

Database Theory: First Assignment

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Each problem is worth 5 points. A correct answer without any proof or explanation will earn you at most 2-3 points.

1. Let SPCU^\neq denote SPCU algebra extended with inequality in selection formulae: $\sigma_{i \neq j} E$, $\sigma_{i \neq a} E$. Determine containment relations between SPCU, SPCU^\neq , and RA.
2. The *division* operator, denoted $:$, is added to the named algebra as follows. For relations I and J with $\text{sort}(I) \subseteq \text{sort}(J)$, the value of $J : I$ is the set of tuples $t \in \pi_{\text{sort}(J) \setminus \text{sort}(I)} J$ such that $(\{t\} \bowtie I) \subseteq J$. Show that relational algebra can simulate the division operator.
3. Consider the following simple game played by Adam and Eve on a finite directed graph whose vertices are divided between the players. The game starts with a token being placed in a starting position. Next, the players move the token along the edges of the graph: the next position is chosen by the owner of the current position. Eve wins the play (finite or infinite) iff at some point the token reaches a designated target vertex, or the token gets trapped in a vertex belonging to Adam with no outgoing edges. Otherwise the play is won by Adam.

Assume the arena is represented by relations: *Move*, storing edges between positions, *Adam*, storing Adam's positions, and *Eve*, storing Eve's positions. Write a FIXPOINT (i.e., WHILE⁺ or CALC+ μ^+) query that returns pairs of vertices (v, w) such that Eve has a winning strategy in the game described above with the starting position v and the target position w .

Is there an equivalent CALC query?

4. Show that WHILE \equiv CALC on input databases with unary relations only (the program can use relations of arbitrary arity).