Computer aided verification

Lecture 3: ω-automata

What are ω-automata useful for?

Example:

$$\phi = \neg G(p \implies \mathsf{X} Fq) \qquad \mapsto \qquad A_\phi \quad = \qquad \longrightarrow \begin{array}{c} s_0 \xrightarrow{\ p\ } s_1 \end{array}$$

$$\text{LTL} \ \subseteq \ \omega\text{-automata}$$

I. ω-automata

Def.: ω-automaton (**Büchi** automaton) $\mathcal{A} = \langle \Sigma, S, S_{\text{init}}, \sigma, F \rangle$

- $S_{\text{init}} \subseteq S$ nonempty subset of initial states

- $\sigma \subseteq S \times \Sigma \times S$ transition relation

- $F \subseteq S$ nonempty subset of accepting states

$\mathcal{A}$ is deterministic when $|S_{\text{init}}| = 1$ i $\forall s, a.\ |\sigma(s, a)| \leq 1$.

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ω-words: $w = a_0 \, a_1 \, a_2 \, \ldots$

Def.:   For $w = a_0 \ a_1 \ a_2 \ \ldots$, a run of an automaton $\mathcal{A}$ is
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Def.:   A language is $L \subseteq \Sigma^\omega$ is ω-regular if $L = L_\omega(\mathcal{A})$
for some $\mathcal{A}$.

An akceptujący run looks like:
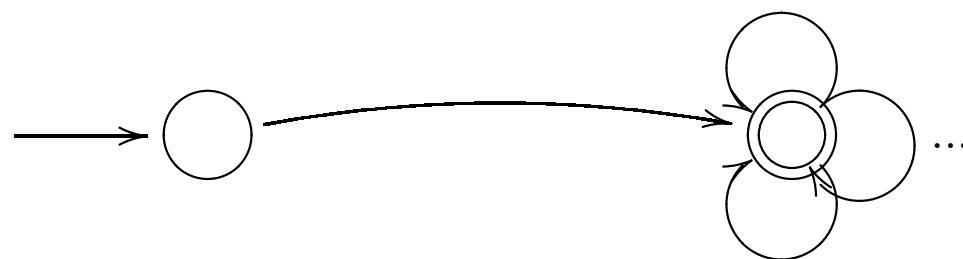
$\Sigma = \{a, b\}$

infinitely often a $\qquad\qquad$ $(b^* a)^\omega$
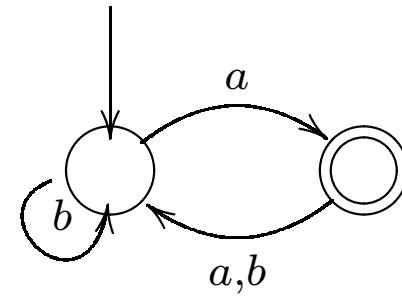
odd(a) $\qquad\qquad$ $(a (a + b))^\omega$
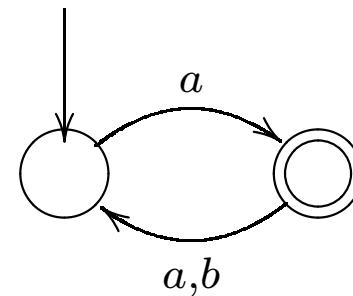
Corollary: $LTL \subsetneq \omega$-automata

– infinitely often a and b

$$b^* \; (aa^* \; bb(bb)^*)^\omega$$

– between any two consecutive a's
  even number of b's

– infinitely often a and b

$$b^* \ (aa^* \ bb(bb)^*)^\omega$$

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   even number of b's

and what about de-
terministic ?

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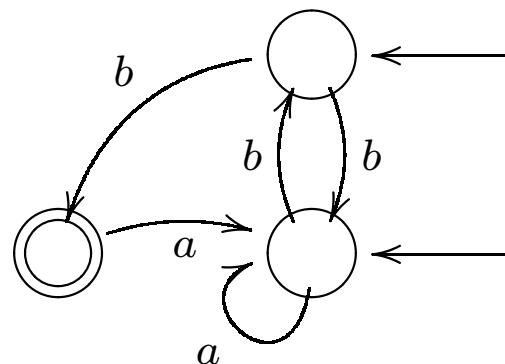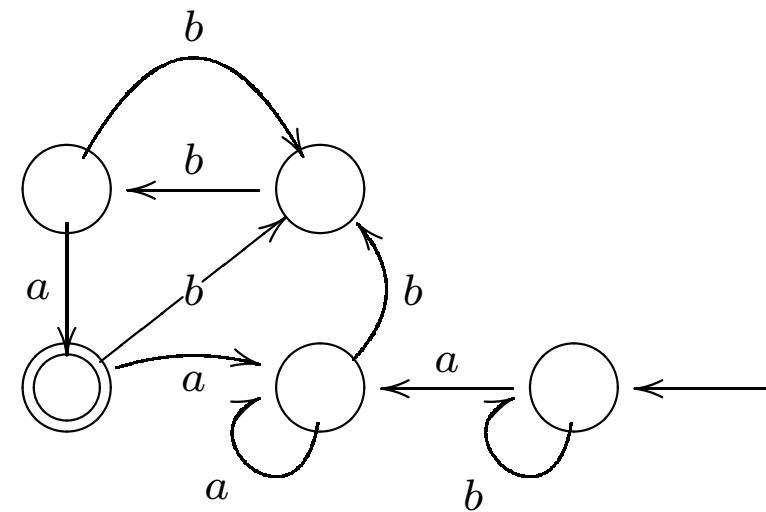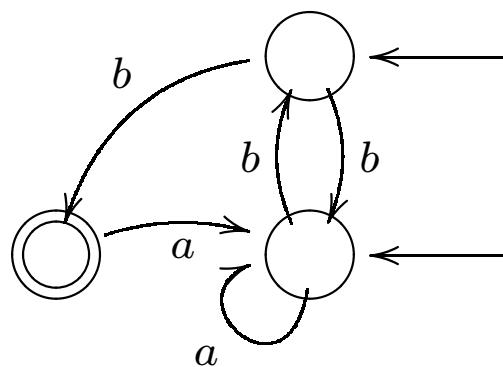
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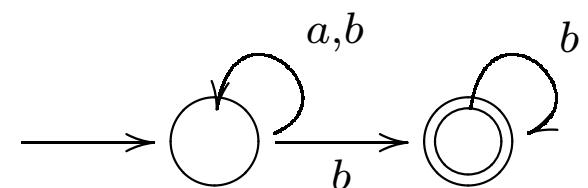
and what about deterministic ?

