1. A set of 8 problems was prepared for an examination. Each student was given 3 of them. No two students received more than one common problem. What is the largest possible number of students?

2. Let \( n \geq 2 \) be a positive integer. Find whether there exist \( n \) pairwise nonintersecting nonempty subsets of \( \{1, 2, 3, \ldots \} \) such that each positive integer can be expressed in a unique way as a sum of at most \( n \) integers, all from different subsets.

3. The numbers 1, 2, \ldots, 49 are placed in a 7 \( \times \) 7 array, and the sum of the numbers in each row and in each column is computed. Some of these 14 sums are odd while others are even. Let \( A \) denote the sum of all the odd sums and \( B \) the sum of all even sums. Is it possible that the numbers were placed in the array in such a way that \( A = B \)?

4. Let \( p \) and \( q \) be two different primes. Prove that

\[
\left\lfloor \frac{p}{q} \right\rfloor + \left\lfloor \frac{2p}{q} \right\rfloor + \left\lfloor \frac{3p}{q} \right\rfloor + \cdots + \left\lfloor \frac{(q-1)p}{q} \right\rfloor = \frac{1}{2}(p-1)(q-1) .
\]

(Here \( \lfloor x \rfloor \) denotes the largest integer not greater than \( x \).)

5. Let 2001 given points on a circle be colored either red or green. In one step all points are recolored simultaneously in the following way: If both direct neighbors of a point \( P \) have the same color as \( P \), then the color of \( P \) remains unchanged, otherwise \( P \) obtains the other color. Starting with the first coloring \( F_1 \), we obtain the colorings \( F_2, F_3, \ldots \) after several recoloring steps. Prove that there is a number \( n_0 \leq 1000 \) such that \( F_{n_0} = F_{n_0+2} \). Is the assertion also true if 1000 is replaced by 999?

6. The points \( A, B, C, D, E \) lie on the circle \( c \) in this order and satisfy \( AB \parallel EC \) and \( AC \parallel ED \). The line tangent to the circle \( c \) at \( E \) meets the line \( AB \) at \( P \). The lines \( BD \) and \( EC \) meet at \( Q \). Prove that \(|AC| = |PQ|\).

7. Given a parallelogram \( ABCD \). A circle passing through \( A \) meets the line segments \( AB, AC \) and \( AD \) at inner points \( M, K, N \), respectively. Prove that

\[
|AB| \cdot |AM| + |AD| \cdot |AN| = |AK| \cdot |AC|.
\]

8. Let \( ABCD \) be a convex quadrilateral, and let \( N \) be the midpoint of \( BC \). Suppose further that \( \angle AND = 135^\circ \). Prove that \(|AB| + |CD| + \frac{1}{\sqrt{2}} \cdot |BC| \geq |AD|\).

9. Given a rhombus \( ABCD \), find the locus of the points \( P \) lying inside the rhombus and satisfying \( \angle APD + \angle BPC = 180^\circ \).

10. In a triangle \( ABC \), the bisector of \( \angle BAC \) meets the side \( BC \) at the point \( D \). Knowing that \(|BD| \cdot |CD| = |AD|^2 \) and \( \angle ADB = 45^\circ \), determine the angles of triangle \( ABC \).

11. The real-valued function \( f \) is defined for all positive integers. For any integers \( a > 1, b > 1 \) with \( d = \gcd(a, b) \), we have

\[
f(ab) = f(d) \left( f\left(\frac{a}{d}\right) + f\left(\frac{b}{d}\right) \right).
\]
Determine all possible values of $f(2001)$.

12. Let $a_1, a_2, \ldots, a_n$ be positive real numbers such that $\sum_{i=1}^{n} a_i^3 = 3$ and $\sum_{i=1}^{n} a_i^5 = 5$. Prove that $\sum_{i=1}^{n} a_i > 3/2$.

13. Let $a_0, a_1, a_2, \ldots$ be a sequence of real numbers satisfying $a_0 = 1$ and $a_n = a_{[7n/9]} + a_{[n/9]}$ for $n = 1, 2, \ldots$. Prove that there exists a positive integer $k$ with $a_k < \frac{k}{2001!}$.
(Here $[x]$ denotes the largest integer not greater than $x$.)

14. There are $2n$ cards. On each card some real number $x$, $1 \leq x \leq 2$, is written (there can be different numbers on different cards). Prove that the cards can be divided into two heaps with sums $s_1$ and $s_2$ so that $\frac{n}{n+1} \leq \frac{s_1}{s_2} \leq 1$.

15. Let $a_0, a_1, a_2, \ldots$ be a sequence of positive real numbers satisfying $i \cdot a_i^2 \geq (i+1) \cdot a_{i-1} a_{i+1}$ for $i = 1, 2, \ldots$ Furthermore, let $x$ and $y$ be positive reals, and let $b_i = xa_i + ya_{i-1}$ for $i = 1, 2, \ldots$. Prove that the inequality $i \cdot b_i^2 > (i+1) \cdot b_{i-1} b_{i+1}$ holds for all integers $i \geq 2$.

16. Let $f$ be a real-valued function defined on the positive integers satisfying the following condition: For all $n > 1$ there exists a prime divisor $p$ of $n$ such that $f(n) = f(n/p) - f(p)$. Given that $f(2001) = 1$, what is the value of $f(2002)$?

17. Let $n$ be a positive integer. Prove that at least $2^{n-1} + n$ numbers can be chosen from the set $\{1, 2, 3, \ldots, 2^n\}$ such that for any two different chosen numbers $x$ and $y$, $x + y$ is not a divisor of $x \cdot y$.

18. Let $a$ be an odd integer. Prove that $a^{2^m} + 2^{2^n}$ and $a^{2^m} + 2^{2^m}$ are relatively prime for all positive integers $n$ and $m$ with $n \neq m$.

19. What is the smallest positive odd integer having the same number of positive divisors as 360?

20. From a sequence of integers $(a, b, c, d)$ each of the sequences

$$(c, d, a, b), \quad (b, a, d, c), \quad (a + nc, b + nd, c, d), \quad (a + nb, b, c + nd, d)$$

for arbitrary integer $n$ can be obtained by one step. Is it possible to obtain $(3, 4, 5, 7)$ from $(1, 2, 3, 4)$ through a sequence of such steps?