Geometric Analysis and Nonlinear PDE
Będlewo, June 3–10, 2007

1 Main talks

LUCA CAPOGNA, University of Arkansas
Mean curvature flow in sub-Riemannian setting

I will discuss the notion of (sub-Riemannian) horizontal mean curvature, the analogue of the Riemannian mean curvature. Most of the talk will be focused on analytical aspects of the mean curvature operator in connection with the mean curvature flow and regularity of minimal surfaces.

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BERNARD DACOROGNA, Federal Technical University of Lausanne
An explicit solution of an implicit PDE: vectorial pyramids and origamis

We construct an explicit solution to the problem

$$\begin{cases} \nabla u\,(x) \in O\,(2) & \text{a.e. } x \in \Omega \\ u\,(x) = \xi x & \text{on } \partial\Omega \end{cases}$$

where Ω is the unit square, $u : \Omega \subset \mathbb{R}^2 \to \mathbb{R}^2$ is the map that we will explicitly construct and $\xi \in \mathbb{R}^{2\times 2}$ is given with appropriate compatibility condition (for example $\xi = 0$).

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FRANK DUZAAR, University of Erlangen–Nuremberg
Non-linear Parabolic Systems with Polynomial Growth and Regularity

In my talk I will address certain regularity features of non-linear parabolic systems with polynomial growth such as partial regularity, dimension reduction of the singular sets of weak solutions and certain Calderón-Zygmund type estimates for such non-linear parabolic systems. All results which will be presented are obtained in collaboration with G. R. Mingione from the University of Parma.

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DANIEL FARACO, Universidad Autónoma de Madrid
Quasiconformal mapping and Tartar's conjecture

A classical question in the vectorial Calculus of Variations is the investigation of the compactness and stability properties of exact and approximate solutions to the differential inclusion

$$Df(x) \in K \quad \text{a.e.}$$

where $f\colon \Omega \to \mathbb{R}^2$ and $K \subset \mathcal{M}^{2\times 2}$.

The systematic investigation of these questions can be traced back at least to the work of Murat and Tartar on oscillating solutions to nonlinear PDE which led to the theory of compensated compactness.

An important example arises in the variational model for phase transitions developed by Ball and James.

Compactness means that we want to give conditions on K which guarantee that a sequence of approximate solutions has a subsequence which converges strongly. **Stability** means that we want to characterize in general the possible weak limits of exact or approximate solutions. Tartar conjectured that a sufficient condition for compactness was the lack of rank one connections, $\det(A-B) \neq 0$ for $A, B \in K$. However he himself discovered a set of four matrices without rank-one connections for which compactness does not hold. These type of sets were called T_4 configurations. It was clear that a necessary condition for compactness was the absence of rank-one connections and T_4 configurations. Work of Székelyhidi showed that to prove that this necessary condition was also sufficient for compactness, new constraints on the possible range of gradient mappings and gradient Young measures were needed. We will discuss how to use quasiconformal mappings to obtain these constraints and hence to prove that Tartar conjecture holds in 2 by 2 matrices. The quasiconformal ideas are based on recent work of Bojarski, D'Onofrio, Iwaniec and Sbordone.

This is a joint work with Lászlö Székelyhidi from ETH (Zurich)

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NICOLA GAROFALO, Purdue University

The stability of embedded minimal surfaces in the Heisenberg group and the Bernstein problem

In recent joint work with Danielli, Nhieu and Pauls we have solved the Bernstein problem for minimal graphs over some plane in the Heisenberg group. In this talk I will discuss a generalization of this result to the topologically more complex embedded minimal surfaces. This constitutes the sub-Riemannian counterpart of the famous result of Fischer-Colbrie and Schoen, and do Carmo and Peng. These authors proved that: Every stable complete minimal surface in $\mathbb{R}^3$ must be a plane. Our main result states that:

> *In the first Heisenberg group the only stable C^2 complete embedded noncharacteristic minimal surfaces without boundary are the vertical planes.*

This is joint work with Danielli, Nhieu and Pauls.

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ANDREAS GASTEL, University of Erlangen–Nuremberg

Poly-energies and polyharmonic maps

We consider energy functionals involving higher order derivatives of mappings between Riemannian manifolds. We discuss existence of critical points (*polyharmonic maps*) by the heat flow method in the critical dimension, as well as other aspects of minimizing such functionals.

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TADEUSZ IWANIEC, University of Syracuse

Invitation to n-harmonic hyperelasticity

We study deformations $h\colon X \longrightarrow Y$ between bounded domains in n-dimensional Euclidean space. The general law of hyperelasticity requires that there exists an energy integral so that the deformations of interest to us are those with smallest energy. We assume here that the integrand is conformally coerced and polyconvex. Some additional regularity conditions are also imposed. Under these conditions we establish the existence and global invertibility of the minimizers. The key tools in obtaining an extremal deformation are the *free Lagrangians*. Finding suitable free Lagrangians and using them for a specific energy functional is truly a work of art. We have done it here for the total harmonic energy and a pair of annuli in the plane.

This is joint work with Jani Onninen.

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PEKKA KOSKELA, University of Jyväskylä

Generalized dimension distortion under mappings of exponentially integrable distortion

A quasiconformal image of a set of Hausdorff dimension strictly less than the dimension n of the underlying Euclidean space is also of dimension strictly less than n. If we replace quasiconformality by a weaker requirement on the distortion function, say by local exponential integrability of the distortion, the conclusion need not hold anymore. We discuss the optimal conclusion in this setting. Analogously to the quasiconformal setting, sharp results appear to require optimal regularity results for the mapping in question.

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JAN KRISTENSEN, University of Oxford

On the regularity of strong local minimizers

We discuss the regularity problem for strong local minimizers of quasiconvex integrals. It is well-known that even under favourable hypotheses such minimizers will only be partially regular, that is, of class C^1 outside some relatively closed subset (singular set) of their domain. Under the assumption of Lipschitz continuity it is shown that the singular set is uniformly porous. The talk is based on joint work with G.R. Mingione (Parma).

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OLLI MARTIO, University of Helsinki

Reflection principle

Let Ω be an open set in the upper half space H^+ of $\mathbf{R}^n$, where $n \geq 1$, such that there is a set $C \subset \partial\Omega \cap \partial H^+$ open in ∂H^+. The reflection principle for the solutions of quasilinear degenerate elliptic equations $\nabla \cdot A(x, \nabla u) = 0$, $p > 1$ and $A(x, h) \cdot h \approx |h|^p$, consists of two steps. The first step is to show that the solution $u \in C(\Omega \cup C) \cap W^{1,p}_{loc}(\Omega)$ with $u = 0$ on C is in $W^{1,p}(\Omega \cap U)$ for some neighborhood U of C and the second step is to show that the reflected function is a solution of the reflected equation in $\Omega \cup C \cup P(\Omega)$ where P is the reflection in ∂H^+. If the equation is the p–harmonic equation $\nabla \cdot (|\nabla u|^{p-2}\nabla u) = 0$, then the equation remains invariant in the reflection process and the reflection principle is the same as the classical reflection principle for harmonic functions.

