist as a quotient of a Hermitian symmetric domain of type IV by an arithmetic group. It is natural to try to determine the Kodaira dimension of each moduli space. I shall outline some of the general theory and discuss how, in many cases, an answer can be found by examining special modular forms.

- Riccardo Grandi

  Wall crossing for Bradlow-Higgs triples

  The aim of the talk is to introduce Bradlow-Higgs triples, the range of stability conditions that one can impose on them and what effects these produce on the moduli problem.

  First, I will try to briefly review what is known for the moduli problem of stable Higgs bundles and for the moduli problem of Bradlow pairs.

  Second, I will present how these two problems can interact to form an entirely new problem and try to point out the reasons for which it is worth studying it.

- Simon Kapfer

  Integer cohomology of compact Hyperkähler manifolds

  Several properties of a compact Hyperkähler manifold can be read off from its cohomology lattice in degree 2. However, for cohomology in other degrees almost nothing is known. An exception is the case of the Hilbert schemes of points on a K3 surface. We will discuss some results comparing the algebra generated by degree-2-classes with the rest of even cohomology.

- Inder Kaur

  Rational points of moduli spaces of vector bundles

  Let K be a complete, discrete valued field with algebraically closed residue field and X_K be a smooth, projective curve defined over K. Consider the moduli space of vector bundles with rank and degree coprime on the curve X_K. It is well known that this moduli space is a Fano variety. However, it is an open problem, whether this variety always has a K-rational point. In this talk, I will show that when the curve X_K has good reduction, this is indeed the case.

- Lore Kesteloot

  The specialization index

  In arithmetic geometry, one of the central questions is to find geometric conditions on a variety that will imply the existence of a rational point. Let X be a variety over the quotient field K of a henselian discrete valuation ring. If X has a rational point, the index $\iota(X)$ is equal to one, but the converse is not necessarily true.

  In this talk, we introduce a new invariant $\iota_{\mathrm{sp}}(X)$, *the specialization index*, such that $\iota(X) \leq \iota_{\mathrm{sp}}(X)$.

  If K has equal characteristic zero and the residue field is algebraically closed, we provide an explicit formula for the specialization index in terms of an *sncd*-model. We use this to construct examples of curves C with $\iota(C) < \iota_{\mathrm{sp}}(C)$.

  If moreover X has trivial coherent cohomology, Colliot-Thélène and Voisin, Esnault and Wittenberg, and Nicaise have independently proven that $\iota(X) = 1$. We generalize this result by also proving that $\iota_{\mathrm{sp}}(X) = 1$.

- Marta Panizzut

  Gonality of Graphs

  In this talk we define the notion of linear systems of divisors on a finite graph, as introduced by Baker and Norine, and state the main results proved by these authors in their groundbreaking paper. These results are combinatorial equivalents of fundamental theorems in the classical theory on algebraic curves, such as the Riemann-Roch Theorem. We also explain the interplay between the notions of divisor on graphs and on curves given by Baker's Specialization Lemma. Let X be a smooth curve over the field of fractions of a complete discrete valuation ring and let $\mathfrak{X}$ be a strongly semistable regular model of X. It is possible to specialize a divisor on the curve to a divisor on the dual graph of the special fiber of $\mathfrak{X}$; through this process the rank of the divisor can only increase. We conclude focusing on results regarding the gonality of graphs obtained from the complete graph on d vertices by omitting at most $d-2$ edges and comparing it with the gonality of plane curves of degree d.

- Andrea Tobia Ricolfi

  Motivic Donaldson-Thomas invariants

  Donaldson-Thomas theory is a refined counting theory for curves on Calabi-Yau threefolds, where curves are viewed as ideal sheaves. The Donaldson-Thomas invariant of a moduli space of such curves is their virtual count, a genuine integer. Donaldson-Thomas invariants can be computed as certain weighted (virtual) Euler characteristics. Sometimes, they almost are Euler characteristics. And this is where motives come into the game: the *motive* of a variety, i.e. its class in the Grothendieck ring of varieties, can be thought of as the universal Euler characteristic of the variety. A virtual motive of a scheme is a class whose Euler characteristic coincides with the virtual Euler characteristic of that scheme: it is a "lift" of the virtual Euler characteristic in the Grothendieck ring, or in a suitably augmented localization of it. We will describe the problem of constructing canonical virtual motives for the moduli spaces of interest in Donaldson-Thomas theory.

- Louis-Philippe Thibault

  (Higher) Preprojective algebras and Gorenstein quotient singularities

  For a finite subgroup G of $\mathrm{SL}(2, \mathbb{C})$, the well-known McKay correspondence establishes a connection between the minimal resolution of the related Kleinian singularity and the representation theory of G, which is encoded in its McKay quiver. This connection can also be expressed in terms of preprojective algebras: The skew-group algebra $k[x, y] * G$ is Morita equivalent to the preprojective algebra of the extended Dynkin quiver associated to G, which is the McKay quiver of G modulo some relations. To generalize this picture in higher dimensions, one has to introduce the notion of higher preprojective algebras, which can be characterized as Calabi-Yau algebras of Gorenstein parameter 1. In this talk, we will explain these ideas and see how preprojective algebras can relate to birational models of Gorenstein quotient singularities.

- Daniele Turchetti

  Lifting Galois covers to characteristic zero with non-Archimedean analytic geometry

  Let k be an algebraically closed field of characteristic $p > 0$ and R a complete discrete valuation ring of characteristic zero, with residue field k. In this talk we deal with the so called lifting problems, that ask what algebraic objects defined over k can be obtained by reduction of algebraic objects over R. More specifically we briefly review results and conjectures about lifting algebraic varieties and finite Galois covers of smooth projective curves. We then focus on this second problem and explain how we can find necessary and sufficient conditions to the lifting of G-Galois covers when G is cyclic with the use of analytic methods. Finally we discuss how these methods provide new perspectives in arithmetic geometry.

- Davide Cesare Veniani

  Lines on quartic

  Counting lines on hypersurfaces has been one of the challenges of algebraic geometers since the Italian school. The fact that on every smooth cubic in the projective space there are exactly 27 lines, combined in a highly symmetrical way, is considered one of the 'gems' of our discipline. Beniamino Segre stated correctly in 1942 that the maximum number of lines on a smooth quartic over an algebraically closed field of characteristic zero is 64, but his proof was wrong. It has recently been corrected by Rams and Schütt using techniques unknown to Segre, such as the theory of elliptic fibrations. The talk will analyse the main ideas involved in the correct proof, also considering the case of positive characteristic, and will present some recent work on mildly singular quartics.

- Maciej Zdanowicz

  On $W_2(k)$-liftings and Frobenius splittings

  In their seminal paper, Deligne and Illusie proved the degeneration of Hodge-de Rham spectral sequence for smooth varieties X/k in purely algebraic manner, using additional assumption of $W_2(k)$-liftability. For smooth varieties this property is governed by a cohomological obstruction, which is known to vanish in case of Frobenius split varieties. We shall shortly elaborate on the singular case, solved by Bhatt by means of the cotangent complex. We will then show an explicit functorial construction of a $W_2(k)$-lifting of a Frobenius split variety. At the end we plan to talk about $W_2(k)$-liftings of a singular variety and its Frobenius morphism.