Chapter 1

Formalization of Default Logic Using CAKE

In this chapter, we formalize a subset of default logic using CAKE method.[1] We present two basic versions of this formalism: *rough default logic* and *rough default logic with strong prerequisites*. The main difference between the versions lies in different treatment of the prerequisite of a default while determining the default's applicability. In the former, a default can be applied if its prerequisite is believed. In the latter, we may require that the prerequisite of a default (or a part of it) has to be known, rather than believed, to make the default applicable. This possibility substantially increases the expressive power of the resulting logic. We also show that both rough default logic and rough default logic with strong prerequisites can be naturally extended to their prioritized versions by slightly changing the voting policy mechanism.

The following discussion assumes familiarity with the material contained in Chapters 4 and 12.

1.1 Preliminaries

A default is said to be *disjunction-free* if its prerequisite and justification are conjunctions of literals and its consequent is a single literal. A default theory is said to be *disjunction-free* if all its defaults are disjunction-free and all its axioms are ground literals.

In the sequel, we restrict ourselves to finite disjunction-free default theories with consistent sets of axioms.[2] We assume that the axioms of a theory are ground

[1] Recall that default logic has been presented in Section 4.2, Chapter 4.

[2] Note that the task of determining whether a set of axioms of a given disjunction-free default theory is consistent is trivial.

literals extracted from the extensional database.[3]

To make our presentation as simple as possible, we start with a number of examples.

Example 1.1.1 Consider the theory $T = \langle \{B(t)\}, \{B(x) : F(x)/F(x)\} \rangle$. This is the "Bird" theory with B, F, t standing for $Bird$, $Flies$ and $Tweety$, respectively. Figure 13.1 represents a diagram corresponding to the theory T.

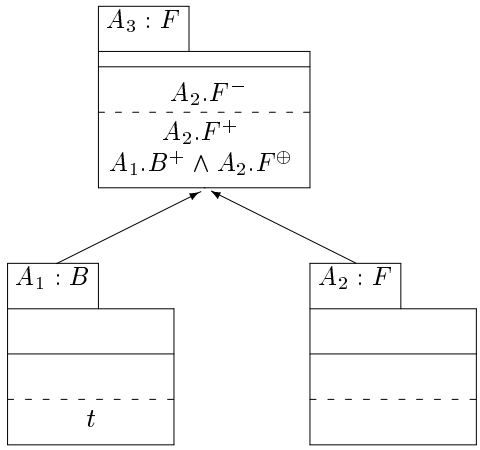

Figure 1.1: Diagram corresponding to the theory of Example 13.1.1.

The bottom agents, A_1 and A_2, are granular agent's representing the extensional database. They are responsible for the relations B and F, respectively. Note that t is in the positive part of the diagram of A_1 and in the boundary region of the diagram of A_2. The agent A_3 represents the default of our theory. It is responsible for the relation F which occurs in the default consequent. There are three methods associated with the agent. The first two, namely $A_2.F^-$ and $A_2.F^+$, allows the agent A_3 to use knowledge of the agent A_2 while computing the relation F. Using them the agent can infer that, for any object x, x lies in in the negative (resp. positive) part of his diagram, provided that x lies in the negative (resp. positive) part of the diagram of A_2. The third method, i.e. $A_1.B^+ \wedge A_2.F^{\oplus}$, represents the default of the theory. Note that the method is placed in the positive part of the diagram. Accordingly, it is to be viewed as the following rule:

[3] That is, ground literals of the form $R(\overline{a})$ (resp. $\neg R(\overline{a})$), where $\overline{a}$ is in the positive (resp. negative) part of the diagram of the extensional database agent responsible for the relation R.

> "For any object x, if x is in the positive part of the diagram of A_1 and x is in the positive/boundary part of the diagram of A_2, infer that x is in the positive part of the diagram of A_3".

The following set of rules is attached to the above diagram:[4]

$$A_2.F^-(x) \rightarrow A_3.F^-(x)$$
$$[A_2.F^+(x) \vee [A_1.B^+(x) \wedge A_2.F^{\oplus}(x)]] \rightarrow A_3.F^+(x)$$
$$A_1.B^+(t).$$

Eliminating the reference to boundary region in $A_2.F^{\oplus}(x)$, i.e. replacing $A_2.F^{\oplus}(x)$ by $\neg A_2.F^-(x)$ (see Chapter 12, section 12.4), we get

$$A_2.F^-(x) \rightarrow A_3.F^-(x)$$
$$[A_2.F^+(x) \vee [A_1.B^+(x) \wedge \neg A_2.F^-(x)]] \rightarrow A_3.F^+(x)$$
$$A_1.B^+(t).$$

The relations $A_3.F^-$, $A_3.F^+$ and $A_1.B^+$ occurring in the heads of the rules are specified by simultaneous fixpoint formula of the form

$$\mathsf{lfp}\ A_3.F^-, A_3.F^+, A_1.B^+.\mathcal{R} \tag{1.1}$$

where $\mathcal{R}$ is the conjunction of the rules.

Below, we compute the relations characterized by the formula (13.1), using an analogon of Algorithm 3.4.2 from Chapter 3. The successive lines consist of sets of literals obtained after performing successive iterations.

$$\{\}$$
$$\{A_1.B^+(t)\}$$
$$\{A_1.B^+(t), A_3.F^+(t)\}.$$

Suppose the query we are interested in is $F(t)$. It is immediately seen that t satisfies the $A_3.F^+$-coordinate of (13.1). Accordingly, the answer to the query is True. ■

Example 1.1.2 Consider the theory

$$T = \langle \{Q(n), R(n)\} \{R(x) : \neg P(x)/\neg P(x), Q(x) : P(x)/P(x)\} \rangle.$$

This is the "Nixon" theory with R, P, Q, n standing for *Republican*, $Pacifist$, $Quaker$ and $Nixon$, respectively. Figure 13.2 represents the diagram corresponding to T.

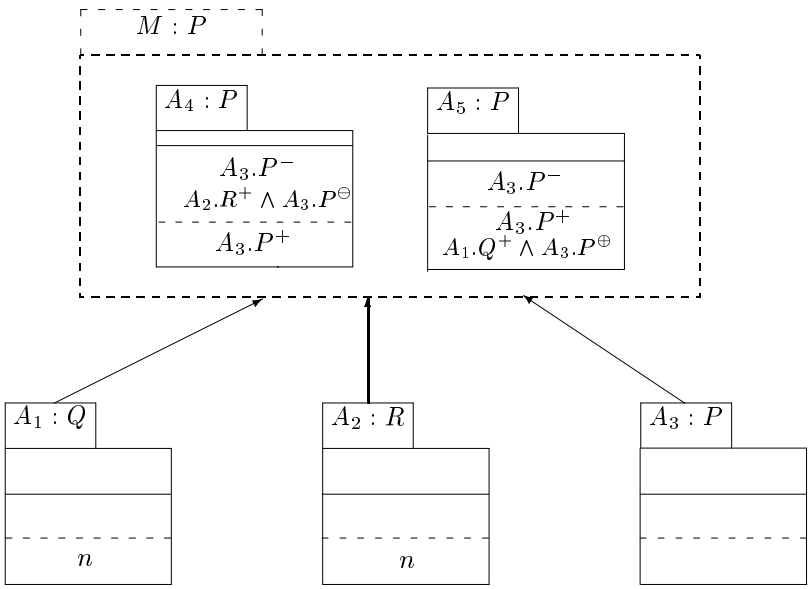

Figure 1.2: Diagram corresponding to the theory of Example 13.1.2.

The agents A_1, A_2 and A_3 represent the extensional database, whereas A_4 and A_5 represent the defaults of the theory.[5] Since both A_4 and A_5 make inferences concerning the relation P, they have been grouped into a single default module.[6] It is the module, not an individual agent, that is responsible for default inferences about P.

The following set of rules is attached to the above diagram:[7]

$$A_1.Q^+(n)$$
$$A_2.R^+(n)$$
$$[A_3.P^-(x) \vee [A_2.R^+(x) \wedge \neg A_3.P^+(x)]] \rightarrow A_4.P^-(x)$$
$$A_3.P^+(x) \rightarrow A_4.P^+(x)$$

[4] Recall that all free variables occurring in the rules are implicitly universally quantified.

[5] Observe that the method $A_2.R^+ \wedge A_3.P^{\ominus}$ used by the agent A_4 is placed in the negative part of its diagram. Accordingly, it has the following reading: "for any object x, if x is in the positive part of the diagram of A_2 and x is in the negative/boundary part of the diagram of A_3, infer that x is in the negative part of the diagram of A_4.

[6] In Chapter 3, a collection of granular agents was referred to as a knowledge module. Here we use the term "a default module", since all agents included in such a module represent defaults.