The talk deals with the reflection principle for quasiminimizers. These are functions $u \in W^{1,p}_{loc}(\Omega)$ which for some $K \geq 1$ satisfy $\int_{\Omega_1} |\nabla u|^p \, dx \leq K \int_{\Omega_1} |\nabla v|^p \, dx$ for all open sets $\Omega_1 \subset\subset \Omega$ and for all functions v such that $u - v \in W^{1,p}_0(\Omega_1)$. Quasiminimizers include all aforementioned solutions. It is shown that the reflection principle holds for quasiminimizers provided that $K < 2$ for $n \geq 2$ and all $K \geq 1$ for $n = 1$. A one dimensional example shows that the constant $K > 1$ is not preserved in the reflection process.

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GIUSEPPE ROSARIO MINGIONE, University of Parma

Calderón-Zygmund estimates around limit cases

Usually, limit cases for a certain theory are those ones beyond which the related results do not any longer hold true; often it even does not make sense to speak about a certain theory when beyond such cases. On the other hand, something I learned from Tadeusz Iwaniec is that our idea of limit cases is strictly related to the functional setting we are dealing with. Therefore, enlarging the function space we are working with allows to view “limit cases” as just special occurences in a wider panorama (*“the most useful function space is the one which still has to be introduced...”*, as Tadeusz very often remarks).

I am not going to introduce new function spaces, but rather to see how considering larger spaces allows one to walk on the borderline of the traditional Calderón-Zygmund theory, observing that in some sense it still works “beyond" those cases traditionally considered as borderline ones; the results are in [5,6]. More specifically, I will present new regularity results for non-linear elliptic problems with measure data of the type

$$-\mathrm{div}\ a(Du) = \mu \quad \text{in } \Omega \qquad \text{and} \qquad u = 0 \quad \text{on } \partial\Omega\ , \tag{1}$$

with $\Omega \subset \mathbb{R}^n$ being a bounded domain, μ being a Radon measure with finite total mass, and $n \geq 2$, assuming, to consider a model case, that

$$\langle Da(Du)\lambda, \lambda\rangle \approx |\lambda|^2 \qquad \text{(ellipticity)}\ .$$

For problem (1) what is basically known is the existence of (suitably defined) weak solution u with

$$Du \in L^s \qquad \text{for every } s < n/(n-1)\ . \tag{2}$$

Such a result is optimal, and this is more or less all is known about the regularity of solutions to elliptic problems with measure data of the type (1), see [1,2,3,4,7] for this and related issues. In particular, while (2) gives a measure of the size of the gradient, nothing is known about the oscillations of Du. The idea is now to show that (2) is actually a consequence of a deeper regularity property of Du, namely its fractional differentiability. To fix the ideas, let us recall that the standard Calderón-Zygmund theory for solution to $\triangle u = f$ asserts that $Du \in W^{1,m}$ if $f \in L^m$, as soon as $m > 1$; via embedding this in turn implies $Du \in L^{n/(n-1)}$, that is the limit case of (2). Clearly such a result does not hold when $m = 1$, that is $Du \notin W^{1,1}$. This is why $W^{1,1}$ is the limit case of Calderón-Zygmund theory. The point now is: not everything is lost in the borderline case (in fact just an ε is lost!!). Indeed, although for a solution to (1) $Du \notin W^{1,1}$, we rather have

$$Du \in W^{1-\varepsilon,1} \qquad \text{for every } \varepsilon > 0\ , \tag{3}$$

and second derivatives of solutions “almost exist”. In a certain sense, by means of fractional Sobolev spaces Calderón-Zygmund theory can be “continued” beyond the limit case $W^{1,1}$. In particular (3) implies (2) via Sobolev embedding. Note that the significance of (3) also lies in that the problem (1) is non-linear, and no integral representation formulae can be used.

This is the type of results I am going to talk about.

References:

[1] Boccardo, L., Gallouët, T.: Nonlinear elliptic and parabolic equations involving measure data. *J. Funct. Anal.* 87 (1989), 149–169.

[2] Dal Maso, G., Murat, F., Orsina, L., Prignet, A.: Renormalized solutions of elliptic equations with general measure data. *Ann. Scuola Norm. Sup. Pisa Cl. Sci. (4)* 28 (1999), 741–808.

[3] Greco, L., Iwaniec, T., Sbordone, C.: Inverting the p-harmonic operator. *manuscripta math.* 92 (1997), 249–258.

[4] Heinonen, J., Kilpeläinen, T., Martio, O.: *Nonlinear potential theory of degenerate elliptic equations.* Oxford Mathematical Monographs., New York, 1993.

[5] Mingione, G.: Calderón-Zygmund estimates for measure data problems. *C. R. Acad. Sci. Paris Sér. I Math.* 344 (2007), 437-442.

[6] Mingione, G.: The Calderón-Zygmund theory for elliptic problems with measure data. *Preprint 2006, available at:* http://www.unipr.it/˜mingiu36

[7] Trudinger, N.S., Wang, X.J.: On the weak continuity of elliptic operators and applications to potential theory. *Amer. J. Math.* 124 (2002), 369–410.

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FRANÇOIS MURAT, Laboratoire Jacques-Louis Lions, Université Paris VI

Existence and a priori estimate for elliptic problems with subquadratic gradient dependent terms

In this lecture I will consider the nonlinear elliptic model problem

$$u \in H^1_0(\Omega), \qquad -\mathrm{div}\, A(x)Du + \alpha_0 u = \gamma |Du|^q + f(x) \ \text{ in } \mathcal{D}'(\Omega),$$

with A a coercive matrix with bounded coefficients, $\alpha_0 \geq 0$, $0 \leq q \leq 2$ and $f \in L^m(\Omega)$ for some suitable m. This is a model problem, and there are many possible variants of it.

In the case where $0 \leq q < 1$, existence is classical for $f \in H^{-1}(\Omega)$. When γ is large, the case where $q = 1$ and $f \in H^{-1}(\Omega)$ is difficult but has been solved by G. Bottaro and M.E. Marina in 1973. On the other hand, the case $q = 2$ has been treated by many authors, including in particular in a series of papers by L. Boccardo, J.-P. Puel and myself. In a more recent paper, V. Ferone and myself proved the existence of a solution u wich further satisfies $e^{\gamma u} - 1 \in H^1_0(\Omega)$, and an a priori estimate for such solutions, when $f \in L^{\frac{N}{2}}(\Omega)$.

In this lecture I will mainly report about recent joint work with Nathalie Grenon and Alessio Porretta, the announcement of which has been published in C. R. Acad. Sci. Paris, Série I, 342, (2006), pp. 23-28. When $1 + \frac{2}{N} \leq q < 2$ and $f \in L^m(\Omega)$ with $m = \frac{N(q-1)}{q}$ (we also solved the case where $1 \leq q < 1 + \frac{2}{N}$, but I will not discuss it since it uses the notion of renormalized solution), and when either $\alpha_0 > 0$ or f is sufficiently small in $L^m(\Omega)$, we prove the existence of a solution u which enjoys the further regularity $|u|^\sigma \in H^1_0(\Omega)$ with $\sigma = \frac{(N-2)(q-1)}{2(2-q)}$, as well as an a priori estimate for any solution which enjoys this further regularity. One of the main interests of our result lies in the priori estimate, the proof of which is non standard.

