# Dependent structures

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# Destruction - match

match $m$ as $x$ in $I$ _ $\vec{y}$ return $(P\ \vec{y}\ x)$ with

$$(c_1\ x_{11}\ \ldots\ x_{1k_1}) \Rightarrow f_1 \mid \ldots \mid (c_n\ x_{n1} \ldots x_{nk_n}) \Rightarrow f_n \text{ end}$$

for $m : I\vec{a}\vec{b}$ the expression above has type $P\vec{b}m$, where

$$I : \forall (p_1 : A_1) \ldots (p_p : A_p)(z_1 : Z_1) \ldots (z_m : Z_m).s$$

$$P : \forall (z_1 : Z_1) \ldots (z_m : Z_m)(c : I\vec{a}\vec{z}).Type$$

If

$$c_i : \forall (p_1 : A_1) \ldots (p_p : A_p)(v_1 : V_1) \ldots (v_{k_i} : V_{k_i}).I\vec{p}\vec{w}$$

then

$$f_i : \forall (v_1 : V_1) \ldots (v_{k_i} : V_{k_i}).P\vec{w}(c_i\vec{a}\vec{v})$$

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# Lists with length: `ilist`

```
Section ilist.
  Variable A : Set.

  Inductive ilist : nat → Set :=
  | Nil : ilist O
  | Cons : ∀ n, A → ilist n → ilist (S n).
```

# Function app on ilist

```
Fixpoint app' n1 (ls1 : ilist n1) n2 (ls2 : ilist n2) : ilist (n1 + n2)
:=
    match ls1 as ls in (ilist n1) return (ilist (n1 + n2)) with
      | Nil ⇒ ls2 ≡ f1
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$$f2 : \forall(n' : nat)(a : A)(l : ilist\ n'), P\ (Sn')\ (Cons\ n'\ a\ l)$$

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$$f1 : ilist(0 + n2)$$

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# Elimination of equality

```
Definition rowne (n,m:nat)(h:n=m)(l:ilist n): ilist m :=
match h in _=m with return (ilist m)
eq_refl => l ≡ f1
```

$$P = fun\ (m : nat)(h : n = m) \Rightarrow ilist\ m$$

Since $\texttt{eq\_refl} : \forall\ (A : Set)(a : A).\texttt{eq}\ A\ a\ a$, one has

$$f1\ :\ P\ n\ (\texttt{eq\_refl}\ nat\ n)$$

therefore

$$f1\ :\ ilist\ n$$

- That is an elimination from `Prop` to `Set` for a singleton type
- That is how tactic `rewrite` works

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# Function hd1 on ilist

```
Definition hd1 n (ls : ilist (S n)) : A :=
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  ls as ls0 in (ilist n0)
  return
      (match n0 with
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with

| Nil => tt ≡ f1 :unit
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# Function hd1 on ilist cont.

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# Function hd2 on ilist

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Definition hd_pom n (ls : ilist n) :=
match ls in (ilist n)
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# Initial subsets of natural numbers

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Section ilist.
  Variable A : Set.

  Inductive ilist : nat → Set :=
  | Nil : ilist O
  | Cons : ∀ n, A → ilist n → ilist (S n).
```

```
  Inductive fin : nat → Set :=
  | First : ∀ n, fin (S n)
  | Next : ∀ n, fin n → fin (S n).
```

Values of type **fin** 3 are: First 2, `Next (First 1)` and `Next (Next (First 0))`.

Note: there are no terms of type **fin** 0 !

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  Inductive ilist : nat → Set :=
  | Nil : ilist O
  | Cons : ∀ n, A → ilist n → ilist (S n).


  Inductive fin : nat → Set :=
  | First : ∀ n, fin (S n)
  | Next : ∀ n, fin n → fin (S n).
```

Values of type **fin** 3 are: First 2, `Next (First 1)` and `Next (Next (First 0))`.

Note: there are no terms of type **fin** 0 !