[7] The relations $A_3.P^{\ominus}$ and $A_3.P^{\oplus}$, occurring in the diagram, have been replaced by $\neg A_3.P^+$ and $\neg A_3.P^-$, respectively.

$$A_3.P^- \rightarrow A_5.P^-(x)$$
$$[A_3.P^+(x) \vee [A_1.Q^+(x) \wedge \neg A_3.P^-(x)]] \rightarrow A_5.P^+(x)$$
$$[A_4.P^-(x) \vee A_5.P^-(x)] \wedge \neg[A_4.P^+(x) \vee A_5.P^+(x)] \rightarrow M.P^-(x)$$
$$[A_4.P^+(x) \vee A_5.P^+(x)] \wedge \neg[A_4.P^-(x) \vee A_5.P^-(x)] \rightarrow M.P^+(x).$$

The last two rules represent the standard voting mechanism used by the module M.

The relations $A_1.Q^+$, $A_2.R^+$, $A_4.P^-$, $A_4.P^+$, $A_5.P^-$, $A_5.P^+$, $M.P^-$ and $M.P^+$, occurring in the heads of the above rules, are defined by the fixpoint formula given by

$$\mathsf{lfp}\ A_1.Q^+, A_2.R^+, A_4.P^-, A_4.P^+, A_5.P^-, A_5.P^+, M.P^-, M.P^+.\mathcal{R} \qquad (1.2)$$

where $\mathcal{R}$ denotes the conjunction of the rules.

Below, we compute the relations characterized by the formula (13.2).

$$\{\}$$
$$\{A_1.Q^+(n), A_2.R^+(n)\}$$
$$\{A_1.Q^+(n), A_2.R^+(n), A_4.P^-(n), A_5.P^+(n)\}.$$

Suppose the query of interest is $P(n)$. It is immediately seen that the agent A_4 answers False and the agent A_5 answers True to the query. However, since it is the default module that is responsible for default inferences about P, and since n neither satisfies $M.P^+$-coordinate of (13.2) nor the $M.P^-$-coordinate of (13.2), we conclude that the answer to the query $P(n)$ is Unknown. ■

What typifies the above examples is that agents reasoning by default obtained all the relevant information from the extensional database. The next example shows that the situation can be more complex.

Example 1.1.3 Consider the theory T given by (here B, F, W and t stand for $Bird$, $Flies$, $HasWings$ and $Tweety$, respectively)

$$T = \langle \{B(t)\}, \{B(x) : F(x)/F(x), F(x) : W(x)/W(x)\} \rangle.$$

T leads to the diagram given in Figure 13.3.

Note that there is an arrow between A_4 and A_5. This is because A_4 is responsible for the relation F that is used by A_5 while computing the relation W.

The set of rules associated with the diagram is the following.

$$A_1.B^+(t)$$

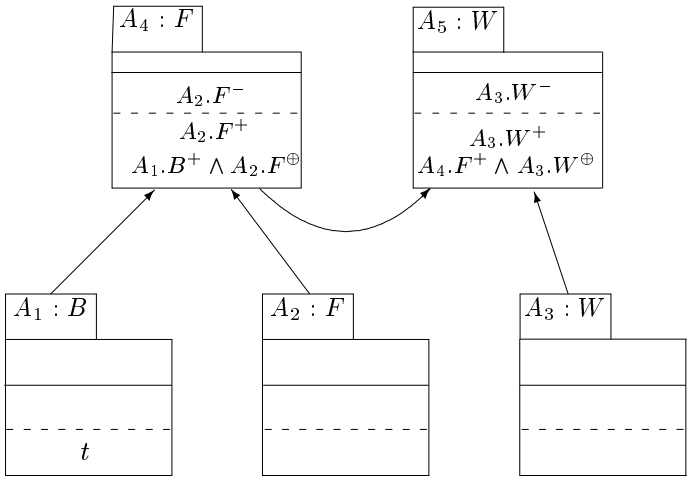

Figure 1.3: Diagram corresponding to the theory of Example 13.1.3.

$$A_2.F^-(x) \rightarrow A_4.F^-(x)$$
$$[A_2.F^+(x) \vee [A_1.B^+(x) \wedge \neg A_2.F^-(x)]] \rightarrow A_4.F^+(x)$$
$$A_3.W^-(x) \rightarrow A_5.W^-(x)$$
$$[A_3.W^+(x) \vee [A_4.F^+(x) \wedge \neg A_3.W^-(x)]] \rightarrow A_5.W^+(x).$$

The relations $A_1.B^+$, $A_4.F^-$, $A_4.F^+$, $A_5.W^-$ and $A_5.W^+$, occurring in the heads of the rules, are defined by the fixpoint formula given by

$$\mathsf{lfp}\ A_1.B^+, A_4.F^-, A_4.F^+, A_5.W^- A_5.W^+.\mathcal{R} \tag{1.3}$$

where $\mathcal{R}$ denotes the conjunction of the rules.

The computation of the relations characterized by the formula (13.3) is given below.

$$\{\}$$
$$\{A_1.B^+(t)\}$$
$$\{A_1.B^+(t), A_4.F^+(t)\}$$
$$\{A_1.B^+(t), A_4.F^+(t), A_5.W^+(t)\}.$$

Suppose the query we are interested in is $W(t)$. Since t satisfies the $A_5.W^+$-coordinate of (13.3), we conclude that the answer to the query $W(t)$ is True.

The next example shows that our version of default logic does not admit ungrounded conclusions.

Example 1.1.4 Consider the theory

$$T = \langle \emptyset, \{B(x) : F(x)/F(x), F(x) : B(x)/B(x)\}\rangle.$$

(Here B and F stand for *Bird* and *Flies*, respectively.) Figure 13.4 presents a diagram corresponding to T.

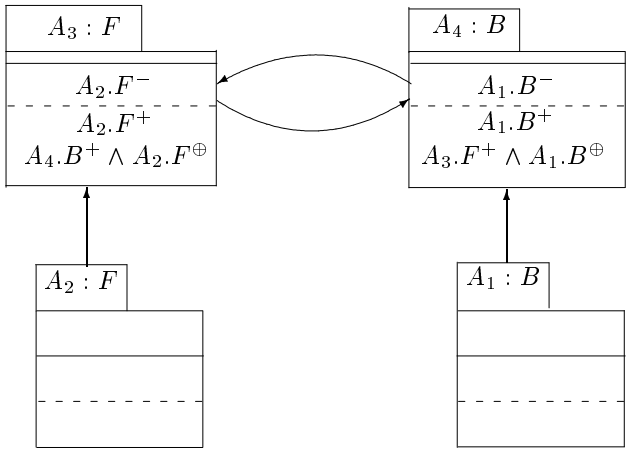

Figure 1.4: Diagram corresponding to the theory of Example 13.1.4.

The following set of rules is attached to the diagram:

$$A_2.F^-(x) \rightarrow A_3.F^-(x)$$
$$[A_2.F^+(x) \vee [A_4.B^+(x) \wedge \neg A_2.F^-(x)]] \rightarrow A_3.F^+(x)$$
$$A_1.B^-(x) \rightarrow A_4.B^-(x)$$
$$[A_1.B^+(x) \vee [A_3.F^+(x) \wedge \neg A_1.B^-(x)]] \rightarrow A_4.B^+(x).$$

The relations occurring in the heads of the rules are specified by the fixpoint formula of the form

$$\mathsf{lfp}\ A_3.F^-, A_3F^+, A_4.B^-, A_4.B^+.\mathcal{R} \tag{1.4}$$

where $\mathcal{R}$ denotes the conjunction of the rules.

Suppose that the query we are interested in is $B(t)$. It is immediately seen that all the relations characterized by the formula (13.4) have empty extensions.[8] Accordingly, the answer to the query is Unknown. ■

[8] Recall that the extension of an n-ary relation is the set of all n-tuples satisfying this relation.

Example 1.1.5 Consider the theory

$$T = \langle \{L(j,b)\}, \{L(x,y) : H(y)/H(y)\} \rangle.$$

(Here L, H, j and b stand for *Likes*, *Happy*, *John* and *Bill*, respectively.) Figure 13.5 presents a diagram corresponding to T.

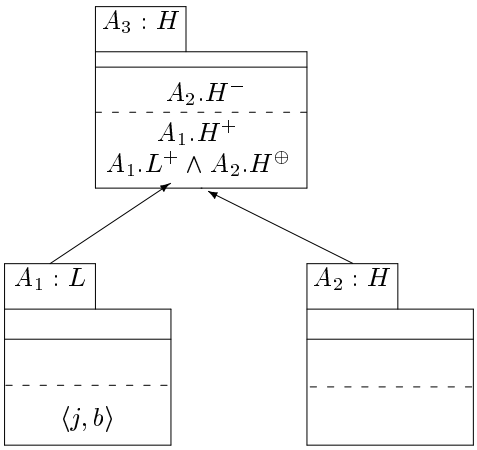

Figure 1.5: Diagram corresponding to the theory of Example 13.1.5.

The rules associated with the diagram are the following.

$$A_1.L^+(j,b)$$
$$A_2.H^-(x) \rightarrow A_3.H^-(x)$$
$$\exists y.[A_1.H^+(x) \vee [A_1.L^+(x,y) \wedge \neg A_2.H^-(x)]] \rightarrow A_3.H^+(x).$$

The relations occurring in the heads of the rules are defined by the fixpoint formula given by

$$\mathsf{lfp}\ A_1.L^+, A_3.H^-, A_3.H^+.\mathcal{R} \tag{1.5}$$

where, as usual, $\mathcal{R}$ is the conjunction of all the rules.