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PABLO PEDREGAL, University of Castilla–La Mancha

On a variational reformulation of micromagnetics. Results in 2D

The model of micromagnetics seeks the equilibrium configuration of the director field (magnetization) as a result of minimizing a certain energy functional where various terms compete. When one of these terms, the exchange energy, is neglected, the functional typically becomes nonconvex so that the understanding of existence/nonexistence of minimizers becomes a relevant issue. Another one of these terms is the external interaction energy coming from an applied constant electric field H. We would like to explore the existence/nonexistence issue and how it depends on this field H in several situations of interest depending on a third contribution to the energy, the anisotropy term:

- the soft case, where there is no anisotropy;
- the uniaxial case, where anisotropy favors a unique direction;
- the cubic case, where anisotropy favors several independent directions.

Our approach is based on a certain variational reformulation to avoid the non-local character of the fourth term of the energy, the magnetostatic energy. It allows for some explicit computations, and the use of some fine existence results for nonconvex variational problems. We will describe specific results in the 2D situation.

This is joint work with Baisheng Yan.

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TRISTAN RIVIÈRE, ETH Zurich
Analytic aspects of Willmore surfaces

We present a new formulation to the Euler-Lagrange equation of the Willmore functional for immersed surfaces in $\mathbb{R}^m$. This new formulation of Willmore equation appears to be of divergence form, moreover, the non-linearities are made of jacobians. Additionally to that, if $\vec{H}$ denotes the mean curvature vector of the surface, this new form writes $\mathcal{L}\vec{H} = 0$ where $\mathcal{L}$ is a well defined locally invertible elliptic self-adjoint operator.

These 3 facts have numerous consequences in the analysis of Willmore surfaces.

One first consequence is that the long standing open problem to give a meaning to the Willmore Euler-Lagrange equation for immersions having only L^2 bounded second fundamental form is now solved. We then establish the regularity of weak Willmore immersion with L^2 bounded second fundamental form. The proof of this result is based on the discovery of conservation laws for Willmore immersions and grad-curl structures which are preserved under weak convergences. We establish then a weak compactness result for Willmore surfaces of energy less than 8π (the Li-Yau condition which ensures the embeddedness of the surface). This theorem is based on a point removability result we prove for Wilmore surfaces in $\mathbb{R}^m$. Finally, we deduce from this point removability result the strong compactness, modulo the Möbius group action, of Willmore tori below the energy level 8π in dimensions 3 and 4.

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CARLO SBORDONE, University of Naples "Federico II"
The weak limit in $W^{1,1}_{\mathbf{loc}}$ of homeomorphisms with finite distortion

This is a report on joint works with L. Greco and C. Trombetti [7] and with N. Fusco and G. Moscariello [6].

Very recently there is a growing interest in studying properties of a homeomorphism $f : \Omega \subset \mathbb{R}^n \longrightarrow f(\Omega) \subset \mathbb{R}^n$of Sobolev class $W^{1,1}_{loc}$ with non negative Jacobian, $J_f \geq 0$, whose differential $Df(x)$ vanishes on the zero set of J_f ([1], [3], [4], [5]). This is equivalent to say that there exists $K : \Omega \longrightarrow [1, \infty[$, a Borel function, such that

$$|Df(x)|^n \leq K(x)J_f(x) \quad a.e.\ \ x \in \Omega. \tag{4}$$

The smallest function $K_f \geq 1$, for which (1) holds is called the <u>distortion</u> of f and f will be referred to as a homeomorphism of finite distortion.

In [3], [4] the inverse f^{-1} of a homeomorphism $f \in W^{1,1}_{loc}(\Omega, \mathbb{R}^n)$ of finite distortion such that $|Df| \in L^{n-1,1}(\Omega)$ is seen to have finite distortion as well. We give a quantitative version of such a result:

$$K_{f^{-1}}(y) \leq c(n)K_f^{n-1}(f^{-1}(y))$$

where, for $n = 2$ we have $c(2) = 1$ hence the stronger result:

$$K_{f^{-1}}(y) = K_f(f^{-1}(y)).$$

We then show that the limit mapping f of a $W^{1,1}$-weakly convergent sequence of homeomorphisms f_j with finite distortion, which converge uniformly to f, has finite distortion, provided f satisfies the Lusin (N) condition. A suitable lower semicontinuity of the distortion follows (see [2] for previous results). However our arguments cannot rely on the weak continuity of the Jacobian determinants.

[1] K. Astala - T. Iwaniec - G. J. Martin -J. Onninen, *Extremal mappings of finite distortion*, Proc. London Math. Soc. (3) 91 (2005), 655-702.

[2] F. W. Gehring - T. Iwaniec, *The limit mapping with finite distortion*, Ann. Acad. Sci. Fenn. Math. 24 (1999), 253-264.

[3] S. Hencl - P. Koskela, *Regularity of the Inverse of a Planar Sobolev Homeomorphism*, Arch. Rational Mech. Anal. 180 (2006), 75-95.

[4] S. Hencl - P. Koskela - J. Malý, *Regularity of the Inverse of a Sobolev Homeomorphism in Space*, (2006) preprint.

[5] S. Hencl - P. Koskela - J. Onninen, *A note on extremal mappings of finite distortion*, Math. Res. Lett. 12 (2005), 231-238.

[6] N. Fusco - G. Moscariello - C. Sbordone, in preparation.

[7] L. Greco - C. Sbordone - C. Trombetti, *A note on planar homeomorphisms*, submitted (2007).

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JEAN VAN SCHAFTINGEN, Catholic University of Louvain
L^1 estimates for vector fields

In a recent work, Bourgain and Brezis have improved regularity estimates for systems with divergence free data of class L^1. I shall discuss these results, together with an elementary proof that I have given, together with some extensions and the relationship between these estimates and Hardy spaces.

2 Short talks

TOMASZ ADAMOWICZ, University of Syracuse
On p-harmonic mappings

We will discuss some recent developments in the geometry of p-harmonic transformations. These are $W^{1,p}_{loc}(\Omega, \mathbb{R}^n)$ solutions to so called p-harmonic system of equations:

$$\operatorname{div}(|Du|^{p-2}Du) = 0 \qquad u : \Omega \subset \mathbb{R}^n \to \mathbb{R}^n, \quad 1 < p < \infty.$$

The maximum principle, Harnack type inequalities and planar p-harmonic mappings will be discussed. Despite similarity of the definition to the scalar counterpart, the setting of mappings turns out to be much more difficult and challenging. Many questions remain open, we will address some of them.

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GILLES ANGELSBERG, ETH Zürich
Large solutions for biharmonic maps in four dimensions

We investigate the existence of large solutions for biharmonic maps from a 4-dimensional Euclidean domain Ω into S^4. Introducing the notion of topological degree for Sobolev maps from $\mathbb{R}^4$ to

S^4, we show that there exist locally minimizing extrinsic biharmonic maps u^* of topological degree -1 and 1. The proof is based upon P.L. Lions' *concentration compactness* principle. This allows us to exclude the phenomena of concentration and vanishing at infinity, for minimizinig sequences for the Hessian energy with prescribed topological degree -1 or 1, up to rescalings and translations. We infer that the degree is preserved in the limit.