Tw: ω-regular languages ae closed under $\cup$, $\cap$ and complementation.

$$\vee, \ \wedge \ \mathsf{i} \ \neg$$

$$\mathcal{A}_1, \mathcal{A}_2 \ \mapsto \ \mathcal{A}$$

$$(1) \ L_\omega(\mathcal{A}) \ = \ L_\omega(\mathcal{A}_1) \ \cup \ L_\omega(\mathcal{A}_2)$$

$$(2) \ L_\omega(\mathcal{A}) \ = \ L_\omega(\mathcal{A}_1) \ \cap \ L_\omega(\mathcal{A}_2)$$

$$\mathcal{A} \ \mapsto \ \bar{\mathcal{A}}$$

$$(3) \ L_\omega(\bar{\mathcal{A}}) \ = \ \Sigma^\omega \ \setminus \ L_\omega(\mathcal{A})$$

(2) $\mathcal{A}_1, \mathcal{A}_2 \ \mapsto \ \mathcal{A}$ $\qquad\qquad\qquad L_\omega(\mathcal{A}) \ = \ L_\omega(\mathcal{A}_1) \ \cap \ L_\omega(\mathcal{A}_2)$

?

(2) $\mathcal{A}_1, \mathcal{A}_2 \mapsto \mathcal{A}$ $\qquad\qquad L_\omega(\mathcal{A}) = L_\omega(\mathcal{A}_1) \cap L_\omega(\mathcal{A}_2)$

$$S = S_1 \times S_2 \times \{1, 2\}$$

$$S_{\mathsf{init}} = S_{1,\mathsf{init}} \times S_{2,\mathsf{init}} \times \{1\}$$

$$F = F_1 \times S_2 \times \{1\}$$

$$((s, t, i), a, (s', t', j)) \in \sigma \iff (s, a, s') \in \sigma_1, \ (t, a, t') \in \sigma_2,$$

$$j = \begin{cases} 2 & \text{if } i = 1, \ s \in F_1 \\ 1 & \text{if } i = 2, \ t \in F_2 \\ i & \text{otherwise} \end{cases}$$

(3) $\mathcal{A} \mapsto \bar{\mathcal{A}}$　　　　　　　　$L_\omega(\bar{\mathcal{A}}) = \Sigma^\omega \setminus L_\omega(\mathcal{A})$
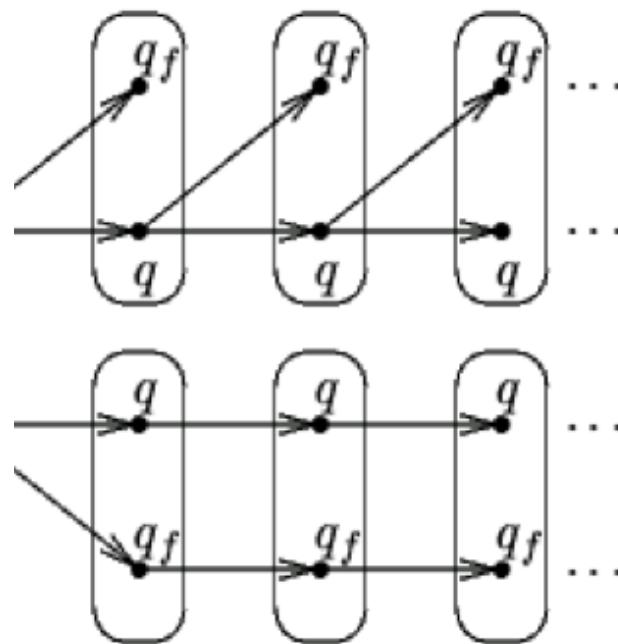
    – no determinization!

Thm:
$(a + b)^*b^\omega$ is not accepted by a deterministic automaton.

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$\ldots$

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...

$\exists i < j$ such that $\sigma(s_0, b^{k_0}ab^{k_1} \ldots ab^{k_i}) = \sigma(s_0, b^{k_0}ab^{k_1} \ldots ab^{k_j})$

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. . .

$\exists i < j$ such that $\sigma(s_0, b^{k_0}ab^{k_1} \ldots ab^{k_i}) = \sigma(s_0, b^{k_0}ab^{k_1} \ldots ab^{k_j})$

Thus $\mathcal{A}$ accepts $b^{k_0}ab^{k_1} \ldots ab^{k_i}(a \ldots ab^{k_j})^\omega$

contradiction!

(3) $\mathcal{A} \mapsto \bar{\mathcal{A}}$ $\qquad\qquad\qquad\qquad\qquad L_\omega(\bar{\mathcal{A}}) = \Sigma^\omega \setminus L_\omega(\mathcal{A})$

- – no determinization

- – a complex construction

- – $|\bar{\mathcal{A}}| = 2^{\mathcal{O}(n \cdot \log n)}$, where $n = |\mathcal{A}|$

Moral: Better to avoid complementation

$$
\begin{array}{ccc}
\phi & \longmapsto & \neg\phi \\
\downarrow & & \downarrow \\
\mathcal{A}_\phi & \dashrightarrow \bar{\mathcal{A}}_\phi \ \equiv \ \mathcal{A}_{\neg\phi}
\end{array}
$$

Question:

How complementation is done if $\mathcal{A}$ is deterministic?

$$\mathcal{A} \longmapsto \bar{\mathcal{A}}$$

$$F \longmapsto \bar{F} = Q \backslash F$$

$$L_\omega(\bar{\mathcal{A}}) = \Sigma^\omega \setminus L_\omega(\mathcal{A}) ?$$

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How complementation is done if $\mathcal{A}$ is deterministic?