Below, is the computations of the relations mentioned in the formula (13.5).

$$\{\}$$
$$\{A_1.L^+(j,b)\}$$
$$\{A_1.L^+(j,b), A_3.H^+(j,b)\}.$$

Suppose that the query under consideration is $H(b)$. Since b satisfies the $A_3.H^+$-coordinate of (13.5), the answer to the query $H(b)$ is True. ■

1.2 Computing Normal Default Theories

We are now ready to describe our version of default logic, referred to as *rough default logic*, in detail. We start with normal default theories.[9]

Two defaults are said to be *similar* if their consequents contain the same relation symbol.

Definition 1.2.1 [Diagram] Let $T = \langle D, W \rangle$ be a normal default theory over a language $\mathcal{L}$. A *diagram* corresponding to T is constructed as follows.

1. For each relation symbol R occurring in $\mathcal{L}$, we construct a granular agent, called *database agent*, representing R. If $R(\overline{t})$ (resp. $\neg R(\overline{t})$) is in W, the tuple $\overline{t}$ occurs in the positive (resp. negative) part of the diagram of the agent. All other tuples of constants are assumed to be in the boundary region of the diagram.[10] Each database agent is labeled by $name : R$, where $name$ is a unique name of the agent and R is the relation symbol it is responsible for.

2. For each default d from D, we construct a granular agent, called *default agent*, representing d. The agent is labeled by $name : R$, where $name$ is a unique name of the agent and R is the relation symbol occurring in the consequent of d. Methods used by default agents will be specified later.

3. All default agents, say $A_1, \ldots A_k$, where $k > 1$, representing similar defaults are grouped into a single knowledge module, referred to as a *default module*. The module is labeled by $name : R$, where $name$ is a unique name of the module and R is the relation symbol the agents $A_1, \ldots A_k$ are responsible for. In what follows, any default agent which is not embedded in a default module will be referred to as an *independent default agent*. Otherwise, the agent will be referred to as a *dependent default agent*.

4. Let A be an independent default agent representing a default d and suppose that the consequent of d contains a relation symbol R. Assume further that $R_1, \ldots, R_k$ are all the relation symbols occurring in the prerequisite of d. For each $1 \leq i \leq k$, we put an arrow to A from a database agent representing R_i and, if exists, from a module (or independent default agent) representing R_i. We also put an arrow to A from a database agent representing R.

[9] Recall that all the theories considered in this chapter are disjunction-free default theories with consistent sets of axioms.

[10] Recall that the boundary part of a diagram is not explicitly drawn.

5. Let M be a module representing a relation R. Let $A_1, \ldots, A_n$ be default agents of M and suppose that A_i corresponds to a default d_i. Assume further that $R_1, \ldots, R_k$ are all the relation symbols occurring in the prerequisites of $d_1, \ldots, d_n$. For each $1 \leq i \leq k$, we put an arrow to M from a database agent representing R_i and, if exists, from a module (or independent default agent) representing R_i. Moreover, we put an arrow to M from a database agent representing R.

6. To each default agent (depended or not) we now assign methods it will employ. Assume that the agent represents a default of the form $l_1 \wedge \cdots \wedge l_n : l/l$, where $n \geq 0$, and suppose that $R_1, \ldots, R_n, R$ are the relation symbols occurring in $l_1, \ldots, l_n, l$, respectively. First of all, there are two methods, namely $A_k.R^+$ and $A_k.R^-$, where A_k is the database agent responsible for the relation R. These methods, which will be referred to as *database methods*, are placed in the positive and negative parts of the diagram, respectively. In addition, there is one method corresponding to the default represented by the agent. This method, referred to as a *default method*, is placed in the positive part of the agent's diagram if l is a positive literal and in the negative part otherwise. The default method is constructed as follows.

   (a) Suppose first that none of $R_1, \ldots, R_n$ is represented either by a default module or by an independent default agent. Then the default method is given by

   $$A_1.X_1 \wedge \cdots A_n.X_n \wedge A.X, \tag{1.6}$$

   where $A_1, \ldots, A_n, A$ are database agents responsible for $R_1, \ldots, R_n, R$, respectively, each X_i is ${R_i}^+$ (if l_i is positive) or ${R_i}^-$ (if l_i is negative) and X is $R^{\oplus}$ (if l is positive) or $R^{\ominus}$ (if l is negative).

   (b) Suppose now that at least one of $R_1, \ldots, R_n$ is represented by a default module or by an independent default agent. Let $R_i, \ldots R_j$ be all relations from $R_1, \ldots, R_n$ that are represented by default modules (independent default agents) and suppose that $M_i, \ldots, M_j$ are the corresponding modules (independent default agents). Then the default method is (13.6) with $A_i, \ldots, A_j$ replaced by $M_i, \ldots, M_j$, respectively. ■

The set of rules corresponding to a diagram of a default theory is specified using CAKE's standard methodology. (See Chapter 12, section 12.5.3.) We also use the standard voting policy to resolve conflicts, i.e. adjudicating agents are not included in default modules.

Theorem 1.2.2 The set of rules R corresponding to a diagram $\mathcal{D}$ of a normal default theory is stratified.

Proof Let R_{dba}, R_{da} and R_m be sets of rules associated with database agents of $\mathcal{D}$, default agents of $\mathcal{D}$ and default modules of $\mathcal{D}$, respectively. It is easily

verified that the relation $\preceq$ specified by

$$R_{dba} \preceq R_{da} \preceq R_m$$

provides a stratification of R.

We shall need the following notion.

Definition 1.2.3 [Provability] Let T be a normal default theory and suppose that $\mathbf{R}$ is the set of rules corresponding to the diagram of T. Let $\mathcal{R}$ be the conjunction of the rules from $\mathbf{R}$ and assume that $R_1, \ldots, R_k$ are all relation symbols occurring in the heads of the rules. Consider the fixpoint formula given by

$$\mathsf{lfp}\ R_1, \ldots, R_k.\mathcal{R} \tag{1.7}$$

A ground literal of the form $P(\overline{c})$ (resp. $\neg P(\overline{c})$) is said to be *provable* in T iff $\overline{c}$ satisfies the R_i-coordinate of (13.7), R_i is of the form $A.P^+$ (resp. $A.P^-$) and A is the name associated with a database agent, an independent default agent or a default module.

The central concept of default logic is that of an extension of a default theory. In rough default logic this notion can be defined as follows.

Definition 1.2.4 [Extension of default theory] Let T be a normal default theory. An *extension* of T is the set of all ground literals which are provable in T.

In view of earlier examples, one may suspect that a ground literal $l(\overline{t})$ belongs to an extension of T iff $l(\overline{t})$ is a member of every extension of T in Reiter's default logic. However, as the next example shows, this is not always the case.

Example 1.2.5 Consider the theory $T = \langle W, D\rangle$, where

$$W = \{Q(n), R(n)\}$$

$$D = \left\{ \frac{Q(x) : P(x)}{P(x)},\ \frac{R(x) : \neg P(x)}{\neg P(x)},\ \frac{P(x) : V(x)}{V(x)},\ \frac{\neg P(x) : V(x)}{V(x)} \right\}.$$

This is an extended version of the "Nixon" theory with R, P, Q, V, n standing for $Republican$, $Pacifist$, $Quaker$, $Voter$ and $Nixon$, respectively. Figure 13.6 represents the diagram corresponding to T.

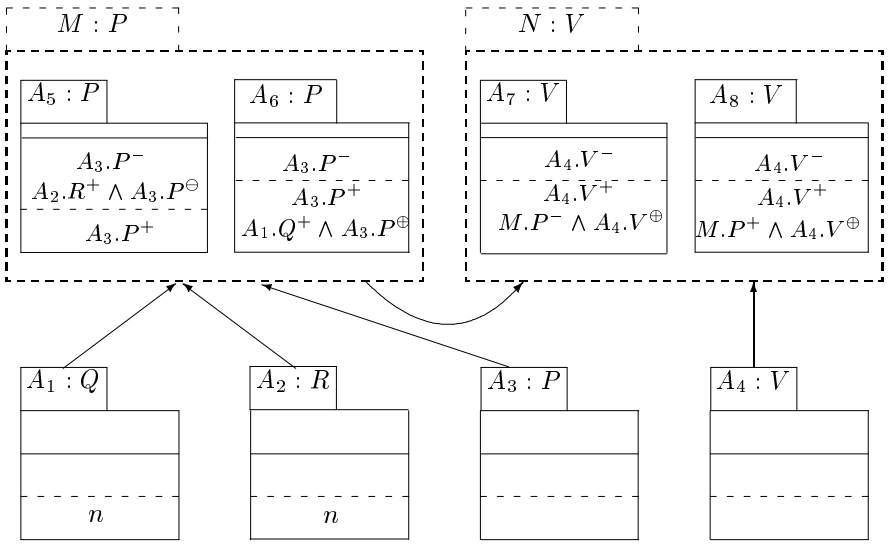

Figure 1.6: Diagram corresponding to the theory of Example 13.2.5.