Then, for $\Omega = B_1$ unit ball in $\mathbb{R}^4$, we show the existence of two non homotopic biharmonic maps for certain Dirichlet boundary data. The key step is a *sphere attaching lemma* stating the existence of a map u, non homotopic to the absolute minimizer $\underline{u}$ of the Dirichlet problem, having less energy than the sum of the energies of $\underline{u}$ and u^*. Thus, we can exclude bubbling of minimizing sequences in the considered homotopy class in order to conclude compactness.

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GIUSEPPINA AUTUORI, University of Florence
Asymptotic stability for nonlinear Kirchhoff systems

We study the asymptotic stability for solutions of the nonlinear damped Kirchhoff system, with homogeneous Dirichlet boundary conditions, under fairly natural assumptions on the external force f and the distributed damping Q. One of the goals of this paper is to formulate a rational definition of solution which is independent of detailed properties of the functions f and Q. The main asymptotic stability results are based on the a priori existence of a suitable auxiliary function $k = k(t)$, introduced by P. Pucci and J. Serrin in [*Precise damping conditions for global asymptotic stability of second order systems*, Acta Math. **170** (1993), 275–307]. Then the results are extended to a more delicate problem, the so called strongly damped Kirchhoff system, involving also an internal dissipation of higher order which represents the internal material damping of Kelvin–Voigt type of the body structure. Finally, the study is further extended to strongly damped Kirchhoff–polyharmonic systems, which model several interesting problems of Woinowsky–Krieger type.

(Joint work with P. Pucci and M.C. Salvatori)

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SIMON BLATT, RWTH Aachen
Chord-Arc Condition for Objects of Arbitrary Codimension

A rectifiable Jordan curve Γ that goes through ∞ is called a *chord-arc curve with constant κ*, if and only if

$$|s - t| \leq (1 + \kappa)|z(s) - z(t)|$$

for all $s, t \in \mathbb{R}$. Here $z(\cdot)$ denotes an arc length parametrization of Γ. In [1-3] S. Semmes introduces a chord-arc condition for hypersurfaces that preserves many of the well-studied relations of chord-arc curves to operator and function theory.

We extend the notion of chord-arc surfaces and constants to submanifolds of arbitrary codimension. Following ideas of Semmes we can show that they contain big pieces of Lipschitz graphs, if the chord-arc constant is sufficiently small. Using this and a smoothing argument, we are able to show that n-dimensional submanifolds with small chord-arc constants are not only homeomorphic to Euclidean n-space, but even unknotted.

[1] Semmes, S. Chord-arc surfaces with small constant. I. *Adv. Math.*, 85(2):198–223, 1991.

[2] Semmes, S. Chord-arc surfaces with small constant. II. Good parameterizations. *Adv. Math.*, 88(2):170–199, 1991.

[3] Semmes, S. Hypersurfaces in $\mathbf{R}^n$ whose unit normal has small BMO norm. *Proc. Amer. Math. Soc.*, 112(2):403–412, 1991.

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VERENA BÖGELEIN, University of Erlangen–Nuremberg

Higher integrability for weak and very weak solutions of degenerate parabolic systems

We consider higher order nonlinear parabolic systems whose simplest model is the second order parabolic p-Laplacean system

$$u_t - \text{div}(|Du|^{p-2}Du) = 0, \qquad u\colon \Omega \times (-T,0) \supset \mathbb{R}^{n+1} \to \mathbb{R}^N, \quad p > \frac{2n}{n+2}.$$

Its space part is singular if $p < 2$ and degenerate if $p > 2$. Contrary to the time-independent case, i.e. the case of singular respectively degenerate elliptic systems, the parabolic p-Laplacean system behaves non-homogeneously, in the sense that solutions are not invariant under multiplication by constants.

We are interested in integrability properties of solutions to such systems and investigate the question whether weak solutions lying in the parabolic Sobolev-space $L^p(-T,0;W^{1,p}(\Omega,\mathbb{R}^N))$ are higher integrable. On the other hand, we also go below the "natural" integrability exponent and consider so called very weak solutions $u \in L^{p-\epsilon}(-T,0;W^{1,p-\epsilon}(\Omega,\mathbb{R}^N))$ for some $\epsilon > 0$. It turns out that for suitably small ϵ any very weak solution already is a weak solution.

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MIROSLAV BULÍČEK, Charles University, Prague

Navier-Stokes-Fourier system for incompressible fluids with temperature dependent material coefficients

We consider a complete thermodynamic model for unsteady flows of incompressible homogeneous Newtonian fluids in a fixed bounded threedimensional domain. The model leads to equations for the velocity, pressure and temperature fields that satisfy the system of evolutionary equations balancing the linear momentum and the energy on any (measurable) subset of the domain, and is completed by the incompressibility constraint. Finding a solution to such a formulation is tantamount to looking for a *weak solution* to the relevant equations of continuum physics. If in addition the entropy inequality is required to hold on any subset of the domain, the solution that fulfills all these requirements is called *suitable weak solution*. In our setting, both the viscosity and the coefficient of the thermal conductivity are functions of the temperature. We deal with Navier's slip boundary conditions for the velocity that leads to a globally integrable pressure, and we consider zero heat flux across the boundary. For such model we establish the long-time and large-data existence of weak solution.

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ALBERT CLOP, Universitat Autonoma de Barcelona & Jyväskylän Yliopisto

Beltrami equations with coefficient in the Sobolev space $W^{1,2}$

We are interested in the removable singularities for bounded solutions to the planar Beltrami equation,

$$\overline{\partial} f - \mu\,\partial f = 0$$

where $\mu \in L^\infty(\mathbb{C})$ satisfies $\|\mu\|_\infty < 1$.

When $\mu = 0$, we recover the classical *Painlevé problem* on removable singularities for bounded analytic functions. When $\mu \in W^{1,2}$ is compactly supported, it turns out that the situation is quite similar. In this case, we show that if E is a compact set with σ-finite length, then

$$E \text{ is removable for } \overline{\partial} - \mu\,\partial \qquad \Longleftrightarrow \qquad E \text{ is removable for } \overline{\partial}.$$

Moreover, we show that the above equivalence holds even when $\mu \in W^{1,p}$ and $p < 2$ is suitably chosen.

This is a joint work with D. Faraco, J. Mateu, J. Orobitg and X. Zhong.

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LUIGI D'ONOFRIO, University of Naples "Partenope"

On weak Hessian determinants

In this talk, we consider and study several weak formulations of the Hessian determinant, arising by formal integration by parts. Our main concern are their continuity properties. We also compare them with the Hessian measure.

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ALEXANDER EGOROV, Sobolev Institute of Mathematics, Novosibirsk

Self-improving regularity of solutions of an differential inequality with a null Lagrangian

We prove a theorem on the self-improving integrability of solutions of inequality $F(v') \leq KG(v')$ with a null Lagrangian G. This result is an extension of a theorem, due to Iwaniec and Martin [3] and Iwaniec [2], on the self-improving integrability of quasiregular mappings.