# Initial subsets of natural numbers

```
Section ilist.
  Variable A : Set.

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# Initial subsets of natural numbers

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```

Values of type **fin** 3 are: First 2, `Next (First 1)` and `Next (Next (First 0))`.

Note: there are no terms of type **fin** 0 !

# Function get (1)

```
Fixpoint get n (ls : ilist n) : fin n → A :=
  match ls with
    | Nil ⇒ fun idx : fin O ⇒ ?
    | Cons x ls' ⇒ fun idx ⇒
      match idx with
        | First _ ⇒ x
        | Next idx' ⇒ get ls' idx'
      end
  end.
```

# Function get (2)

```
Fixpoint get n (ls : ilist n) : fin n → A :=
  match ls in ilist k return fin k → A with
    | Nil ⇒ fun idx : fin O ⇒
      match idx in fin n' return (match n' with
                                  | O ⇒ A
                                  | S _ ⇒ unit
                                  end) with
        | First _ ⇒ tt
        | Next _ ⇒ tt
      end
    | Cons x ls' ⇒ fun idx ⇒
      match idx in fin n' return A with
        | First _ ⇒ x
        | Next idx' ⇒ get ls' idx'
      end
  end.
```

There is a problem with type-checking the recursive call to get.

# Function get (2)

```
Fixpoint get n (ls : ilist n) : fin n → A :=
  match ls in ilist k return fin k → A with
    | Nil ⇒ fun idx : fin O ⇒
      match idx in fin n' return (match n' with
                                  | O ⇒ A
                                  | S _ ⇒ unit
                                  end) with
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        | Next _ ⇒ tt
      end
    | Cons x ls' ⇒ fun idx ⇒
      match idx in fin n' return A with
        | First _ ⇒ x
        | Next idx' ⇒ get ls' idx'
      end
  end.
```

There is a problem with type-checking the recursive call to get.

# Function get (3)

```
Fixpoint get n (ls : ilist n) : fin n → A :=
  match ls with
    | Nil ⇒ fun idx ⇒
      match idx in fin n' return (match n' with
                                    | O ⇒ A
                                    | S _ ⇒ unit
                                  end) with
        | First _ ⇒ tt
        | Next _ ⇒ tt
      end
    | Cons x ls' ⇒ fun idx ⇒
      match idx in fin n' return (fin (pred n') → A) → A with
        | First _ ⇒ fun _ ⇒ x
        | Next idx' ⇒ fun get_ls' ⇒ get_ls' idx'
      end (get ls')
  end.
End ilist.
```

# Examples

`Arguments Nil [A]. Arguments Cons [A n].`
`Arguments First [n]. Arguments Next [n].`

`Check Cons 0 (Cons 1 (Cons 2 Nil)).`

```
Cons 0 (Cons 1 (Cons 2 Nil))
    : ilist nat 3
```

`Eval simpl in get (Cons 0 (Cons 1 (Cons 2 Nil))) First.`

```
    = 0
    : nat
```

`Eval simpl in get (Cons 0 (Cons 1 (Cons 2 Nil))) (Next First).`

```
    = 1
    : nat
```

`Eval simpl in get (Cons 0 (Cons 1 (Cons 2 Nil))) (Next (Next First)).`

```
    = 2
    : nat
```

# Examples

`Arguments` Nil [$A$]. `Arguments` Cons [$A$ $n$].
`Arguments` First [$n$]. `Arguments` Next [$n$].

`Check` Cons 0 (Cons 1 (Cons 2 Nil)).

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Cons 0 (Cons 1 (Cons 2 Nil))
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```
     = 0
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```
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     = 2
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# Examples

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# Heterogenic lists

```
Section hlist.
  Variable A : Type.
  Variable B : A → Type.

  Inductive hlist : list A → Type :=
  | HNil : hlist nil
  | HCons : ∀ (x : A) (ls : list A), B x → hlist ls → hlist (x :: ls).

  Variable elm : A.

  Inductive member : list A → Type :=
  | HFirst : ∀ ls, member (elm :: ls)
  | HNext : ∀ x ls, member ls → member (x :: ls).
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  | HFirst : ∀ ls, member (elm :: ls)
  | HNext : ∀ x ls, member ls → member (x :: ls).
```

# Function hget(1)

```
Fixpoint hget ls (mls : hlist ls) : member ls → B elm :=
  match mls with
    | HNil ⇒ fun mem : member nil⇒
      match mem in member ls' return (match ls' with
                                        | nil ⇒ B elm
                                        | _ :: _ ⇒ unit
                                      end) with
        | HFirst _ ⇒ tt
        | HNext _ _ ⇒ tt
      end
```

# Function hget(2)

```
    | @HCons a l e mls' ⇒ fun mem : member ( a :: l) ⇒
      match mem in member ls' return (match ls' with
                                        | nil ⇒ Empty_set
                                        | x' :: ls'' ⇒
                                    B x' → (member ls'' → B elm) → B elm
                                      end) with
        | HFirst _ ⇒ fun e' _ ⇒ e'
        | HNext _ mem' ⇒ fun _ get_mls' ⇒ get_mls' mem'
      end e (hget mls')
  end.
End hlist.
```

# Examples of heterogenic lists

```
Arguments HCons [A B x ls].
Arguments HNil [A B].