$$\mathcal{A} \dashrightarrow \bar{\mathcal{A}}$$

$$F \dashrightarrow \bar{F} = Q \backslash F$$

$$L_\omega(\bar{\mathcal{A}}) = \Sigma^\omega \setminus L_\omega(\mathcal{A}) ? \quad \textbf{NO!}$$

co-Büchi: a run $r = s_0\ s_1\ s_2\ \ldots$ is accepting when $s_i \in \bar{F}$ for almost all i $(\inf(r) \subseteq \bar{F})$.

| problem for finite automata	problem for ω-automata	complexity	cost of algorithm
$L(A) \neq \emptyset$	$L_\omega(A) \neq \emptyset$	NLOGSPACE	$\mathcal{O}(n)$
$L(A) = \Sigma^*$	$L_\omega(A) = \Sigma^\omega$	PSPACE	$2^{\mathcal{O}(n \cdot \log n)}$
$L(A) \subseteq L(B)$	$L_\omega(A) \subseteq L_\omega(B)$	PSPACE	$2^{\mathcal{O}(n \cdot \log n)}$
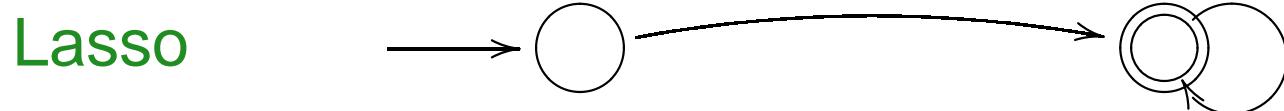
Lasso

**Thm:** $L_\omega(A) \neq \emptyset$ iff $\mathcal{A}$ has a lasso.

II. LTL $\mapsto$ BA

```
$ spin -f "[](p U q)"
never {
T0:
        if
        :: (p) -> goto T0
        :: (q) -> goto accept
        fi;
accept:
        if
        :: ((p) || (q)) -> goto T0
        fi
}
```
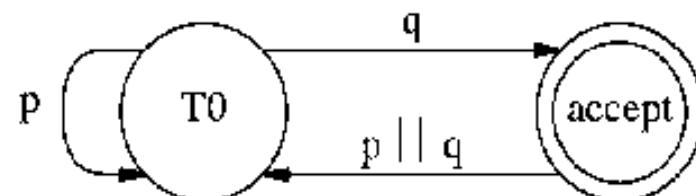
```
$ spin -f "[]<>p"
never {
T0:
        if
        :: (true) -> goto T0
        :: (p) -> goto accept
        fi;
accept:
        if
        :: (true) -> goto T0
        fi
}
```
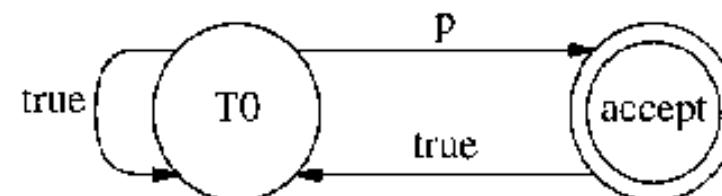

SPIN's doc

Generalized ω-automata (GBA)

– $\{F_1, \ldots, F_n\}$ instead of F

– a run r is accepting when $\forall i.\ \inf(r) \cap F_i \neq \emptyset$

Question: Are generalized automata more expressive?

Generalized ω-automata (GBA)

- $\{F_1, \ldots, F_n\}$ instead of F

- a run r is accepting when $\forall i.\ \inf(r) \cap F_i \neq \emptyset$

Question: Are generalized automata more expressive?

$$\mathcal{A}_{F_1 \ldots F_n} \ \mapsto \ \mathcal{A}_F$$

$$L_\omega(\mathcal{A}_{F_1 \ldots F_n}) \ = \ L_\omega(\mathcal{A}_F) \ \subseteq \ L_\omega(\mathcal{A}_{F_1}) \cap \ldots \cap L_\omega(\mathcal{A}_{F_n})$$

$$|\mathcal{A}_F| = \mathcal{O}(|\mathcal{A}_{F_1,\ldots,F_n}| \cdot n)$$

– **SPIN:**   LTL $\mapsto$ GBA $\mapsto$ BA

– **LTL2BA:** LTL $\mapsto$ ABA $\mapsto$ GBA' $\mapsto$ BA

– On-the-fly verification

LTL$^+$:

$$\phi \; := \; p \; | \; \neg p \; | \; \phi_1 \wedge \phi_2 \; | \; \phi_1 \vee \phi_2 \; | \; \mathsf{X}\,\phi \; | \; \phi_1 \, \mathsf{U} \, \phi_2 \; | \; \phi_1 \, \mathsf{R} \, \phi_2 \; |$$

$$\texttt{true} \; | \; \texttt{false}$$

Intuition: $\phi \; \equiv \; \mathsf{now}(\phi) \; \overset{\wedge}{\underset{\vee}{}} \; \mathsf{later}(\phi)$

$$\phi \, \mathsf{U} \, \psi \; \equiv \; \psi \; \vee \; (\phi \wedge \mathsf{X}(\phi \, \mathsf{U} \, \psi))$$

$$\phi \, \mathsf{R} \, \psi \; \equiv \; \psi \; \wedge \; (\phi \vee \mathsf{X}(\phi \, \mathsf{R} \, \psi))$$

(fixed points)

...do not think about tomorrow! :)

$$\alpha \;\mapsto\; \text{today}(\alpha) \;-\; \text{boolean formula over } P \cup \bar{P} \cup \{\,\mathsf{X}\,\phi : \phi \ldots\}$$

$$\bar{P} = \{\neg p : p \in P\}$$

$$
\begin{aligned}
\text{today}(\alpha) &= \alpha, \;\; \text{gdy} \;\; \alpha = p, \neg p, \mathsf{X}\,\beta, \mathtt{true}, \mathtt{false} \\
\text{today}(\alpha \vee \beta) &= \text{today}(\alpha) \;\vee\; \text{today}(\beta) \\
\text{today}(\alpha \wedge \beta) &= \text{today}(\alpha) \;\wedge\; \text{today}(\beta) \\
\text{today}(\alpha \,\mathsf{U}\, \beta) &= \text{today}(\beta) \;\vee\; (\text{today}(\alpha) \wedge \mathsf{X}\,(\alpha \,\mathsf{U}\, \beta)) \\
\text{today}(\alpha \,\mathsf{R}\, \beta) &= \text{today}(\beta) \;\wedge\; (\text{today}(\alpha) \vee \mathsf{X}\,(\alpha \,\mathsf{R}\, \beta))
\end{aligned}
$$

$$\alpha \;\mapsto\; \text{today}(\alpha) \;\mapsto\; \text{dnf}(\alpha) \;\subseteq\; \mathcal{P}(P \cup \bar{P} \cup \{\mathsf{X}\,\phi : \phi \ldots\})$$

$$\boxed{\text{today}(\alpha) \;\equiv\; \vee_{X \in \text{dnf}(\alpha)}\,\left(\,\wedge X\right)}$$