In Reiter's default logic T has two extensions, E_1 and E_2, given by

$$E_1 = Cn(\{Q(n), R(n), V(n), P(n)\})$$
$$E_2 = Cn(\{Q(n), R(n), V(n), \neg P(n)\}).$$

Note that the literal $V(n)$ is a member of both E_1 and E_2. However, in rough default logic the literal $V(n)$ does not belong to the extension. To see this, consider the set of rules associated with the diagram of the theory T:

$$A_1.Q^+(n)$$
$$A_2.R^+(n)$$
$$[A_3.P^-(x) \vee [A_2.R^+(x) \wedge \neg A_3.P^+(x)]] \rightarrow A_5.P^-(x)$$
$$A_3.P^+(x) \rightarrow A_5.P^+(x)$$
$$A_3.P^-(x) \rightarrow A_6.P^-(x)$$
$$[A_3.P^+(x) \vee [A_1.Q^+(x) \wedge \neg A_3.P^-(x)]] \rightarrow A_6.P^+(x)$$
$$A_4.V^-(x) \rightarrow A_7.V^-(x)$$
$$[A_4.V^+(x) \vee [M.P^-(x) \wedge \neg A_4.V^-(x)]] \rightarrow A_7.V^+(x)$$
$$A_4.V^-(x) \rightarrow A_8.V^-(x)$$
$$[A_4.V^+(x) \vee [M.P^+(x) \wedge \neg A_4.V^-(x)]] \rightarrow A_8.V^+(x)$$
$$[A_5.P^-(x) \vee A_6.P^-(x)] \wedge \neg[A_5.P^+(x) \vee A_6.P^+(x)] \rightarrow M.P^-(x)$$
$$[A_5.P^+(x) \vee A_6.P^+(x)] \wedge \neg[A_5.P^-(x) \vee A_6.P^-(x)] \rightarrow M.P^+(x)$$

$$[A_7.V^-(x) \vee A_8.V^-(x)] \wedge \neg[A_7.V^+(x) \vee A_8.V^+(x)] \rightarrow N.V^-(x)$$
$$[A_7.V^+(x) \vee A_8.V^+(x)] \wedge \neg[A_7.V^-(x) \vee A_8.V^-(x)] \rightarrow N.V^+(x).$$

The last four rules represent the standard voting mechanism used by the modules M and N.

The relations occurring in the heads of the above rules are defined by the following fixpoint formula:

$$\mathsf{lfp}\ \overline{X}.\mathcal{R} \tag{1.8}$$

where $\overline{X}$ is the tuple $A_1.Q^+, A_2R^+, A_5.P^-, A_5.P^+, A_6.P^-, A_6.P^+, A_7.V^-, A_7.V^+, A_8.V^-, A_8.V^+, M.P^-, M.P^+, N.V^-, N.V^+$ and $\mathcal{R}$ is the conjunction of all the rules.

The computation of the relations defined by the formula (13.8) is given below.

$$\{\}$$
$$\{A_1.Q^+(n), A_2.R^+(n)\}$$
$$\{A_1.Q^+(n), A_2.R^+(n), A_5.P^-(n), A_6.P^+(n)\}.$$

Now, it is immediately seen that n does not satisfy the $N.V^+$-coordinate of (13.8). Accordingly, $V(n)$ is not provable in the theory T and hence $V(n)$ does not belong to the extension of T. ■

The above examples should give the reader some feeling of how our formalism works. The crucial observation is that defaults whose consequents refer to the same relation symbol are grouped in a module. In contrast to Reiter's logic, where these defaults can interact individually, in our formalism the interaction goes via modules.

The next two results are straightforward.

Theorem 1.2.6 In rough default logic every normal default theory has one extension. ■

Theorem 1.2.7 In rough default logic every normal default theory with consistent set of axioms has consistent extension. ■

It should be noticed that the property of semi-monotonicity generally does not hold in rough default logic. To see this, consider the normal theory $T = \langle W, D\rangle$, where $W = \{Q(n), R(n)\}$ and $D = \{Q(x) : P(x)/P(x)\}$. The extension of this theory is $E = \{Q(n), R(n), P(n)\}$. Now, if we add to D a new default $R(x) : \neg P(x)/\neg P(x)$, the extension of the expanded theory is $E' = \{Q(n), R(n)\}$.

1.3 Default Logic with Strong Prerequisites

Both standard default logic and our approach introduced in the previous section lack tools to distinguish between facts known to be true, i.e. derivable from axioms, and those believed to be true, i.e. derivable by applying defaults. In consequence, both these formalisms are unable to properly deal with many practically occurring settings. The next example will help to illustrate this.

Example 1.3.1 Suppose we are given:

> John is a person.
>
> Normally, a person is honest.
>
> Normally, a person known to be honest can be safely lent money.

Given these facts, we are prepared to believe that John is honest, but we do not want to infer that he can be safely lent money. The reason, of course, is that John's honesty is a default conclusion rather, than an iron-clad fact.

This simple common-sense facts cannot be properly formalized in the framework of default logic we have introduced. The best one can do is to use the default theory T given by

$$W = \{P(j)\} \qquad D = \left\{ \frac{P(x) : H(x)}{H(x)}, \ \frac{H(x) : M(x)}{M(x)} \right\}.$$

where P, H and M stand for "Person", "Honest" and "Can be safely lent money", respectively.

Figure 13.7 represents the diagram corresponding to T.

It is easily checked that, contrary to our intention, the literal $M(j)$ is a member of the extension of T.

Fortunately, CAKE method gives us a simple way to block the unwanted conclusion. All we have to do is to remove the arrow from A_4 to A_5, put an additional arrow from A_2 to A_5 and to replace the method $A_4.H^+ \wedge A_3.M^{\oplus}$, assigned to A_5, by $A_2.H^+ \wedge A_3.M^{\oplus}$ (see Figure 13.8).

This simple trick makes the second default inapplicable for John, because his honesty cannot be inferred directly from the database. ■

We now formalize the above idea in a systematic way by introducing a new variant of rough default logic. This variant, called *rough default logic with strong prerequisites* SP default logic (SP rough default logic, for short), explicitly distinguishes between facts known to be true and those believed to be true.

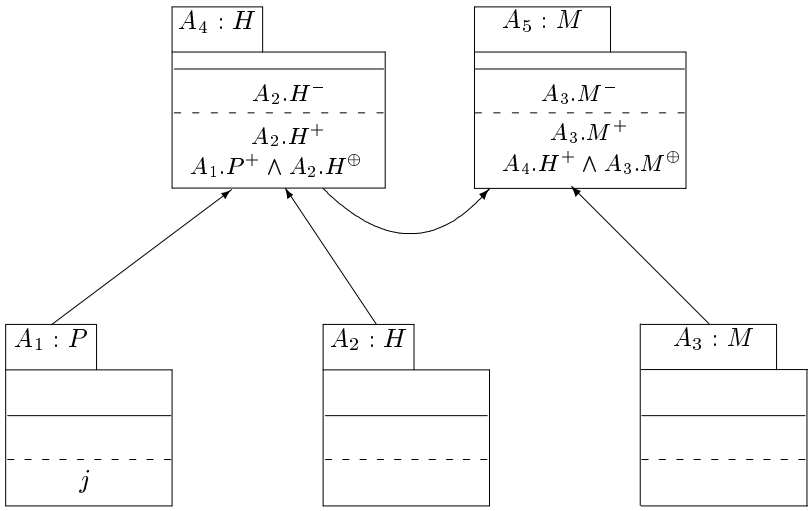

Figure 1.7: Diagram corresponding to the theory of Example 13.3.1.

We extend the language of rough default logic by adding a new operator **K** which can be applied to literals. A formula of the form **K**l is to be read "l is known to be true".[11]

As before, we limit ourselves to disjunction-free default theories. However, in SP rough default logic any conjunct occurring in the prerequisite a of a default can be preceded by the operator **K**. More specifically, we consider defaults of the form

$$\frac{B_1 \wedge \cdots \wedge B_n : l}{l}$$

where each B_i is either a literal or a literal preceded by the operator **K** and l is a literal.

The formal difference between rough and SP rough default logic lies in the definition of a diagram. In the latter, any conjunct of the form **K**l occurring in the prerequisite of a default must be supported by the axioms of the theory under consideration, rather than by the consequent of an applied default.

Definition 1.3.2 Let T be a disjunction-free normal default theory of SP rough default logic. A diagram corresponding to T is constructed as before (see Definition 13.2.1) with points 4, 5 and 6 replaced by:

[11] It should be emphasized that the **K** operator we use here is a syntactic sugar rather than the knowledge operator employed in modal logics.

