14.5 Consider the family of linear Gaussian networks, as illustrated on page 502.
   a. In a two-variable network, let \( X_1 \) be the parent of \( X_2 \), let \( X_1 \) have a Gaussian prior, and let \( P(X_2|X_1) \) be a linear Gaussian distribution. Show that the joint distribution \( P(X_1, X_2) \) is a multivariate Gaussian, and calculate its covariance matrix.
   b. Prove by induction that the joint distribution for a general linear Gaussian network on \( X_1, \ldots, X_n \) is also a multivariate Gaussian.

14.6 The probit distribution defined on page 503 describes the probability distribution for a Boolean child, given a single continuous parent.
   a. How might the definition be extended to cover multiple continuous parents?
   b. How might it be extended to handle a *multivalued* child variable? Consider both cases where the child’s values are ordered (as in selecting a gear while driving, depending on speed, slope, desired acceleration, etc.) and cases where they are unordered (as in selecting bus, train, or car to get to work). [Hint: Consider ways to divide the possible values into two sets, to mimic a Boolean variable.]

14.7 This exercise is concerned with the variable elimination algorithm in Figure 14.10.
   a. Section 14.4 applies variable elimination to the query
      \[ P(\text{Burglary} | \text{JohnCalls} = \text{true}, \text{MaryCalls} = \text{true}) . \]
      Perform the calculations indicated and check that the answer is correct.
   b. Count the number of arithmetic operations performed, and compare it with the number performed by the enumeration algorithm.
   c. Suppose a network has the form of a chain: a sequence of Boolean variables \( X_1, \ldots, X_n \) where \( \text{Parents}(X_i) = \{X_{i-1}\} \) for \( i = 2, \ldots, n \). What is the complexity of computing \( P(X_1|X_n = \text{true}) \) using enumeration? Using variable elimination?
   d. Prove that the complexity of running variable elimination on a polytree network is linear in the size of the tree for any variable ordering consistent with the network structure.

14.8 Investigate the complexity of exact inference in general Bayesian networks:
   a. Prove that any 3-SAT problem can be reduced to exact inference in a Bayesian network constructed to represent the particular problem and hence that exact inference is NP-hard. [Hint: Consider a network with one variable for each proposition symbol, one for each clause, and one for the conjunction of clauses.]
   b. The problem of counting the number of satisfying assignments for a 3-SAT problem is \#P-complete. Show that exact inference is at least as hard as this.

14.9 Consider the problem of generating a random sample from a specified distribution on a single variable. You can assume that a random number generator is available that returns a random number uniformly distributed between 0 and 1.
   a. Let \( X \) be a discrete variable with \( P(X = x_i) = p_i \) for \( i \in \{1, \ldots, k\} \). The cumulative distribution of \( X \) gives the probability that \( X \in \{x_1, \ldots, x_j\} \) for each possible \( j \). Ex-