For example:

$$
\begin{array}{lcl}
\text{dnf}(\alpha) & = & \{\{\alpha\}\}, \;\; \text{gdy} \;\; \alpha = p, \neg p, \mathsf{X}\,\beta \\
\text{dnf}(\alpha \vee \beta) & = & \text{dnf}(\alpha) \;\cup\; \text{dnf}(\beta) \\
\text{dnf}(\alpha\,\mathsf{U}\,\beta) & = & \text{dnf}(\beta) \;\cup\; \text{dnf}(\alpha \wedge \mathsf{X}\,(\alpha\,\mathsf{U}\,\beta))) \\
\text{dnf}(\texttt{true}) & = & \{\emptyset\} \qquad\qquad\quad {\color{green}\wedge\emptyset \;\equiv\; \texttt{true}} \\
\text{dnf}(\texttt{false}) & = & \emptyset \qquad\qquad\qquad {\color{green}\vee\emptyset \;\equiv\; \texttt{false}}
\end{array}
$$

GBA $\mathcal{A}_\phi = \langle \Sigma, S, S_{\mathsf{pocz}}, \sigma, F \rangle$:

   – $S = \mathcal{P}(P \cup \bar{P} \cup \{\,\mathsf{X}\,\phi : \phi \ldots\})$

   – $\Sigma = \mathcal{P}(P)$

   – $S_{\mathsf{pocz}} = \mathsf{dnf}(\phi)$

   – $X \xrightarrow{A} Y$ **iff**

      – $X \cap P \subseteq A$ non-contradictory

      – $(X \cap \bar{P}) \cap A = \emptyset$ X i A today

      – $Y \in \mathsf{dnf}(\,\wedge\,\{\alpha \mid \mathsf{X}\,\alpha \in X\})$ possible tomorrow

   – $F = ?$

$\phi = \neg a \cup b$

$S = \mathcal{P}(a, \neg a, b, \neg b, X(\neg a \cup b))$

$\Sigma = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

$F = ?$

$\phi = \neg a \cup b$
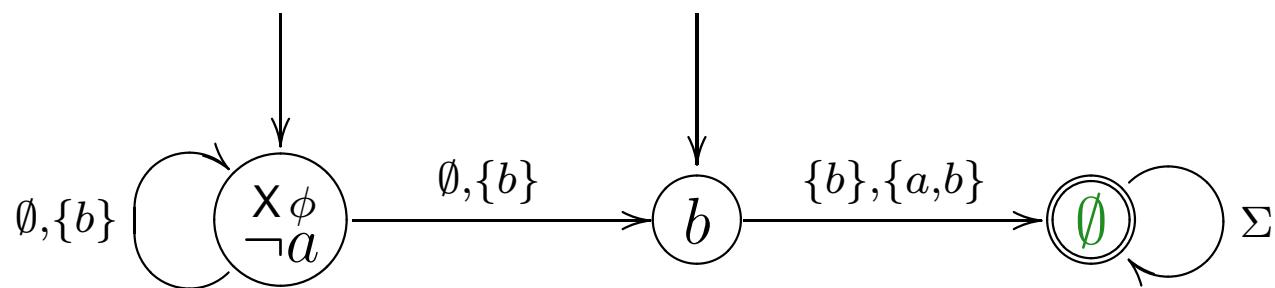
$S = \mathcal{P}(a, \neg a, b, \neg b, \mathsf{X}(\neg a \cup b))$

$\Sigma = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$
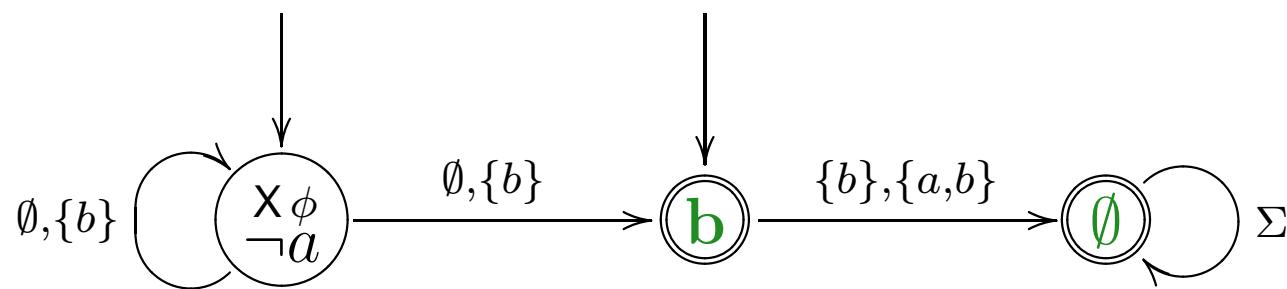
$F = \{\{\emptyset, \{b\}\}\}$

– $F_i = \{A \mid \alpha_i \, \mathsf{U} \, \beta_i \notin A \, \vee \, \beta_i \in A\}, \quad i = 1, \ldots, n$

$$\{\alpha_i \, \mathsf{U} \, \beta_i \mid i = 1, \ldots, n\} \subseteq \mathsf{subformula}(\phi)$$

– $F_i = \{X \in S \mid \alpha_i \, \mathsf{U} \, \beta_i \notin \mathsf{cons}(X) \, \vee \, \beta_i \in \mathsf{cons}(X)\}$

$X \subseteq \mathsf{cons}(X)$

$\alpha \vee \beta \in \mathsf{cons}(X) \quad$ jeśli $\quad \alpha \in \mathsf{cons}(X)$ lub $\beta \in \mathsf{cons}(X)$