We also present some applications to stability problems for classes of solutions which we consider in [1].

References

[1] Egorov A.A. Stability of classes of mappings, quasiconvexity, and null Lagrangians // Dokl. Akad. Nauk (to appear).

[2] Iwaniec T. p-Harmonic tensors and quasiregular mappings // Ann. of Math. (2). 1992. V. 136, no. 3. P. 589-624.

[3] Iwaniec T., Martin G. Quasiregular mappings in even dimensions // Acta Math. 1993. V. 170, no. 1. P. 29-81.

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ANNA FÖGLEIN, University of Erlangen–Nuremberg

Higher integrability of minimizers of functionals with (2,q)-growth in the Heisenberg Group

We study minimizers of functionals of the form

$$\mathcal{F}(u) = \int_\Omega F(\mathfrak{X}u)\,dx$$

where Ω is a bounded domain in the Heisenberg Group $\mathbb{H}^n$, the symbol $\mathfrak{X}$ stands for the horizontal derivative on $\mathbb{H}^n$, and F is a C^2 function with $(2,q)$-growth

$$\nu|z|^2 \leq F(z) \leq L(1+|z|^2)^{\frac{q}{2}},$$

which satisfies the convexity condition

$$\nu|\xi|^2 \leq \langle D^2F(z)\xi, \xi\rangle \leq L(1+|z|^2)^{\frac{q-2}{2}}|\xi|^2.$$

We show local boundedness of the horizontal and vertical derivative of u under certain conditions on the upper growth exponent q. The proof involves repeated interplay between estimates for the vertical and horizontal derivatives. Techniques needed include fractional difference quotients, approximation by more regular functionals and a variant of the Moser iteration.

References

[1] A. Domokos, *Differentiability of solutions for the non-degenerate p-Laplacian in the Heisenberg group.* J. Differ. Equations 204, 2004.

[2] L. Esposito, F. Leonetti and G. Mingione, *Regularity results for minimizers of irregular integrals with (p, q) growth.* Forum Math. 14, 2002.

[3] J.J. Manfredi, G. Mingione: *Regularity results for quasilinear elliptic equations in the Heisenberg group.* Mathematische Annalen, to appear.

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PAWEŁ GOLDSTEIN, University of Warsaw

Calderon–Zygmund type estimates for nonlinear systems with quadratic growth on the Heisenberg group

We prove the Calderón-Zygmund–type estimates for nonlinear subelliptic systems with quadratic growth, of the type

$$\mathrm{div}_{\mathcal{X}} A(z, \mathcal{X}u(z)) = \mathrm{div}_{\mathcal{X}} F(z),$$

with $|A(z, \xi)| \approx L(1 + |\xi|)$, where $\mathcal{X}$ is the horizontal gradient on Heisenberg group.

This is joint work with Anna Zatorska–Goldstein.

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PIOTR GWIAZDA, University of Warsaw

Existence of solutions to non-Newtonian flow in generalized Orlicz spaces

We will present existence results for non-Newtonian fluids with the stress tensor of very fast (faster than polynomial) and non-homogenous (in x) growth. Consequently L^p spaces framework is not suitable to formulate growth conditions of the stress tensor. Instead, the concept of the so-called *N-function* is introduced, which leads to the definition of the generalized Orlicz space (known also as *the Musielak–Orlicz space*). Example of the generalized Orlicz spaces are the spaces $L^{p(x)}$. They constitute the appropriate framework in analysis of situations when we deal with a problem of modeling strongly inhomogeneous physical behavior — like the so-called electrorheological fluids.

Part of the difficulties in mathematical analysis of such models are caused by the generalized Orlicz spaces themselves. Although they create a natural framework for such analysis, they exhibit new problems arising from the fast growth of an *N-function*, like the lack of reflexivity of the space or the lack of the density of smooth functions. The literature provides numerous results on the existence of solutions to abstract elliptic and parabolic problems in Orlicz spaces, but the framework of Musielak-Orlicz spaces for non-Newtonian flows is still a developing field.

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JENS HABERMANN, University of Erlangen-Nuremberg
Regularity for minimizers of integral functionals and solutions of elliptic PDE with $p(x)$ growth

The aim of my talk is to introduce the audience to the regularity theory for quasiconvex variational integrals and elliptic partial differential equations with so-called $p(x)$ growth. More precisely this means that the growth exponents of the appearing integrands and coefficients, respectively, are functions in space. In the past ten years, many investigations have been made on such type of problems. On one hand they are interesting from the mathematical point of view, since they represent the borderline case between problems with standard p growth and problems with (p, q) growth. On the other hand problems of $p(x)$ type can also be interesting in applications. They appear in the mathematical modeling of physical problems, such as for example electrorheological fluids.

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DARIA V. ISANGULOVA, Sobolev Institute of Mathematics, Novosibirsk
Sharp geometric rigidity on Heisenberg groups

We consider a Heisenberg group $\mathbb{H}^n$ with Carnot–Carathéodory distance d. Consider a nonconstant mapping $f\colon U \to \mathbb{H}^n$ of the Sobolev class $W^1_{1,\mathrm{loc}}$. The mapping f is a *quasiisometry* if f does not change orientation on U and there is a constant $L \geq 1$ such that and $L^{-1}|\xi| \leq |D_h f(x)\xi| \leq L|\xi|$ for all horizontal vectors ξ for almost all $x \in U$. We denote by $I(L, U)$ the class of quasiisometries on U with the constant L. If $L = 1$ then f is an isometry.

A domain $U \subset \mathbb{H}^n$ is a *John domain* $J(\alpha, \beta)$ [1], $0 < \alpha \leq \beta$, if there exists a point $x_0 \in U$ such that every $x \in U$ can be joined in U to x_0 by a rectifiable curve γ parameterized by the arc length, $\gamma(0) = x$, $\gamma(l) = x_0$, $l \leq \beta$, and $\mathrm{dist}(\gamma(s), \partial U) \geq \frac{\alpha s}{l}$ for all $s \in [0, l]$.

THEOREM. *Let U be a John domain $J(\alpha, \beta)$ on $\mathbb{H}^n$, $n > 1$. There exist constants $N_1, N_2 > 0$ depending only on n such that for any mapping $f \in I(1 + \varepsilon, U)$ there is an isometry φ such that*

$$\sup_{x \in U} d(f(x), \varphi(x)) \leq N_1 \frac{\beta^2}{\alpha}(\sqrt{\varepsilon} + \varepsilon)$$

and

$$\int_U \exp\biggl(\Bigl(\frac{\beta}{\alpha}\Bigr)^{2n+3} \frac{N_2}{\varepsilon} |D_h f(x) - D_h \varphi(x)|\biggr)\, dx \leq 2|U|.$$

The dilation $\delta_{1+\varepsilon}$ shows that the orders of closeness in the theorem are asymptotically sharp.

Proof of the theorem explores the method of Yu. G. Reshetnyak [2] in Euclidean spaces. It relies on constructing some first-order differential operator with finite dimensional kernel. In the case of Heisenberg group $\mathbb{H}^1$ some qualitative rigidity was obtained in [3].