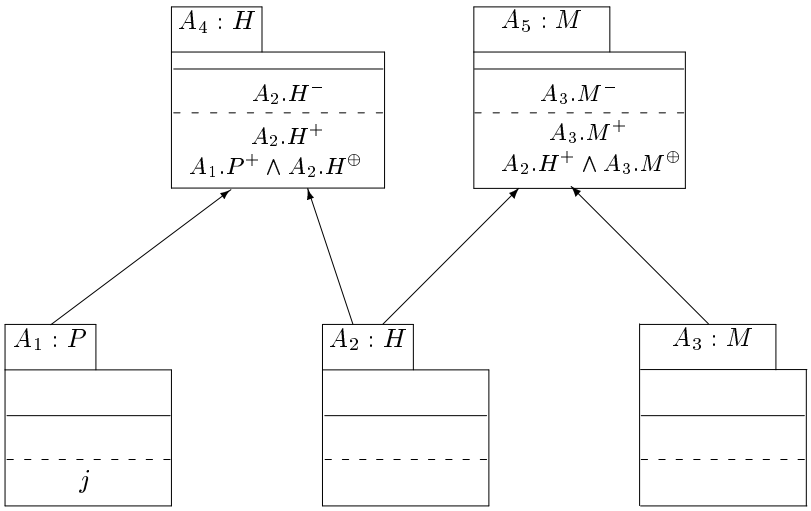

Figure 1.8: Modified diagram corresponding to the theory of Example 13.3.1.

4. Let A be an independent default agent representing a default d and suppose that the consequent of d contains a relation symbol R. Assume further that $R_1,\ldots,$ R_k are all the relation symbols occurring in the prerequisite of d and let $R'_1,\ldots R'_l$ be those elements from $R_1,\ldots,$ R_k that are not preceded by the operator $\mathbf{K}$. For each $1 \leq i \leq k$, we put an arrow to A from a database agent representing R_i and, for each $1 \leq i \leq l$, we put an arrow to A from a module (or independent default agent), if exists, representing R'_i. We also put an arrow to A from a database agent representing R.

5. Let M be a module representing a relation R. Let A_1, $\ldots$, A_n be default agents of M and suppose that A_i corresponds to a default d_i. Assume further that $R_1,\ldots,$ R_k are all the relation symbols occurring in the prerequisites of $d_1,\ldots,d_n$ and $R'_1,\ldots,R'_l$ are those elements from $R_1,\ldots,$ R_k that are not preceded by the operator $\mathbf{K}$. For each $1 \leq i \leq k$, we put an arrow to M from a database agent representing R_i and, for each $1 \leq i \leq l$, we put an arrow to M from a module (or independent agent), if exists, representing R'_i. Moreover, we put an arrow to M from a database agent representing R.

6. To each default agent we assign methods it will employ. Assume that the agent represents a default of the form $B_1 \wedge \cdots \wedge B_n : l/l$, where $n \geq 0$, and suppose that $R_1,\ldots,R_n,R$ are relation symbols occurring in $l_1,\ldots,l_n,l$, respectively. Assume further that $B'_1,\ldots,B'_k$ are those conjuncts of $B_1 \wedge \cdots \wedge B_n$ which are not preceded by $\mathbf{K}$ and let $R'_1,\ldots,R'_k$ be

the relation symbols occurring in $B'_1, \ldots, B'_k$. First of all, there are two database methods, namely $A'.R^+$ and $A'.R^-$, where A' is the database agent responsible for the relation R. These methods are placed in the positive and the negative part of the diagram, respectively. In addition, there is one default method corresponding to the default represented by the agent. This method is placed in the positive part of the agent's diagram if l is positive and in the negative part otherwise. The default method is specified as follows.

(a) Suppose first that none of $R'_1, \ldots, R'_k$ is represented either by a default module or by an independent default agent. Then the default method used by the agent is given by

$$A_1.X_1 \wedge \cdots A_n.X_n \wedge A'.X, \tag{1.9}$$

where $A_1, \ldots, A_n, A'$ are database agents responsible for $R_1, \ldots, R_n, R$, respectively, each X_i is ${R_i}^+$ (if l_i is positive) or ${R_i}^-$ (if l_i is negative) and X is $R^\oplus$ (if l is positive) or $R^\ominus$ (if l is negative).

(b) Suppose now that at least one of $R'_1, \ldots, R'_k$ is represented by a default module or by an independent default agent. Let $R'_i, \ldots R'_j$ be all relation symbols from $R'_1, \ldots, R'_k$ that are represented by default modules (independent default agents) and suppose that $M_i, \ldots, M_j$ are the corresponding modules (independent default agents). Then the default method used by the agent is (13.9) with $A_i, \ldots, A_j$ replaced by $M_i, \ldots, M_j$, respectively. ■

The set of rules corresponding to a diagram of a normal default theory of SP rough default logic is specified as usual. We also use the standard voting policy.

It is obvious that the set of rules corresponding to a normal default theory of rough default logic is stratified.

The notion of provability in a normal default theory and the notion of an extension of a normal default theory are specified as before. (See Definitions 13.2.3 and 13.2.4.

Example 1.3.3 Suppose we are given:

John is a person.

Normally, a person is honest.

Normally, if a person is believed to be honest, she/he can be safely lent a small amount of money.

Normally, if a person is known to be honest, she/he can be safely lent a large amount of money.

In SP rough default logic these common-sense facts can be naturally represented by the following theory T:

$$W = \{P(j)\}$$
$$D = \left\{ \frac{P(x) : H(x)}{H(x)},\ \frac{H(x) : SM(x)}{SM(x)},\ \frac{\mathbf{K}H(x) : LM(x)}{LM(x)} \right\}.$$

where j, P, H, SM and LM stand for "John", "Person", "Honest", "Can be safely lent a small amount of money" and "Can be safely lent a large amount of money", respectively.

The diagram corresponding to T is presented in Figure 13.9.

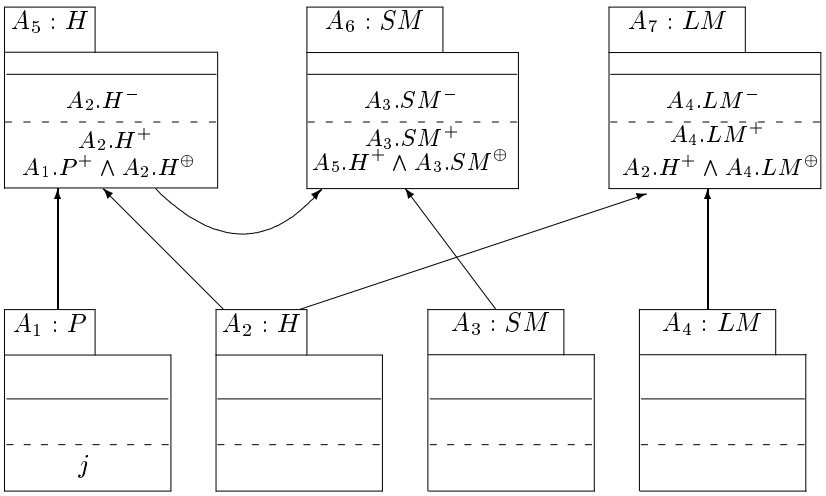

Figure 1.9: Diagram corresponding to the theory of Example 13.3.3.

The set of rules corresponding to the diagram of the considered theory is the following.

$$A_1.P^+(j)$$
$$A_2.H^-(x) \to A_5.H^-(x)$$
$$[A_2.H^+(x) \vee [A_1.P^+(x) \wedge \neg A_2.H^-(x)]] \to A_5.H^+(x)$$
$$A_3.SM^-(x) \to A_6.SM^-(x)$$
$$[A_3.SM^+(x) \vee [A_5.H^+(x) \wedge \neg A_3.SM^-(x)]] \to A_6.SM^+(x)$$
$$A_4.LM^-(x) \to A_7.LM^-(x)$$
$$[A_4.LM^+(x) \vee [A_2.H^+(x) \wedge \neg A_4.LM^-(x)]] \to A_7.LM^+(x).$$

The relations occurring in the heads of the above rules are defined by the fixpoint formula given by

$$\mathsf{lfp}\ \overline{X}.\mathcal{R} \tag{1.10}$$

where $\overline{X}$ is the tuple $A_1.P^+$, $A_5.H^-$, $A_5.H^+$, $A_6.SM^-$, $A_6.SM^+$, $A_7.LM^-$, $A_7.LM^+$ and $\mathcal{R}$ is the conjunction of all the rules.

Below is the computation of the relations specified by the formula (13.10).

$\{\}$
$\{A_1.P^+(j)\}$
$\{A_1.P^+(j), A_5.H^+(j)\}$
$\{A_1.P^+(j), A_5.P^+(j), A_6.SM^+(j)\}$.

The queries we are interested in are $SM(j)$ and $LM(j)$. It is immediately seen that, according to our intuition, $SM(j)$ is in the extension of the theory T, whereas $LM(j)$ is not. ■

We conclude this section by observing that analogons of Theorems 13.2.6 and 13.2.7 obviously hold for SP rough default logic.

1.4 Prioritized Default Logic

Both rough default logic and SP rough default logic can be easily extended to their prioritized versions. (See Chapter 4, section 4.2.6.)

