Rozwiązanie zadania 1.
Zakładamy, że $Var = FId$. Nie dokonujemy deallokacji pamięci. Niech:

$$\begin{array}{llll} l & \in & Loc & = & \mathbb{N} \\ (\varrho, x, i) & \in & Fun & = & Env \times Var \times Ins \\ \varrho & \in & Env & = & Var \to Loc \cup Fun \cup \{undef\} \\ s & \in & S & = & Loc \to \mathbb{Z} \cup \{free\} \\ flag & \in & FLAG & = & \{\,\texttt{normal}\,\} \cup \{\,\texttt{return}\ n \mid n \in \mathbb{Z}\,\} \end{array}$$

Powyższe rekurencyjne równania mają najmniejsze rozwiązanie.

$$\boxed{__ \vdash_e __ \Rightarrow __ \subseteq Env \times (S \times Exp) \times (S \times \mathbb{Z})}$$

$$\varrho \vdash_e s, \texttt{n} \Rightarrow s, \texttt{n}$$

$$\frac{\varrho x \in Loc \text{ oraz } s(\varrho x) \in \mathbb{Z}}{\varrho \vdash_e s, x \Rightarrow s, s(\varrho x)}$$

$$\frac{\begin{array}{c}\varrho \vdash_e s, e_1 \Rightarrow s_1, n_1 \\ \varrho \vdash_e s_1, e_2 \Rightarrow s_2, n_2\end{array}}{\varrho \vdash_e s, e_1 + e_2 \Rightarrow s_2, n_1 + n_2}$$

$$\frac{\begin{array}{c}\varrho f = (\varrho_0, x, i) \\ \varrho \vdash_e s, e \Rightarrow s', n \\ l \in Loc \text{ oraz } s'l = free \\ \varrho_0[x \mapsto l, f \mapsto (\varrho_0, x, i)] \vdash_i s'[l \mapsto n], i \Rightarrow s'', \texttt{normal}\end{array}}{\varrho \vdash_e s, f(e) \Rightarrow s'', 0}$$

$$\frac{\begin{array}{c}\varrho f = (\varrho_0, x, i) \\ \varrho \vdash_e s, e \Rightarrow s', n \\ l \in Loc \text{ oraz } s'l = free \\ \varrho_0[x \mapsto l, f \mapsto (\varrho_0, x, i)] \vdash_i s'[l \mapsto n], i \Rightarrow s'', \texttt{return}\ n\end{array}}{\varrho \vdash_e s, f(e) \Rightarrow s'', n}$$

$$\boxed{\vdash_d __ \Rightarrow __ \subseteq (Env \times S \times Dec) \times (Env \times S)}$$

$$\frac{l \in Loc \text{ oraz } sl = free}{\vdash_d \varrho, s, \texttt{var}\ x = 0 \Rightarrow \varrho[x \mapsto l], s[l \mapsto 0]}$$

$$\frac{\begin{array}{c}\vdash_d \varrho, s, d_1 \Rightarrow \varrho_1, s_1 \\ \vdash_d \varrho_1, s_1, d_2 \Rightarrow \varrho_2, s_2\end{array}}{\vdash_d \varrho, s, d_1; d_2 \Rightarrow \varrho_2, s_2}$$

$$\vdash_d \varrho, s, \texttt{fun}\ f(x)\ \{\ i\ \} \Rightarrow \varrho[f \mapsto (\varrho, x, i)], s$$

$$\boxed{_ \vdash_i _ \Rightarrow __ \subseteq Env \times (S \times Ins) \times (S \times FLAG)}$$

$$\frac{\varrho \vdash_e s,\ e \Rightarrow s',\ n \qquad \varrho x \in Loc}{\varrho \vdash_i s,\ x := e \Rightarrow s'[\varrho x \mapsto n],\ \texttt{normal}}$$

$$\frac{\varrho \vdash_i s,\ i_1 \Rightarrow s_1,\ \texttt{return}\ n}{\varrho \vdash_i s,\ i_1 ; i_2 \Rightarrow s_1,\ \texttt{return}\ n}$$

$$\frac{\varrho \vdash_i s,\ i_1 \Rightarrow s_1,\ \texttt{normal} \qquad \varrho \vdash_i s_1,\ i_2 \Rightarrow s_2,\ \mathit{flag}}{\varrho \vdash_i s,\ i_1 ; i_2 \Rightarrow s_2,\ \mathit{flag}}$$

$$\varrho \vdash_i s,\ \texttt{skip} \Rightarrow s,\ \texttt{normal}$$

$$\frac{\vdash_d \varrho,\ s,\ d \Rightarrow \varrho',\ s' \qquad \varrho' \vdash_i s',\ i \Rightarrow s'',\ \mathit{flag}}{\varrho \vdash_i s,\ \texttt{begin}\ d\ \texttt{in}\ i\ \texttt{end} \Rightarrow s'',\ \mathit{flag}}$$

$$\frac{\varrho \vdash_e s,\ e \Rightarrow s',\ n \qquad n \neq 0}{\varrho \vdash_i s,\ \texttt{if}\ e = 0\ \texttt{then}\ i\ \texttt{fi} \Rightarrow s',\ \texttt{normal}}$$

$$\frac{\varrho \vdash_e s,\ e \Rightarrow s',\ 0 \qquad \varrho \vdash_i s',\ i \Rightarrow s'',\ \mathit{flag}}{\varrho \vdash_i s,\ \texttt{if}\ e = 0\ \texttt{then}\ i\ \texttt{fi} \Rightarrow s'',\ \mathit{flag}}$$

$$\frac{\varrho \vdash_e s,\ e \Rightarrow s',\ 0}{\varrho \vdash_i s,\ \texttt{while}\ e \neq 0\ \texttt{do}\ i\ \texttt{done} \Rightarrow s',\ \texttt{normal}}$$

$$\frac{\varrho \vdash_e s,\ e \Rightarrow s',\ n \qquad n \neq 0 \qquad \varrho \vdash_i s',\ i \Rightarrow s'',\ \texttt{return}\ n}{\varrho \vdash_i s,\ \texttt{while}\ e \neq 0\ \texttt{do}\ i\ \texttt{done} \Rightarrow s'',\ \texttt{return}\ n}$$

$$\frac{\varrho \vdash_e s,\ e \Rightarrow s',\ n \qquad n \neq 0 \qquad \varrho \vdash_i s',\ i \Rightarrow s'',\ \texttt{normal} \qquad \varrho \vdash_i s'',\ \texttt{while}\ e \neq 0\ \texttt{do}\ i\ \texttt{done} \Rightarrow s''',\ \mathit{flag}}{\varrho \vdash_i s,\ \texttt{while}\ e \neq 0\ \texttt{do}\ i\ \texttt{done} \Rightarrow s''',\ \mathit{flag}}$$

$$\frac{\varrho \vdash_e s,\ e \Rightarrow s',\ n}{\varrho \vdash_i s,\ \texttt{return}\ e \Rightarrow s',\ \texttt{return}\ n}$$