The talk is based on a joint work with S. Vodopyanov [4]. The work was partially supported by the Russian Foundation for Basic Research (grant 06–01–00735).

References

[1] John F., *Rotation and strain*, Comm. Pure Appl. Math. 14 (1961), 391–413.

[2] Reshetnyak Yu. G., *Stability theorems in geometry and analysis*, Mathematics and its Applications (Dordrecht). 304. Kluwer Academic Publishers, Dordrecht, 1994.

[3] Arcozzi N. and Morbidelli D., *Stability of isometric maps in the Heisenberg group*, Preprint, 2005.

[4] Isangulova D. and Vodopyanov S., *Sharp geometric rigidity on Heisenberg groups*, Preprint, 2007.

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AGNIESZKA KAŁAMAJSKA, University of Warsaw
Lusin type theorem with convex integration and quasiconvex hulls of sets

We obtain Lusin type theorem showing that after extracting an open set of an arbitrary small measure one can apply the variant of convex integration theory dealing with quasiconvex hulls of sets. The result can be used to construct approximate solutions of the Partial Differential Inclusion $Du \in K$, with given boundary data.

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MARIA KARMANOVA, Sobolev Institute of Mathematics, Novosibirsk
Coarea Formula for Contact $C^{1,1}$-Mappings of Carnot Manifolds

Abstract: In Euclidean spaces, there are many applications of the coarea formula, namely, in the theory of exterior forms and currents, and in the problem of minimal surfaces. In particular, Stokes formula and can be easily proved with using of the coarea formula. There is a question to extend this formula to structures of more general nature in comparison with Euclidean ones, especially to structures of non-holonomic geometry. In 1982, P. Pansu proved the coarea formula for functions defined on a Heisenberg group. Then, J. Heinonen has proved the coarea formula in the case of real-valued smooth function defined on Carnot group. Other result, concerning the proof of this formula, belongs to V. Magnani. He proved in 2000 the coarea inequality for mappings of Carnot groups. The equality was proved only for the case of the mapping defined on a Carnot group with values in $\mathbb{R}^k$. Until now, the question about the coarea formula for mappings of nilpotent groups is still opened even is such case as Heisenberg groups. We prove the coarea formula for contact $C^{1,1}$-mappings of Carnot manifolds. In particular, we investigate the level sets of such mappings, and the relation of Riemannian and Hausdorff measures on them.

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MIKHAIL KOROBKOV, Sobolev Institute of Mathematics, Novosibirsk
On necessary and sufficient conditions for a curve to be the gradient range of a C^1 function

Many mathematicians (J.Ball, S.Müller, V.Sverak, B.Kirchheim and others) studied the following problem: under which conditions on a set K the differential relation $Dv \in K$ has a nontrivial Lipschitz solution?

We study the similar problem for C^1-smooth (not only Lipschitz) solutions to the differential relations. In a particular, we prove that if the gradient range of a C^1-smooth function v is a continuous arc, then this arc has tangents in a weak sense and these tangents are rank-one matrices. Moreover, the direction of these tangents is a function of bounded variation.

In this talk we extend our previous plane results to the multidimensional case.

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MIKA KOSKENOJA, University of Helsinki
Functions preserving plurisubharmonicity under holomorphic mappings

We study functions having the property that plurisubharmonicity is preserved under holomorphic mappings between domains in $\mathbb{C}^n$. Both pull backs and push forwards are considered. As an application we prove a capacity inequality for proper holomorphic mappings.

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DENIS KOVTONYUK, IAMM, Ukrainian National Academy of Sciences

On finitely bi–Lipschitz homeomorphisms

We investigate the removability of isolated singularities and the boundary behavior of finitely bi-Lipschitz mappings, which are a natural generalization of the well-known classes of isometries and quasi-isometries in $\mathbb{R}^n$.

Let Ω be an open set in $\mathbb{R}^n$, $n \geqslant 2$, and suppose the mappings $f : \Omega \to \mathbb{R}^n$ are continuous. Given a mapping $f : \Omega \to \mathbb{R}^n$ and a point $x \in \Omega \subseteq \mathbb{R}^n$, let

$$l(x,f) = \liminf_{\substack{y \to x \\ y \in \Omega}} \frac{|f(x)-f(y)|}{|x-y|}\,, \quad L(x,f) = \limsup_{\substack{y \to x \\ y \in \Omega}} \frac{|f(x)-f(y)|}{|x-y|}\,.$$

We say that a mapping $f : \Omega \to \mathbb{R}^n$ is *finitely bi-Lipschitz* if $0 < l(x,f) \leqslant L(x,f) < \infty$ holds for all $x \in \Omega$.

Recall that the *outer dilatation* of mapping f at x is defined by

$$K_O(x,f) = \begin{cases} \frac{||f'(x)||^n}{|J(x,f)|}\,, & \text{if } J(x,f) \neq 0 \\ 1\,, & \text{if } f'(x) = 0 \end{cases}$$

and $K_O(x,f) = \infty$ otherwise. As usual, here $f'(x)$ denotes the Jacobian matrix of f at the point of differentiability x and $J(x,f)$ is its determinant, $||f'(x)||$ is the operator norm of $f'(x)$, i.e.

$$||f'(x)|| = \max\{\, |f'(x)h| :\ h \in \mathbb{R}^n,\ |h| = 1\}\,.$$

Theorem 1. *Let D be a domain in $\mathbb{R}^n$, $n \geq 2$, $x_0 \in D$, and let $f : D \setminus \{x_0\} \to \mathbb{R}^n$ be a finitely bi-Lipschitz homeomorphism. Suppose that*

$$\int\limits_0^{\varepsilon_0} \frac{dr}{r \cdot k_{n-1}(r)} = \infty\,,$$

where $\varepsilon_0 < dist(x_0, \partial D)$ and

$$k_{n-1}(r) = \left(\fint_{|x-x_0|=r} K_O^{n-1}(x,f)\, d\mathcal{A} \right)^{\frac{1}{n-1}}\,.$$

Then f has a continuous extension to D.

Theorem 2. *Let D be a domain in $\mathbb{R}^n$, $n \geq 2$, and let $f : D \to \mathbb{R}^n$ be a finitely bi-Lipschitz homeomorphism. Suppose that D and $D' = f(D)$ are bounded convex domains. If at every point $x_0 \in \partial D$*

$$\int\limits_0^{\delta(x_0)} \frac{dr}{||\, K_O||_{\,n-1}(x_0,r)} = \infty$$

for some $\delta(x_0) \in (0, d(x_0))$*, where* $d(x_0) = \sup_{x \in D} |x - x_0|$*, and*

$$|| K_O||_{n-1}(x_0, r) = \left(\int_{D \cap S(x_0,r)} K_O^{n-1}(x, f)\, d\mathcal{A} \right)^{\frac{1}{n-1}},$$

where $S(x_0, r) = \{x \in \mathbb{R}^n : |x - x_0| = r\}$*, then* f *has a homeomorphic extension to* $\overline{D}$*.*

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VISA LATVALA, University of Joensuu

Minimizers of the variable exponent non-uniformly convex Dirichlet energy

We deal with energy minimizing properties of the function $u = \lim_{\lambda_j \to 1^+} u_{\lambda_j}$, where u_{λ_j} is the solution to the $p_{\lambda_j}(\cdot)$-Laplacian Dirichlet problem with given boundary values. Here $p \colon \Omega \to [1, \infty)$ is a variable exponent and $p_{\lambda_j}(x) = \max\{p(x), \lambda_j\}$ for $\lambda_j > 1$. This problem leads in a natural way to a mixture of Sobolev and total variation norms. The main results are obtained under the assumption that p is strongly log-Hölder continuous and bounded. The results are related to the analysis of a model for image restoration combining total variation and isotropic smoothing.