We start with some terminology.

Let $<$ be a strict partial order on a finite set X. For all $x, y \in X$, y is said to be a *predecessor* of x iff $x < y$.

Let $<$ be a strict partial order on a finite set X. With each $x \in X$ we associate a natural number, denoted by $depth(x)$, as follows. If x has no predecessors then $depth(x) = 1$. Otherwise, $depth(x) = 1 + max(depth(x_1), \ldots, depth(x_k))$, where $x_1, \ldots, x_k$ are all the members from X which are predecessors of x. Intuitively, $depth(x)$ is the length of the longest sequence $x_0, x_1, \ldots, x_k$ such that $x_0 = x$ and $x_i < x_{i+1}$, for $i = 0, 1, \ldots, k-1$.

Let $<$ be a strict partial order on a finite set X. We define a sequence $X_1, \ldots X_n$ of subsets of X by

$$X_i = \{x \in X : depth(x) = i\}$$

It is obvious that the sets $X_1, \ldots X_n$ form a partition of the set X. In the sequel, this partition will be referred to as $<$-*partition* of X.

Definition 1.4.1 By a normal prioritized theory of rough default logic (resp. SP rough default logic) we mean a triple $\langle D, W, < \rangle$, where $\langle D, W \rangle$ is a normal default theory of rough default logic (resp. SP rough default logic) and $<$ is a strict partial order on D. Intuitively, $d_1 < d_2$ means that d_2 has higher priority than d_1. ■

The only difference between rough default logic (resp. SP rough default logic) and prioritized version of rough default logic (resp. prioritized version of SP rough default logic) lies in different voting policy.

Definition 1.4.2 [Modified Voting Policy] Let M be a default module associated with a relation symbol R and suppose that $d_1, \ldots, d_n$ are all the defaults of the considered theory containing R in their consequents. Assume further that $D_1, \ldots, D_k$ is a $<$-partition of the set $\{d_1, \ldots, d_n\}$.

The modified voting process starts by applying the standard voting policy to all the agent's representing the defaults from D_1. If the answer is True or False, this is the final answer of the modified voting process. If the answer is Unknown because at least one agent representing the defaults from D_1 answered True and at least one of those agents answered False, the final answer of the modified voting process is also Unknown. Otherwise, i.e. if all the agents representing defaults from D_1 answered Unknown, the standard voting policy is applied to the agents representing the defaults from D_2. This may continue for $D_3, \ldots, D_k$ unless at least one agent answers True or False. ■

To specify the notion of a diagram of prioritized default logic, we need the following notation.

Suppose that $A_1, \ldots, A_n$ are default agents responsible for a relation R. We write $V^+(A_1, \ldots, A_n; R)$, $V^-(A_1, \ldots, A_n; R)$ and $V^\pm(A_1, \ldots, A_n; R)$ as the abbreviations for the formulas

$$[A_1.R^+ \vee \cdots \vee A_n.R^+] \wedge \neg[A_1.R^- \vee \cdots \vee A_n.R^-]$$
$$[A_1.R^- \vee \cdots \vee A_n.R^-] \wedge \neg[A_1.R^- \vee \cdots \vee A_n.R^-]$$
$$A_1.R^\pm \wedge \cdots \wedge A_n.R^\pm$$

respectively.[12]

Definition 1.4.3 Let $\langle D, W, < \rangle$ be a prioritized normal default theory of rough default logic (resp. SP rough default logic). The diagram corresponding to T is specified as before, see Definition 13.2.1 (resp. Definition 13.3.2) with the following additions.

[12] Observe that $V^+(A_1, \ldots, A_n; R)$ (resp. $V^-(A_1, \ldots, A_n; R)$) holds if the standard voting policy about R applied to the agents $A_1, \ldots, A_n$ returns the answer True (resp. False). The formula $V^\pm(A_1, \ldots, A_n; R)$ holds iff the answer of all the agents is Unknown.

7. Assume that agents $A_1, \ldots, A_n$, representing defaults $d_1, \ldots, d_n$, respectively, are grouped in a default module M. Let R be the relation symbol occurring in the consequents of $d_1, \ldots, d_n$ and let $D_1, \ldots, D_k$ be a $<$-partition of the set $\{d_1, \ldots, d_n\}$. If $k > 1$, then adjudicating agent, labeled by $name : R$, is introduced to the module M.[13] As usual, *name* is a unique name of the introduced agent. For each $1 \leq i \leq n$, we put an arrow to the adjudicating agent from A_i.

8. Methods used by the new adjudicating agent are specified as follows. Assume that, for each $1 \leq j \leq k$, $d_{j1}, d_{j2}, \ldots d_{j,i_j}$ are all the defaults from the set D_j. The following methods are put in the negative part of the diagram of the agent:

$$
\begin{aligned}
& V^-(A_{11}, \ldots, A_{1i_1}; R) \\
& V^{\pm}(A_{11}, \ldots, A_{1i_1}; R) \wedge V^-(A_{21}, \ldots, A_{2i_2}; R) \\
& \ldots \\
& V^{\pm}(A_{11}, \ldots, A_{1i_1}; R) \wedge \cdots \wedge V^{\pm}(A_{(k-1)1}, \ldots, A_{(k-1)i_{k-1}}; R) \wedge \\
& \qquad\qquad V^-(A_{k1}, \ldots, A_{ki_k}; R).
\end{aligned}
$$

   The following methods are put in the positive part of the diagram of the agent:

$$
\begin{aligned}
& V^+(A_{11}, \ldots, A_{1i_1}; R) \\
& V^{\pm}(A_{11}, \ldots, A_{1i_1}; R) \wedge V^+(A_{21}, \ldots, A_{2i_2}; R) \\
& \ldots \\
& V^{\pm}(A_{11}, \ldots, A_{1i_1}; R) \wedge \cdots \wedge V^{\pm}(A_{(k-1)1}, \ldots, A_{(k-1)i_{k-1}}; R) \wedge \\
& \qquad\qquad V^+(A_{k1}, \ldots, A_{ki_k}; R).
\end{aligned}
$$

The set of rules corresponding to a diagram of a prioritized normal default theory of rough default logic (SP rough default logic) is specified employing the usual CAKE's methodology.[14]

We have the following analogon of Theorem 13.2.2.

Theorem 1.4.4 The set of rules R corresponding to a diagram $\mathcal{D}$ of a prioritized normal default theory is stratified.

Proof Let R_{dba}, R_{da}, R_{adj} and R_m be sets of rules associated with database agents of $\mathcal{D}$, default agents of $\mathcal{D}$, adjudicating agents of $\mathcal{D}$ and default modules

[13] Note that $k > 1$ implies that there are priorities among the defaults represented in the module M.

[14] It should be emphasized that the presence of adjudicating agents changes the rules attached to modules. (See Chapter 12, section 12.5.3.)

of $\mathcal{D}$, respectively. It is easily verified that the relation $\preceq$ specified by

$$R_{dba} \preceq R_{da} \preceq R_{adj} \preceq R_m$$

provides a stratification of R. ■

The notion of provability in a normal default theory and the notion of an extension of a normal default theory are specified as before. (See Definitions 13.2.3 and 13.2.4.

Example 1.4.5 Consider the theory

$$W = \{A(j) \wedge FTS(j)\} \qquad D = \left\{ \frac{FTS(x) : \neg E(x)}{\neg E(x)}, \ \frac{A(x) : E(x)}{E(x)} \right\}.$$

This is non-prioritized theory from Example 4.2.23 (Chapter 4, section4.2.6) with A, FTS, E and j standing for "Adult", "FullTimeStudent", "Employed" and "John", respectively. The diagram corresponding to T is presented in Figure 13.10.

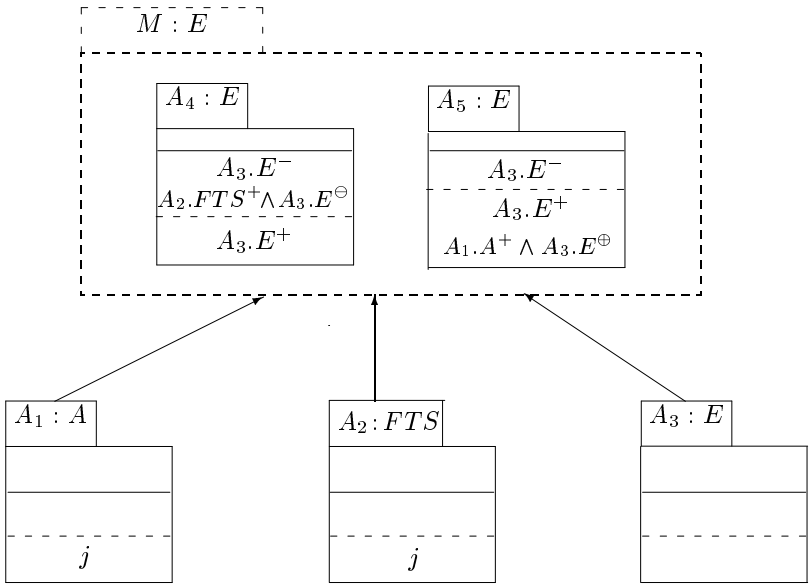

Figure 1.10: Diagram corresponding to the theory of Example 13.4.5.