$\alpha \wedge \beta \in \mathsf{cons}(X) \quad$ jeśli $\quad \alpha \in \mathsf{cons}(X)$ i $\beta \in \mathsf{cons}(X)$

$\alpha \, \mathsf{U} \, \beta \in \mathsf{cons}(X) \quad$ jeśli $\quad \beta \, \vee \, (\alpha \wedge \mathsf{X}(\alpha \, \mathsf{U} \, \beta)) \in \mathsf{cons}(X)$

$\alpha \, \mathsf{R} \, \beta \in \mathsf{cons}(X) \quad$ jeśli $\quad \beta \, \wedge \, (\alpha \vee \mathsf{X}(\alpha \, \mathsf{R} \, \beta)) \in \mathsf{cons}(X)$

$\phi \ = \ \neg a \, \textsf{U} \, b$

$S \ = \ \mathcal{P}(a, \ \neg a, \ b, \ \neg b, \ \textsf{X}(\neg a \, \textsf{U} \, b))$

$\Sigma \ = \ \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$
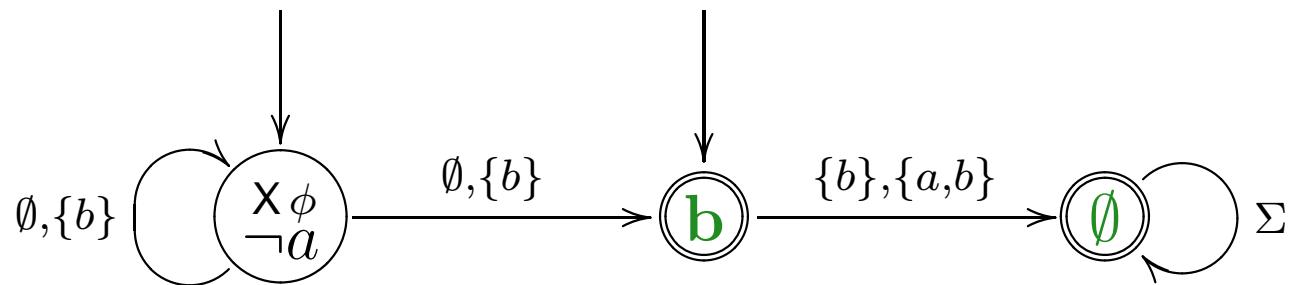
Can automaton $\mathcal{A}_\phi$ be smaller?

$\phi = \neg a \;\mathsf{U}\; b$

$S = \mathcal{P}(a,\; \neg a,\; b,\; \neg b,\; \mathsf{X}(\neg a \;\mathsf{U}\; b))$

$\Sigma = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$
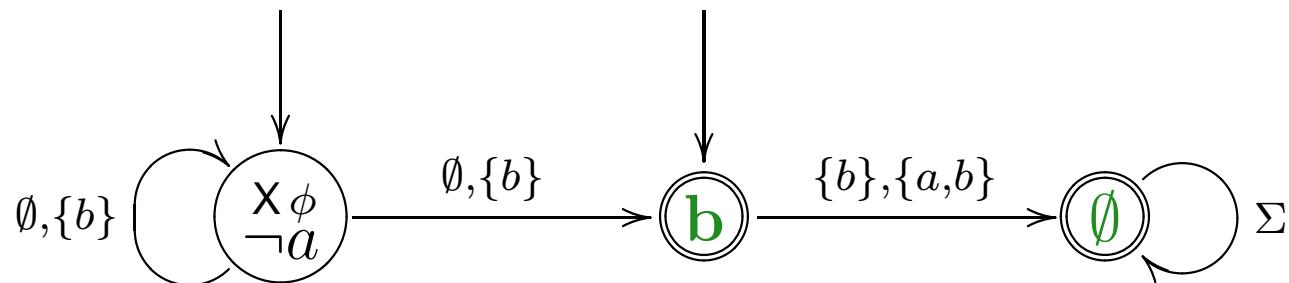
Can automaton $\mathcal{A}_\phi$ be smaller?   **YES!**
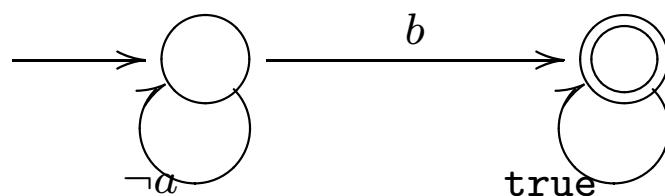
$$\theta = \neg\, \mathsf{G}\,(q \implies \mathsf{F}\,r) \equiv \mathsf{F}\,(q \wedge \mathsf{G}\,\neg r)$$

$$\mathsf{dnf}(\mathsf{F}\,\alpha) = \mathsf{dnf}(\alpha) \cup \{\{\mathsf{X}\,\mathsf{F}\,\alpha\}\} \qquad\qquad \mathsf{F}\,\alpha \equiv \alpha \vee \mathsf{X}\,\mathsf{F}\,\alpha$$

$$\mathsf{dnf}(\mathsf{G}\,\alpha) = \mathsf{dnf}(\alpha \wedge \mathsf{X}\,\mathsf{G}\,\alpha) \qquad\qquad \mathsf{G}\,\alpha \equiv \alpha \wedge \mathsf{X}\,\mathsf{G}\,\alpha$$

$$S = \mathcal{P}(q,\ \neg q,\ r,\ \neg r,\ \mathsf{X}(\mathsf{F}\,(q \wedge \mathsf{G}\,\neg r)), \mathsf{X}\,\mathsf{G}\,\neg r) \qquad\qquad \mathsf{F} = ?$$

$$\mathsf{dnf}(\mathsf{F}\,(q \wedge \mathsf{G}\,\neg r)) = \{\{\mathsf{X}\,\mathsf{F}\,(q \wedge \mathsf{G}\,\neg r)\},\ \{q, \neg r, \mathsf{X}\,\mathsf{G}\,\neg r\}\}$$

$\theta = \neg(\mathsf{G}\,\mathsf{F}\,p \implies \mathsf{G}\,(q \implies \mathsf{F}\,r)) \equiv \mathsf{G}\,\mathsf{F}\,p \wedge \mathsf{F}\,(q \wedge \mathsf{G}\,\neg r)$