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VALENTINO MAGNANI, University of Pisa

Contact equations and Lipschitz extensions

We will present the relationship between Lipschitz extensions of mappings in stratified groups and the solution of a first order nonlinear system of Partial Differential Equations.

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ANNA OCHAL, Jagiellonian University

Nonsmooth Nonlinearities in Contact Mechanics

Mathematical modeling of problems arising in mechanics is a fascinating field of research which combine analysis, differential equations, theoretical mechanics and numerical simulations. During the last twenty years variational methods based on the formulations with nonconvex energy functionals and in particular hemivariational inequalities play an important role in the understanding of many physical phenomena.

The phenomena of friction and adhesion related to contact between deformable bodies can be frequently found in everyday life (e.g. contact of train wheels with the rails, a shoe with the floor, tectonic plates, the car's braking system, etc). On the other hand, the adhesive processes are important in industry where nonmetallic parts are glued together. In particular such phenomena are important in the description of composite materials which are light and strong, and are useful in aviation, space exploration and in the automotive industry.

The multivalued laws between stresses and strains, reactions and displacements and between generalized velocities and forces appear in these problems in a natural way. Variational formulations are based on the theory of evolution inclusions of first and second order. Nonconvex, nonsmooth and locally Lipschitz superpotentials which subdifferentials model multivalued boundary conditions lead to hemivariational inequalities. The hemivariational inequalities make use of the generalized gradient of Clarke-Rockafellar and they are generalizations of variational inequalities. They appear when the constitutive laws and/or the boundary conditions are derived from the convex and nondifferentiable potentials.

We consider dynamical model of the contact of viscoelastic body with a rigid foundation in the unilateral friction and adhesion as well as dynamical friction problem with termal and/or piezo effects. We present some results concerning the existence of weak solutions, we provide conditions on the uniqueness of for systems governed by hemivariational inequalities. We establish the above results using the theory of monotone and pseudomonotone operators and fixed point theorems.

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DARIUSZ PĄCZKA, Szczecin University of Technology

Weak and Young measure solutions for hyperbolic initial-boundary value problems of elastodynamics with nonconvex energy in the Orlicz–Sobolev space setting

Let $\Omega \subset \mathbb{R}^n$ be a bounded Lipschitz domain and $T, \mu \in (0, \infty)$. We establish a first existence results of global weak solutions (respectively, Young measure solutions) in the Orlicz–Sobolev space setting for the nonlinear hyperbolic problems $u''_{tt} = \operatorname{div}(\sigma(Du)) + \mu(\triangle u)'_t$ (respectively for $u''_{tt} = \operatorname{div}(\sigma(Du))$), subject to the classical Cauchy initial condition and the Dirichlet boundary condition, where $u\colon \Omega \times [0,T] \to \mathbb{R}^d$ is a given function and the Piola–Kirchhoff stress function $\sigma = \partial\Phi/\partial F$ is continuous and $\Phi\colon \mathbb{M}^{d\times n} \to \mathbb{R}$ is the stored-energy C^1-function.

This is joint work with Hôǹg Thái Nguyêñ.

References

[1] J.M. Ball, *A version of the fundamental theorem for Young measures*, in: PDE's and Continuum Models of Phase Transitions (Nice, 1988) (D. Serre M. Rascle and M. Slemrod, eds.), Lect. Notes in Physics, vol. 344, Springer, Berlin, 1989, pp. 207–215.

[2] G. Friesecke and G. Dolzmann, *Implicit time discretization and global existence for a quasi-linear evolution equation with nonconvex energy*, SIAM J. Math. Anal. **28** (1997), 363–380.

[3] H.T. Nguyêñ and D. Pączka, *Young parametrized measures and weak-type limits of sequences in non-L_p-type spaces of measurable functions*, Preprint, University of Szczecin, Poland, 2001.

[4] H.T. Nguyêñ and D. Pączka, *Existence theorems for the Dirichlet elliptic inclusion involving exponential-growth-type multivalued right-hand side*, Bull. Polish Acad. Sci. Math. **53** (2005), 361–375.

[5] P. Pedregal, *Parametrized Measures and Variational Principles*, Birkhäuser, Basel, 1997.

[6] M.O. Rieger, *Young measure solutions for nonconvex elastodynamics*, SIAM J. Math. Anal. **34** (2003), 1380–1398.

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ANDREI POKROVSKII, National Academy of Sciences of Ukraine

Local approximations by solutions of elliptic equations and removable singularities

For a linear second order uniformly elliptic operator $\mathfrak{L}$ in divergence form with bounded and measurable real coefficients in a bounded domain $G \subset \mathbb{R}^n$ ($n \geq 2$), we establish a metric criterion for removability of closed subsets of G for generalized solutions of the equation $\mathfrak{L}f = 0$ in classes of functions which have locally square integrable distributional derivatives in G and given order of local approximations in the corresponding Sobolev norm by generalized solutions of this equation on balls in G. The dependence of differential and difference properties of functions from these classes on the coefficients of $\mathfrak{L}$ are investigated.

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WALDEMAR POMPE, Warsaw University
Convex integration and the martingale convergence theorem

We present a variant of the convex integration method, in which we do not require to construct an in-approximation, nor any other explicit approximation of the set K. Our assumptions are given only in terms of the (local) geometry of the set $\partial(K^{pc}) \setminus K$

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ANTTI RASILA, Helsinki University of Technology
Canonical domains and p-harmonic functions

A classical result by J. Hadamard (1896), known as the three circles theorem, gives the a relation between the maximum absolute values of an analytic function on three concentric circles. It is a natural question, what results of this type can be proved for other classes of functions. We will discuss a three spheres type theorem for the p-harmonic functions defined on canonical domains in the Euclidean n-dimensional space proved in [1]. In addition, we will outline the results on stagnation zones and Phragmen-Lindelöf type theorems from [2]. This presentation is based on joint research with V.M. Miklyukov and M. Vuorinen.

[1] V.M. MIKLYUKOV, A. RASILA and M. VUORINEN: *Three spheres theorem for p-harmonic functions*, Helsinki University of Technology, Institute of Mathematics Research Reports A502, 2006, 1-17. Houston J. Math. (to appear) arXiv math.AP/0701439

[2] V.M. MIKLYUKOV, A. RASILA and M. VUORINEN: *Stagnation zones for A-harmonic functions on canonical domains*, in preparation.