Consider the query $E(j)$. It is easily checked that $E(j)$ is not a member of the extension of T. This is because the query is answered False by the agent A_4 and True by the agent A_5. Accordingly, using the standard voting policy, the final answer is Unknown. As we remarked earlier, see Example 4.2.23, this answer is intuitively problematic. Given that John is both a full time student and an adult the answer to the query should be False. To achieve this effect, we should use a prioritized version of rough default logic putting $\frac{A(x):E(x)}{E(x)} < \frac{FTS(x):\neg E(x)}{\neg E(x)}$.

The diagram of the prioritized theory is similar as before, but the module M should be replaced by that from Figure 13.11.

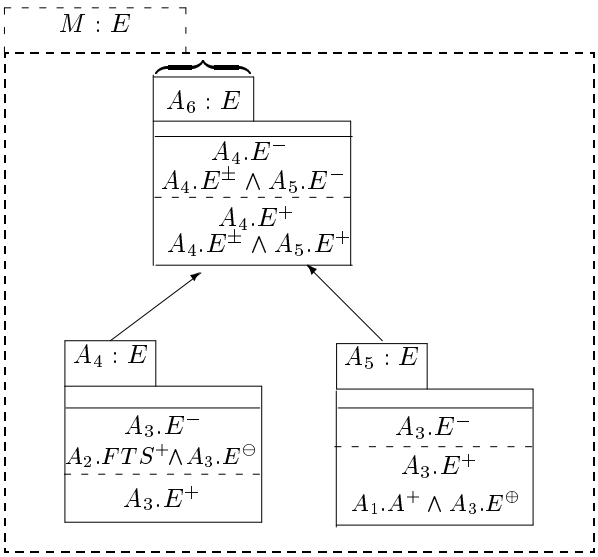

Figure 1.11: New module corresponding to the prioritized theory of Example 13.4.5.

The following set of rules is attached to the prioritized theory:

$$A_1.A^+(j)$$
$$A_2.FTS^+(j)$$
$$[A_3.E^-(x) \vee [A_2.FTS^+(x) \wedge \neg A_3.E^+(x)]] \rightarrow A_4.E^-(x)$$
$$A_3.E^+(x) \rightarrow A_4.E^+(x)$$
$$A_3.E^-(x) \rightarrow A_5.E^-(x)$$
$$[A_3.E^+(x) \vee [A_1.A^+(x) \wedge \neg A_3.E^-(x)]] \rightarrow A_5.E^+(x)$$
$$[A_4.E^-(x) \vee [\neg A_4.E^+(x) \wedge \neg A_4.E^-(x) \wedge A_5.E^-(x)]] \rightarrow A_6.E^-(x)$$

$$[A_4.E^+(x) \vee [\neg A_4.E^+(x) \wedge \neg A_4.E^-(x) \wedge A_5.E^+(x)]] \rightarrow A_6.E^+(x)$$
$$A_6.E^+(x) \rightarrow M.E^+(x)$$
$$A_6.E^-(x) \rightarrow M.E^-(x).$$

The relations occurring in the heads of the above rules are specified by the fixpoint formula given by

$$\mathsf{lfp}\ \overline{X}.\mathcal{R} \tag{1.11}$$

where $\overline{X}$ is the tuple $A_1.A^+$, $A_2.FTS^+$, $A_4.E^+$, $A_4.E^-$, $A_5.E^+$, $A_5.E^-$, $A_6.E^+$, $A_6.E^-$, $M.E^+$, $M.E^-$ and $\mathcal{R}$ is the conjunction of all the rules.

Computing the relations defined by the formula (13.11), we get

$$\{\}$$
$$\{A_1.A^+(j), A_2.FTS^+(j)\}$$
$$\{A_1.A^+(j), A_2.FTS^+(j), A_4.E^-(j), A_5.E^+(j)\}$$
$$\{A_1.A^+(j), A_2.FTS^+(j), A_4.E^-(j), A_5.E^+(j), A_6.E^-(j)\}$$
$$\{A_1.A^+(j), A_2.FTS^+(j), A_4.E^-(j), A_5.E^+(j), A_6.E^-(j), M.E^-(j)\}.$$

Since j satisfies the $M.E^-$-coordinate of (13.11), we conclude that $\neg E(j)$ belongs to the extension of the prioritized theory. ■

It is obvious that analogons of Theorems 13.2.6 and 13.2.7 hold for prioritized versions of rough default logic and SP rough default logic.

1.5 Semi-Normal Default Theories

In this section we investigate disjunction-free semi-normal non-prioritized default theories of rough default logic.[15]

Recall that a disjunction-free semi-normal default is any default of the form

$$\frac{l_1 \wedge \cdots \wedge l_n : l'_1 \wedge \cdots l'_k \wedge l}{l}$$

where $l_1, \ldots, l_n, l'_1, \ldots, l'_k, l$ are literals.

To deal with semi-normal default theories, we proceed analogously as before. The only difference is that additional conjuncts occurring in justifications of semi-normal defaults have to be taken under consideration. The next example will help to illustrate this.

[15] The considerations of this section can be easily generalized to SP rough default logic and to prioritized versions of both rough default logic and SP rough default logic.

Example 1.5.1 Consider the theory

$$W = \{D(b)\} \qquad D = \left\{ \frac{D(x) : A(x)}{A(x)}, \ \frac{A(x) : \neg D(x) \wedge E(x)}{E(x)} \right\}.$$

Here A, D, E, and b stand for "Adult", "HighSchoolDropout", "Employed" and "Bill", respectively. (See Example 4.2.17, Chapter 4, section 4.2.4.)

The diagram corresponding to T is presented in Figure 13.12.

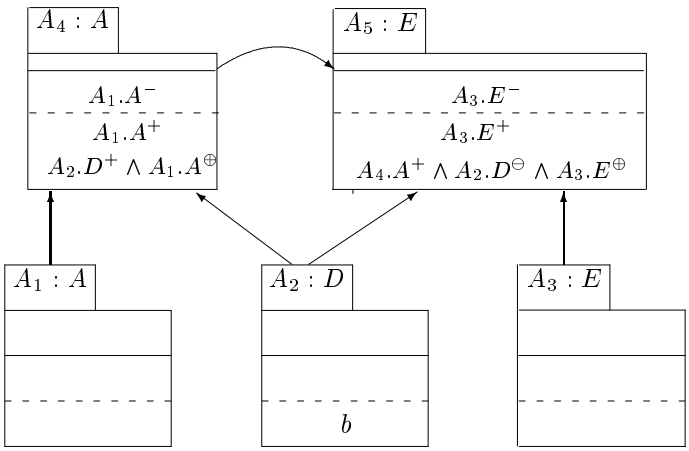

Figure 1.12: Diagram corresponding to the theory of Example 13.5.1.

Consider the agent A_5 representing the semi-normal default of our theory. Note that its default method refers to the relation D which occurs in the justification of the default.

The set of rules attached to the above diagram is the following.

$$A_2.D^+(b)$$
$$A_1.A^-(x) \rightarrow A_4.A^-(x)$$
$$[A_1.A^+(x) \vee [A_2.D^+(x) \wedge \neg A_1.A^-(x)]] \rightarrow A_4.A^+(x)$$
$$A_3.E^-(x) \rightarrow A_5.E^-(x)$$
$$[A_3.E^+(x) \vee [A_4.A^+(x) \wedge \neg A_2.D^+(x) \wedge \neg A_3.E^-(x)]] \rightarrow A_5.E^+(x).$$

It is easily verified that the above set of rules is stratified. The fixpoint formula specifying the relations occurring in the heads of the rules is given by

$$\mathsf{lfp}\ A_2.D^+, A_4.A^-, A_4.A^+, A_5.E^-, A_5.E^+.\mathcal{R} \tag{1.12}$$

where $\mathcal{R}$ denotes the conjunction of the rules.

Below is the computation of the relations specified by the formula (13.12).

$$\{\}$$
$$\{A_2.D^+(b)\}$$
$$\{A_2.D^+(b), A_4.A^+(b)\}.$$

Suppose that the query of interest is $E(b)$. It is immediately seen that, according to our expectations, b neither satisfies the $A_5.E^+$-coordinate nor the $A_5.E^-$-coordinate of the fixpoint formula (13.12). Accordingly, the answer to the query $E(b)$ is Unknown. ■

To deal with semi-normal default theories, we have to adjust the notion of a diagram.

Definition 1.5.2 Let T be a disjunction-free semi-normal default theory. A diagram corresponding to T is constructed as that from Definition 13.2.1 with points 4, 5 and 6 replaced by:

4. Let A be an independent default agent representing a default d and suppose that the consequent of d contains a relation symbol R. Assume further that $R_1,\ldots,$ R_k are all the relation symbols occurring in the prerequisite of d and let $R_{k+1},\ldots,R_m$ be all the relation symbols, except R, occurring in the justification of d. For each $1 \leq i \leq m$, we put an arrow to A from a database agent representing R_i and, if exists, from a module (or independent default agent) representing R_i. We also put an arrow to A from a database agent representing R.