$\mathsf{dnf}(\mathsf{F}\,(q \wedge \mathsf{G}\,\neg r)) = {\color{red}\mathsf{X}\,\mathsf{F}\,(q \wedge \mathsf{G}\,\neg r)} \vee ({\color{red}q \wedge \neg r \wedge \mathsf{X}\,\mathsf{G}\,\neg r})$

$\mathsf{dnf}(\mathsf{G}\,\mathsf{F}\,p) = \mathsf{dnf}((p \vee \mathsf{X}\,\mathsf{F}\,p) \wedge \mathsf{X}\,\mathsf{G}\,\mathsf{F}\,p) =$
$\qquad\qquad\qquad {\color{red}(p \wedge \mathsf{X}\,\mathsf{G}\,\mathsf{F}\,p)} \vee ({\color{red}\mathsf{X}\,\mathsf{F}\,p \wedge \mathsf{X}\,\mathsf{G}\,\mathsf{F}\,p})$

$\mathsf{dnf}(\mathsf{G}\,\mathsf{F}\,p \wedge \mathsf{F}\,(q \wedge \mathsf{G}\,\neg r)) = \ldots \vee \ldots \vee \ldots \vee \ldots$

$\mathsf{X}\,\mathsf{F}\,(q \wedge \mathsf{G}\,\neg r), p, \mathsf{X}\,\mathsf{G}\,\mathsf{F}\,p \qquad\qquad q, \neg r, \mathsf{X}\,\mathsf{G}\,\neg r, p, \mathsf{X}\,\mathsf{G}\,\mathsf{F}\,p$

$\mathsf{X}\,\mathsf{F}\,(q \wedge \mathsf{G}\,\neg r), \mathsf{X}\,\mathsf{F}\,p, \mathsf{X}\,\mathsf{G}\,\mathsf{F}\,p \qquad q, \neg r, \mathsf{X}\,\mathsf{G}\,\neg r, \mathsf{X}\,\mathsf{F}\,p, \mathsf{X}\,\mathsf{G}\,\mathsf{F}\,p$

$$\theta_n = \neg((\mathsf{G}\,\mathsf{F}\,p_1 \wedge \ldots \wedge \mathsf{G}\,\mathsf{F}\,p_n) \implies \mathsf{G}\,(q \implies \mathsf{F}\,r)) \equiv$$

$$\mathsf{G}\,\mathsf{F}\,p_1 \wedge \ldots \wedge \mathsf{G}\,\mathsf{F}\,p_n \wedge \mathsf{F}\,(q \wedge \mathsf{G}\,\neg r)$$
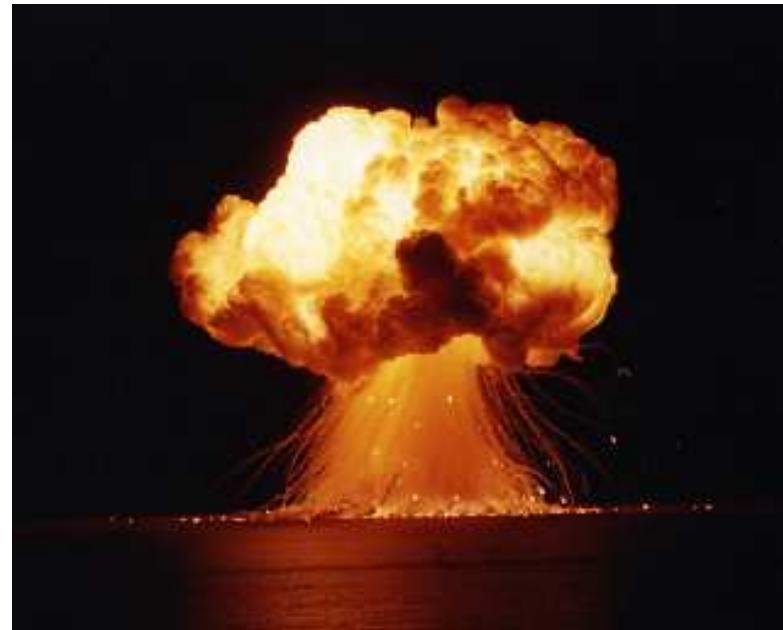
$$\theta_n = \neg((\mathsf{G}\,\mathsf{F}\,p_1 \wedge \ldots \wedge \mathsf{G}\,\mathsf{F}\,p_n) \implies \mathsf{G}\,(q \implies \mathsf{F}\,r))$$

	Spin		Wring		EQLTL	LTL2BA–		LTL2BA	
	time	space	time	space	time	time	space	time	space
θ_1	0.18	460	0.56	4,100	16	0.01	9	0.01	9
θ_2	4.6	4,200	2.6	4,100	16	0.01	19	0.01	11
θ_3	170	52,000	16	4,200	18	0.01	86	0.01	19
θ_4	9,600	970,000	110	4,700	25	0.07	336	0.06	38
θ_5			1,000	6,500	135	0.70	1,600	0.37	48
θ_6			8,400	13,000	N/A	12	8,300	4.0	88
θ_7			72,000[†]	43,000[†]		220	44,000	32	175
θ_8						4,200	260,000	360	250
θ_9						97,000	1,600,000	3,000	490
θ_{10}								36,000	970