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RUSLAN SALIMOV, IAMM, Ukrainian National Academy of Sciences
On ACL-homeomorphisms

Let G and G' be domains in $\mathbb{R}^n$, $n \geq 2$, and let $Q : G \to [1, \infty]$ be a measurable function. A homeomorphism $f : G \to G'$ is called a $Q-$*homeomorphism* if

$$M(f\Gamma) \leq \int\limits_G Q(x) \cdot \varrho^n(x)\, dm(x)$$

for every family Γ of paths in G and every admissible function ϱ for Γ. Here m denotes the Lebesgue measure in $\mathbb{R}^n$. This conception is a natural generalization of the geometric definition of a quasiconformal mapping.

Recall that, given a family of paths Γ in $\mathbb{R}^n$, a Borel function $\varrho : \mathbb{R}^n \to [0, \infty]$ is called *admissible* for Γ, abbr. $\varrho \in adm\,\Gamma$, if

$$\int\limits_\gamma \varrho\, ds \ \geq \ 1$$

for all $\gamma \in \Gamma$. The (conformal) *modulus* of Γ is the quantity

$$M(\Gamma) \ = \ \inf_{\varrho \in adm\,\Gamma} \int\limits_G \varrho^n(x)\, dm(x)\ .$$

Theorem. Let G and G' be domains in $\mathbb{R}^n$, $n \geq 2$, and $f : G \to G'$ is Q–homeomorphism with $Q \in L^1_{loc}$. Then f belongs to ACL and f is differentiable a.e. in G.

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HÉLIA SERRANO, University of Castilla–La Mancha

Non-periodic Γ-convergence of quadratic functionals

The explicit characterization of the density of the Γ-limit for quadratic functionals, with linear perturbations, is achieved by means of the joint Young measure associated with relevant sequences, in the case of non-periodic coefficients.

This is a joint work with Pablo Pedregal.

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EVGENY A. SEVOST'YANOV, IAMM, Ukrainian National Academy of Sciences

Quasilinear Beltrami Equation

Let $\Delta = \{z \in \mathbb{C} : |z| < 1\}$. The equation

$$f_{\overline{z}} = \nu\,(z,\, f(z))\; f_z\,, \tag{5}$$

where $z = x + iy\,,\quad f_{\overline{z}} = (f_x + if_y)\,/2$ and $f_z = (f_x - if_y)\,/2$, is said to be a *quasilinear Beltrami equation.* The function $\nu = \nu\,(z,\, w) : D \times \mathbb{C} \to \Delta$ satisfies the Carathéodory conditions if ν is measurable w.r.t. $z \in D$ for all fixed $w \in \mathbb{C}$ and continuous w.r.t. $w \in \mathbb{C}$ for a.e. $z \in D$. Set

$$K_\nu\,(z,w) = \frac{1 + |\nu\,(z,w)|}{1 - |\nu\,(z,w)|}\,.$$

We say that $\varphi : D \to \mathbb{C}$ has *finite mean oscillation* at a point $z_0 \in D$, in short $\varphi \in$ *FMO*(z_0), if and only if

$$\overline{\lim_{\varepsilon\to 0}}\; ⨍_{B(z_0,\varepsilon)} |\varphi(z) - \overline{\varphi}_\varepsilon|\; dm(z) < \infty\,,$$

and we say that $\varphi \in FMO(\Delta)$ if and only if $\varphi \in FMO(z_0)$ for every $z_0 \in \Delta$.

Theorem. *Assume that* $\nu(z,w) : \Delta \times \mathbb{C} \to \Delta$ *satisfies the Carathéodory conditions. If* $K_\nu(z,\, w) \leq Q(z) \in FMO(\Delta)$, *then the equation* (5) *has regular homeomorphic* $W^{1,1}_{loc}$ *solution in* Δ.

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GRAHAM SMITH, MPI Leipzig

Positive Special Legendrian Submanifolds and Weingarten Problems

We introduce a compactness result for Special Legendrian (and Lagrangian) submanifolds subject to a positivity condition, and we show how this may be used to study families of hypersurfaces satisfying certain curvature conditions.

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MARTA SZUMAŃSKA, Technical University of Warsaw

Integral energies based on Menger curvature for closed rectifiable curves

Menger curvature of a triple of points is defined as the reciprocal of the radius of the circle passing through these points. We will discuss how finiteness of certain integral energies related to this concept affects regularity and self-avoidance of closed rectifiable curves.

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AGNIESZKA ŚWIERCZEWSKA–GWIAZDA, University of Warsaw

Lipschitz truncations method for non-Newtonian flows with discontinuous Cauchy stress

Our interest concentrates on non-Newtonian phenomena exhibited by incompressible liquids, in particular on problems related with the presence of yield stress, which is also an example of discontinuous Cauchy stress. We can view the model with the yield stress as a possible and reasonable approximation of more realistic fluid response — example being the model of influence of platelets activation on blood rheology. The existence of weak solutions will be presented. As the most effective method, the approach of truncations of the test function will be used, in particular the method of Lipschitz truncations basing on the theory of maximal functions, Hardy-Littlewood theorem, etc. These are the methods originally arising from the regularity theory and have been recently exploited in the existence of solutions considerations. The discontinuous problem will be presented in the multi-valued graphs formulation and the tools of Young measures will be applied.

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SERGEY VODOPYANOV, Sobolev Institute of Mathematics, Novosibirsk

Foundations of the geometry of Carnot-Carathéodory spaces and differentiability of mappings

The main goal of the talk is to show a new approach to the geometry of Carnot-Caratheodory spaces to study. It is based on a new result which characterizes the closeness of geometries of different local tangent cones [1]. From here we obtain basic results of the theory: estimates of the closeness of geometries of a Carnot manifold and a local tangent cone, Rashevskiy-Chow type Connectivity Theorem, Local Approximation Theorem, Ball-Box Theorem, estimates for a comparison of metrics and others.

As an application [2], we prove differentiability of curves and mappings of Carnot-Caratheodory spaces. In particular, we obtain analytic characterization of quasiconformal mappings between two such spaces, and some other results.

[1] Vodopyanov S. K., Karmanova M. B. Local Geometry of Carnot manifolds under minimal smoothness, *Doklady Mathematics*, 2007, 413, no. 3, P. 305–311.

[2] Vodopyanov S. K. Geometry of Carnot–Carathéodory Spaces and Differentiability of Mappings, *The Interaction of Analysis and Geometry*. Contemporary Mathematics, **424** (2007), P. 247–302.

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ANNA ZATORSKA-GOLDSTEIN, University of Warsaw

On a nonlinear fourth order elliptic system with critical growth in first order derivatives

We prove that all bounded weak solutions of the fourth order system

$$\Delta^2 u = Q(x, u, \nabla u), \qquad u \in W^{2,2}(\Omega, \mathbb{R}^N),$$

where the nonlinearity grows critically with the gradient of a solution, i.e., $|Q(x, u, \nabla u)| \lesssim |\nabla u|^4$, are regular once an appropriate smallness condition (expressed in terms of Morrey norms and guaranteeing that u is small in BMO) is satisfied. The result holds in every dimension.

This is joint work with P. Strzelecki.

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