Definition someTypes : list Set := nat :: bool :: nil.

Example someValues : hlist (fun T : Set ⇒ T) someTypes :=
  HCons 5 (HCons true HNil).

Eval simpl in hget someValues HFirst.

     = 5
     : (fun T : Set ⇒ T) nat

Eval simpl in hget someValues (HNext HFirst).

     = true
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```

# Interpreter of simply typed lambda calculus (1)

```
Inductive type : Set :=
| Unit : type
| Arrow : type → type → type.
```

```
Inductive exp : list type → type → Set :=
| Const : ∀ ts, exp ts Unit
| Var : ∀ ts t, member t ts → exp ts t
| App : ∀ ts dom ran, exp ts (Arrow dom ran) → exp ts dom → exp ts
ran
| Abs : ∀ ts dom ran, exp (dom :: ts) ran → exp ts (Arrow dom ran).
```

```
Arguments Const [ts].
```

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Arguments Const [ts].
```

# Interpreter of simply typed lambda calculus (2)

```
Fixpoint typeDenote (t : type) : Set :=
  match t with
    | Unit ⇒ unit
    | Arrow t1 t2 ⇒ typeDenote t1 → typeDenote t2
  end.
```

```
Fixpoint expDenote ts t (e : exp ts t) : hlist typeDenote ts →
typeDenote t :=
  match e with
    | Const _ ⇒ fun _ ⇒ tt
    | Var mem ⇒ fun s ⇒ hget s mem
    | App e1 e2 ⇒ fun s ⇒ (expDenote e1 s) (expDenote e2 s)
    | Abs e' ⇒ fun s ⇒ fun x ⇒ expDenote e' (HCons x s)
  end.
```

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  end.
```

# Interpreter of simply typed lambda calculus - examples

```
Eval simpl in expDenote Const HNil.
    = tt : typeDenote Unit

Eval simpl in expDenote (Abs (dom := Unit) (Var HFirst)) HNil.
    = fun x : unit ⇒ x
    : typeDenote (Arrow Unit Unit)

Eval simpl in expDenote (Abs (dom := Unit)
  (Abs (dom := Unit) (Var (HNext HFirst)))) HNil.
    = fun x _ : unit ⇒ x
    : typeDenote (Arrow Unit (Arrow Unit Unit))

Eval simpl in expDenote (Abs (dom := Unit) (Abs (dom := Unit) (Var
HFirst))) HNil.
    = fun _ x0 : unit ⇒ x0
    : typeDenote (Arrow Unit (Arrow Unit Unit))

Eval simpl in expDenote (App (Abs (Var HFirst)) Const) HNil.
    = tt : typeDenote Unit
```

# Interpreter of simply typed lambda calculus - examples

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Eval simpl in expDenote Const HNil.
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```
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```
Eval simpl in expDenote (Abs (dom := Unit) (Var HFirst)) HNil.
```

```
    = fun x : unit ⇒ x
    : typeDenote (Arrow Unit Unit)
```

```
Eval simpl in expDenote (Abs (dom := Unit)
  (Abs (dom := Unit) (Var (HNext HFirst)))) HNil.
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```
    = fun x _ : unit ⇒ x
    : typeDenote (Arrow Unit (Arrow Unit Unit))
```

```
Eval simpl in expDenote (Abs (dom := Unit) (Abs (dom := Unit) (Var
HFirst))) HNil.
```

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    : typeDenote (Arrow Unit (Arrow Unit Unit))
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```

```
    = tt : typeDenote Unit
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# Interpreter of simply typed lambda calculus - examples

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# Interpreter of simply typed lambda calculus - examples

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    : typeDenote (Arrow Unit (Arrow Unit Unit))

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# Interpreter of simply typed lambda calculus - examples

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Eval simpl in expDenote Const HNil.
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Eval simpl in expDenote (App (Abs (Var HFirst)) Const) HNil.
    = tt : typeDenote Unit
```

# Interpreter of $\lambda^{\rightarrow}$— summary

- syntax, typing rules and semantics of evaluation for $\lambda^{\rightarrow}$
- interpreter = implementation of denotational semantics
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