[Gastin,Oddoux 2001]

$$\phi_n = p_1 \cup (p_2 \cup (\ldots \cup p_n) \ldots)$$

$$\theta_n = \neg\big(p_1 \cup (p_2 \cup (\ldots \cup p_n) \ldots)\big)$$
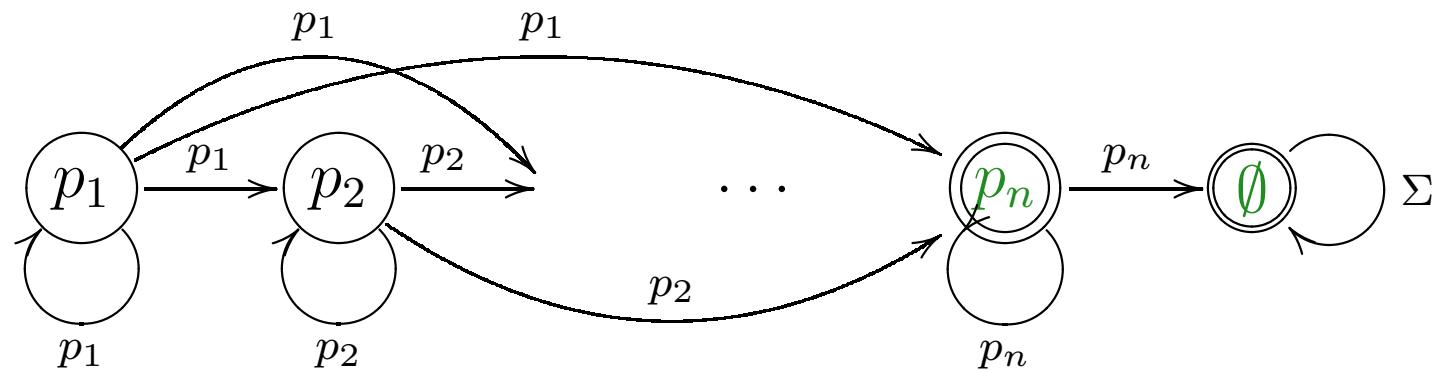
III. LTL ↦ ABA
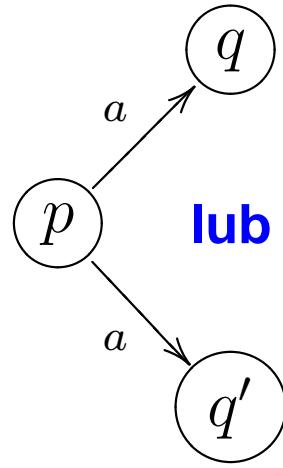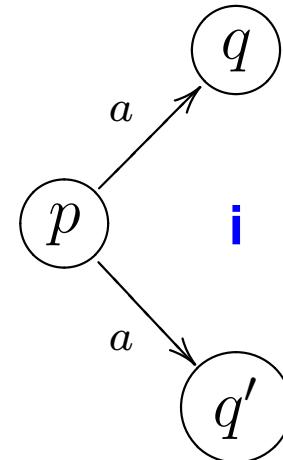
$$\sigma(p,a) \; = \; q \vee q'$$

$$(p,a,q),(p,a,q') \in \sigma$$

$$\sigma(p,a) \; = \; q \wedge q'$$

—

Np.: $\quad \sigma(p,a) \; = \; p_1 \; \vee \; p_2 \wedge p_3 \quad$ (DNF)

Question: run = ?

ABA $\mathcal{A}_\phi = \langle \Sigma, S, S_{\mathsf{pocz}}, \sigma, F \rangle$:

- $S =$ modal subformula ($\mathsf{X}\,\alpha, \ \alpha\,\mathsf{U}\,\beta, \ \alpha\,\mathsf{R}\,\beta$)
  and literals ($p, \ \neg p$)

- $S_{\mathsf{init}} = \mathsf{today}(\phi)$

- $\sigma : S \times \Sigma \rightarrow \mathsf{Bool}^+(S)$

$$\sigma(p, A) \quad = \quad \texttt{true, if } p \in A, \text{ otherwise } \texttt{false}$$
$$\sigma(\neg p, A) \quad = \quad \texttt{true, if } p \notin A, \text{ otherwise } \texttt{false}$$
$$\sigma(\mathsf{X}\,\alpha, A) \quad = \quad \alpha \qquad \textcolor{red}{!!!}$$
$$\sigma(\alpha\,\mathsf{U}\,\beta, A) \quad = \quad \sigma(\beta, A) \ \vee \ (\sigma(\alpha, A) \ \wedge \ \alpha\,\mathsf{U}\,\beta)$$
$$\sigma(\alpha\,\mathsf{R}\,\beta, A) \quad = \quad \sigma(\beta, A) \ \wedge \ (\sigma(\alpha, A) \ \vee \ \alpha\,\mathsf{R}\,\beta)$$
$$\sigma(\mathsf{F}\,\alpha, A) \ = \ \sigma(\alpha, A) \ \vee \ \mathsf{F}\,\alpha$$
$$\sigma(\mathsf{G}\,\alpha, A) \ = \ \sigma(\alpha, A) \ \wedge \ \mathsf{G}\,\alpha$$
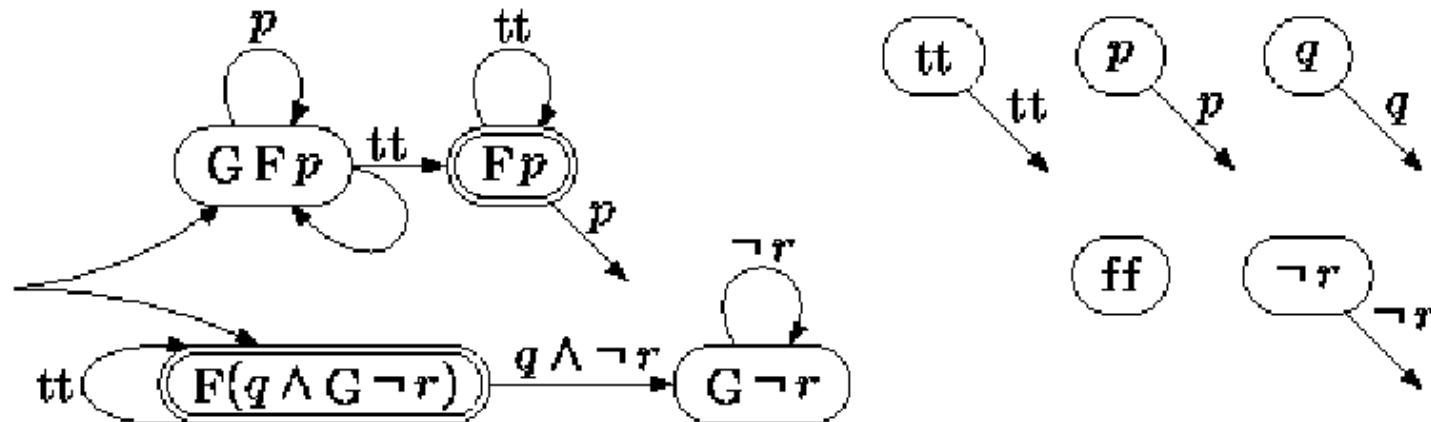
$$\phi \;=\; \neg(\,\mathsf{G}\,\mathsf{F}\,p \implies \mathsf{G}\,(q \implies \mathsf{F}\,r)) \;\equiv\; \mathsf{G}\,\mathsf{F}\,p \;\wedge\; \mathsf{F}\,(q \wedge \mathsf{G}\,\neg r)$$