5. Let M be a module representing a relation R. Let $A_1, \ldots, A_p$ be default agents of M and suppose that A_i corresponds to a default d_i. Assume further that $R_1,\ldots,$ R_k are all the relation symbols occurring in the prerequisites of $d_1,\ldots,d_p$ and let $R_{k+1},\ldots,R_m$ be all the relation symbols, except R, occurring in the justifications of $d_1,\ldots,d_p$. For each $1 \leq i \leq m$, we put an arrow to M from a database agent representing R_i and, if exists, from a module (or independent default agent) representing R_i. Moreover, we put an arrow to M from a database agent representing R.

6. To each default agent (depended or not) we now assign methods it will employ. Assume that the agent represents a default of the form $l_1 \wedge \cdots \wedge l_n : l_{n+1} \wedge \cdots \wedge l_m \wedge l / l$, where $n \geq 0$, and suppose that $R_1,\ldots,R_n,R_{n+1},\ldots,R_m$, R are the relation symbols occurring in $l_1,\ldots,l_n,l_{n+1},\ldots,l_m,l$, respectively. First of all, there are two methods, namely $A_k.R^+$ and $A_k.R^-$, where A_k is the database agent responsible for the relation R. These methods are placed in the positive and negative parts of the diagram, respectively. In addition, there is one method corresponding to the default

represented by the agent. This method, is placed in the positive part of the agent's diagram if l is a positive literal and in the negative part otherwise. The default method is constructed as follows.

(a) Suppose first that none of $R_1, \ldots, R_n, R_{n+1}, \ldots, R_m$ is represented either by a default module or by an independent default agent. Then the default method is given by

$$A_1.X_1 \wedge \cdots A_n.X_n \wedge A_{n+1}.X_{n+1} \wedge \cdots \wedge A_m.X_m \wedge A.X \quad (1.13)$$

where $A_1, \ldots, A_n, A_{n+1}, \ldots A_m, A$ are database agents responsible for $R_1, \ldots, R_n, R_{n+1}, \ldots, R_m, R$, respectively, each X_i $(1 \leq i \leq n)$ is ${R_i}^+$,if l_i is positive, or ${R_i}^-$, if l_i is negative, each X_i $(n+1 \leq i \leq m)$ is $R_i^{\oplus}$, if l_i is positive, or $R_i^{\ominus}$, if l_i is negative. Finally, X is $R^{\oplus}$, if l is positive, or $R^{\ominus}$, if l is negative.

(b) Suppose now that at least one of $R_1, \ldots, R_n, R_{n+1}, \ldots, R_m$ is represented by a default module or by an independent default agent. Let $R_i, \ldots R_j$ be all relations from $R_1, \ldots, R_n, R_{n+1}, \ldots, R_m$ that are represented by default modules (independent default agents) and suppose that $M_i, \ldots, M_j$ are the corresponding modules (independent default agents). Then the default method is (13.13) with $A_i, \ldots, A_j$ replaced by $M_i, \ldots, M_j$, respectively.■

It is natural to ask whether the rules corresponding to a diagram of semi-normal theories are stratified. Unfortunately, as the next example shows, this is not always the case.

Example 1.5.3 Consider the theory T consisting of the empty set of axioms and the following triple of semi-normal defaults:

$$d_1 = \frac{: P(a) \wedge \neg Q(a)}{\neg Q(a)} \qquad d_2 = \frac{: Q(a) \wedge \neg R(a)}{\neg R(a)} \qquad d_3 = \frac{: R(a) \wedge \neg P(a)}{\neg P(a)}.$$

This is the theory from Example 4.2.19 (Chapter 4, section 4.2.4). As we have observed, the theory has no extension in Reiter's default logic. The reason is that applying any one default forces the application of one of the other two, but applying the latter contradicts the justification of this already applied. The diagram corresponding to T is presented in Figure 13.13.

The set of rules corresponding to the above diagram is the following.

$$[A_1.P^-(x) \vee [\neg A_5.R^-(x) \wedge \neg A_1.P^+(x)]] \rightarrow A_4.P^-(x)$$
$$A_1.P^+(x) \rightarrow A_4.P^+$$
$$[A_2.R^-(x) \vee [\neg A_6.Q^-(x) \wedge \neg A_2.R^+(x)]] \rightarrow A_5.R^-(x)$$
$$A_2.R^+(x) \rightarrow A_5.R^+$$
$$[A_3.Q^-(x) \vee [\neg A_4.P^-(x) \wedge \neg A_3.Q^+(x)]] \rightarrow A_6.Q^-(x)$$
$$A_3.Q^+(x) \rightarrow A_6.Q^+.$$

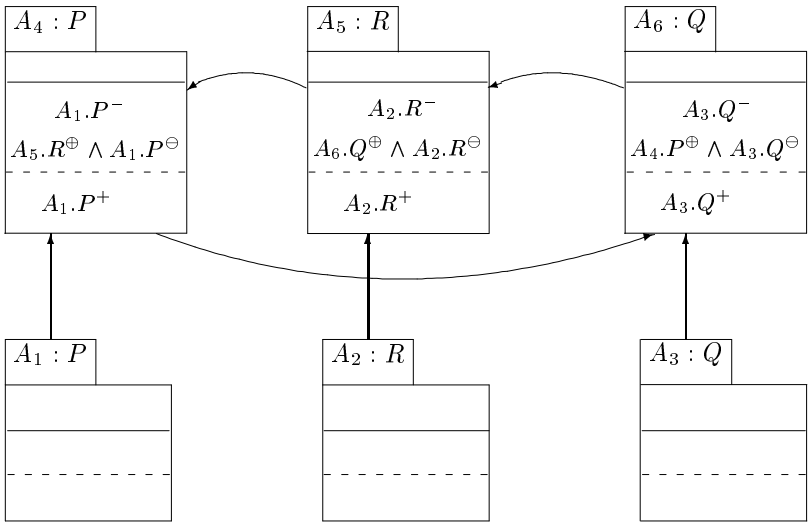

Figure 1.13: Diagram corresponding to the theory of Example 13.5.3.

We leave it to the reader as an exercise to show that the above set of rules is not stratified.

The relations occurring in the heads of the rules are specified by the fixpoint formula given by

$$\mathsf{lfp}\ A_4.P^-, A_4.P^+, A_5.R^-, A_5.R^+, A_6.Q^-, A_6.Q^+.\mathcal{R} \tag{1.14}$$

where $\mathcal{R}$ is the conjunction of the rules.

Although the above set is not stratified, there is the least fixpoint corresponding to the formula (13.14). It is given by the following computation.

$$\{\}$$
$$\{A_4.P^-(a), A_5.R^-(a), A_6.Q^-(a)\}.$$

Following our earlier approach, we could assume that the considered theory has an extension given by $\{\neg P(a), \neg R(a), \neg Q(a)\}$. Unfortunately, it is easily seen that this extension is very problematic from the intuitive point of view. It seems, therefore, that non-stratified theories, even those for which the appropriate least fixpoint exists, should be excluded from the considerations. ■

For stratified semi-normal default theories the notion of an extension is defined

as usual. Also, the Theorems 13.2.6 and 13.2.7 hold for stratified semi-normal default theories.

1.6 Conclusions

In this Chapter, we have showed that a subset of default logic can be easily formalized using the CAKE methodology. The main advantage of this approach is that the computation of an extension of a default theory is tractable. As has been shown in [KS91], for Reiter's default logic, this problem is generally hard, even for propositional disjunction-free theories.

The use of rough set idea has allowed us to distinguish between "hard" and "soft" facts, whereas considering the prerequisite of a default. This allows us to properly model various naturally occurring scenarios that are very difficult to deal with in traditional versions of default logic.

Although semi-normal default theories need not be stratified[16], a good deal of such theories are. These can be easily computed in our version of default logic.[17]

We have not discussed general non-normal default theories here. In our opinion, they are not interesting from the practical point of view. As we argued elsewhere, all naturally occurring scenarios can be properly dealt with in the framework of semi-normal default theories. (See [Lu85], for a detailed discussion.)

To compute an extension of a default theory, we construct an appropriate "global" fixpoint formula. In practice, we are generally interested in single queries. For this reason, it would be reasonable to specify a top-down algorithm which performs only necessary calculations. Such algorithm is quite straightforward, provided that a diagram corresponding to a given theory does not contain any cycles. To specify such algorithm in a general case is an interesting research problem.

1.7 Bibliographic Notes

The version of default logic we have presented here is quite original and no similar version has been discussed in the literature. Of course, our considerations have been heavily influenced by the general literature concerning default logic. (See section 4.4, Chapter 4).

[16] We say that a theory is stratified iff the set of rules corresponding to a diagram of the theory enjoys this property.

[17] It should be emphasized that semi-normal defaults can be often eliminated by introducing priorities between normal defaults.