$$\mathsf{dnf}(\,\mathsf{F}\,(q \wedge \mathsf{G}\,\neg r)) \;=\; \mathsf{X}\,\mathsf{F}\,(q \wedge \mathsf{G}\,\neg r) \;\vee\; (q \wedge \neg r \wedge \mathsf{X}\,\mathsf{G}\,\neg r)$$

$$\mathsf{dnf}(\,\mathsf{G}\,\mathsf{F}\,p) \;=\; (p \wedge \mathsf{X}\,\mathsf{G}\,\mathsf{F}\,p) \vee (\mathsf{X}\,\mathsf{F}\,p \wedge \mathsf{X}\,\mathsf{G}\,\mathsf{F}\,p)$$

$$\phi \;=\; \neg(\,\mathsf{G}\,\mathsf{F}\,p \;\Longrightarrow\; G(q \;\Longrightarrow\; \mathsf{F}\,r)) \;\equiv\; \mathsf{G}\,\mathsf{F}\,p \;\wedge\; \mathsf{F}\,(q \wedge \mathsf{G}\,\neg r)$$
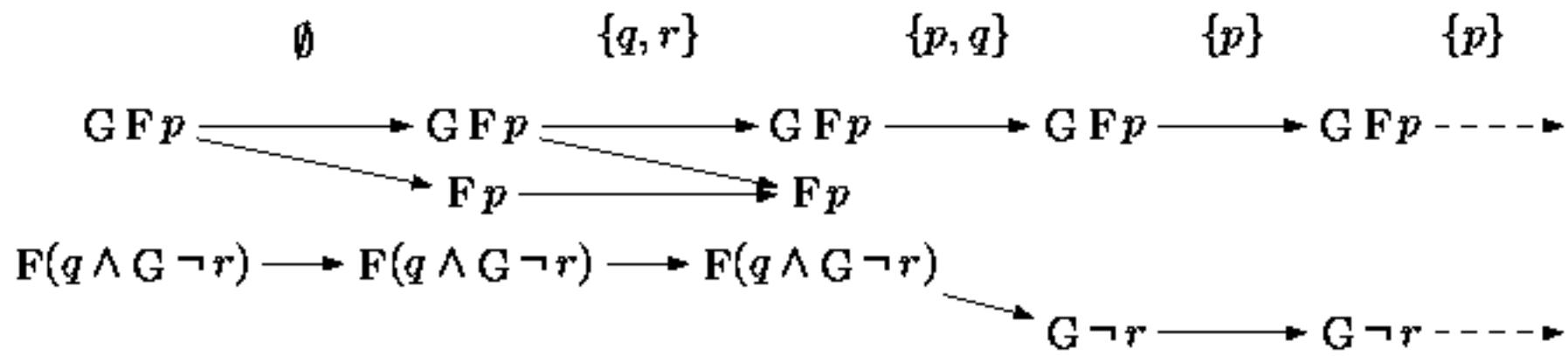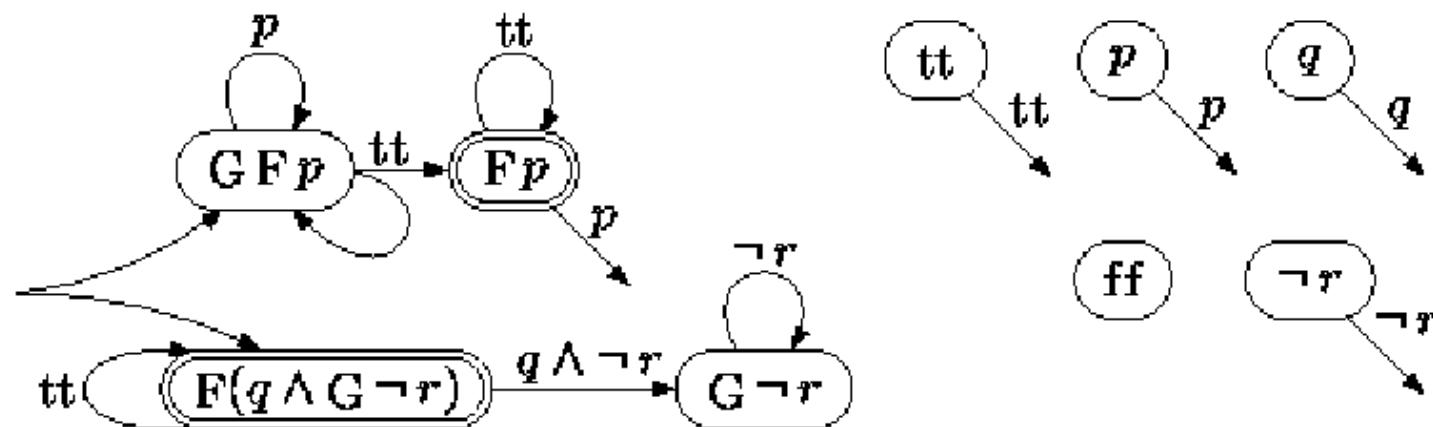
[Gastin,Oddoux 2001]

ABA $\mathcal{A}_\phi \;=\; \langle \Sigma, S, S_{\mathsf{init}}, \sigma, F \rangle$:

- $S \;=\;$ modal subformula $(\,\mathsf{X}\,\alpha,\; \alpha\,\mathsf{U}\,\beta,\; \alpha\,\mathsf{R}\,\beta\,)$
  and literals $(p,\; \neg p)$

- $S_{\mathsf{init}} \;=\;$ today(ϕ)

- $\sigma : S \times \Sigma \to \mathsf{Bool}^{+}(S)$

  $\dots$

- $F \;=\; \{\alpha\,\mathsf{R}\,\beta\}$