Plenary Lectures

Paul Balmer
University of California, Los Angeles

Tensor triangular geometry

Tensor triangular geometry is an umbrella term to designate geometric techniques appearing via tensor triangulated categories in various areas of mathematics, like algebraic geometry, homotopy theory, modular representation theory, noncommutative topology, motivic theory, etc. I will introduce the concept of spectrum of a tensor triangulated category and try to advertise the use of this tool, by giving some applications as well as providing some examples of computations.

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Mladen Bestvina
University of Utah

Bounded Cohomology via Quasi-Trees

My plan is to present an introduction/survey starting with definitions of quasi-homomorphisms and bounded cohomology, basic construction for free groups due to Brooks, main theorems in the subject (Bavard, Epstein-Fujiwara, Burger-Monod) with applications to rigidity, and end with a sketch of a recent result, joint with Bromberg and Fujiwara, that provides proofs that $H^2_b(G)$ is infinite dimensional in all known and several new cases (including with twisted coefficients), by essentially reducing the situation to the Brooks argument.

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Natalia Castellana Vila
Universitat Autonoma de Barcelona

Constructing maps from p-completed classifying spaces

In this talk I will discuss joint works with A. Libman and L. Morales. The main tool used to understand and classify maps between p-completed classifying spaces is the existence of mod p homology decompositions of these spaces in terms of p-local information. Usually one starts with p-local algebraic information (subgroups and associated orbit categories) and tries to construct a map which realizes this data after restriction. Two situations will be described: maps between classifying spaces of p-completed finite groups and maps to the p-completed classifying space of a unitary group.

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Mark Andrea de Cataldo
Stony Brook University, New York

The Hodge theory of character varieties

After a brief review of the statement of the decomposition theorem, I will discuss a geometric description of the perverse filtration on the cohomology of algebraic varieties via the Lefschetz hyperplane theorem (joint with L. Migliorini) and I will discuss work in progress concerning the mixed Hodge theory of character varieties (joint work in progress with T. Hausel and L. Migliorini).

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Alexander N. Dranishnikov
University of Florida

On Gromov's macroscopic dimension

Gromov introduced the notion of macroscopic dimension $\dim_{mc}$ to study large scale properties of universal coverings X of manifolds M with positive scalar curvature. He proposed a conjecture that for n-dimensional manifolds M always $\dim_{mc} X < n-1$. I plan to present partial results (joint with D. Bolotov) towards Gromov's conjecture.

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Nitu Kitchloo
University of California, San Diego

Real Johnson-Wilson Theories

I plan to describe the nature of a family of cohomology theories known as real Johnson-Wilson theories. They can be construted as fixed points of the involution given by complex conjugation acting on the standard Johnson-Wilson theories. I will show how these theories have very rich structure that can be exploited to obtain various interesting results, including new non-immersion results for real projectice spaces. If time permits, I will talk about the question of orientation for these theories, and the relation to Lubin-Tate spectra.

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Pascal Lambrechts
Université Catholique de Louvain

On the rational homology of spaces of smooth embeddings

We consider (a variation of) the space of smooth embeddings $\mathrm{Emb}(M; R^n)$, of a compact manifold M in a large euclidean space. We prove that its homology is an invariant of the rational homotopy type of M. A special case of this is when M is 1-dimensional in which case we get that the homology of this embedding space is the homology of an explicit graph complex. The techniques are Goodwillie-Weiss calculus of embeddings, Weiss orthogonal calculus, and a relative version of Kontsevich's formality of the little disks operad. This is joint work with Greg Arone, Victor Turchin, and Ismar Volic.

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Anatoly Libgober
University of Illinois at Chicago

Applications of elliptic genus

I will discuss apllications of two variable elliptic genus to various problems related to topology of complex manifolds which inlcude algebro-

geometric analogs of Novikov conjecture, invariants of singular real algebraic varieties and topology of loops spaces.

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Piotr Nowak
Texas A&M University

Controlled coarse homology and isoperimetric inequalities

In this talk we will introduce a controlled homology theory for discrete metric spaces. This homology is a quasi-isometry invariant and generalizes the uniformly finite homology of Block and Weinberger. We will present two main results. First we will show that a certain fundamental class vanishes in linearly controlled homology for every infinite, finitely generated group. This is a homological version of the classical Burnside problem in group theory, with a positive answer. Then we characterize vanishing of the fundamental class in our homology in terms of an isoperimetric inequality on G and show how it is related to amenability. As applications we characterize existence of primitives of the volume form with prescribed growth, which generalizes Gromov's answer to Sullivan's question. We also will show that coarse homology classes obstruct weighted Poincaré inequalities of Li and Wang and present applications to Pontryagin classes and distortion of diffeomorphisms.

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Taras Panov From real quadrics to polytopes via manifolds
Moscow University

Manifolds obtained as complete intersections of real quadratic hypersurfaces in a complex space have a natural torus action on them, and are known to toric topologists as moment-angle manifolds. They correspond naturally to combinatorial simple polytopes, and a direct passage from quadrics to polytopes involves some nice convex geometrical reasoning. The quadratic equations or the polytopes

may be very simple, while the corresponding moment-angle manifolds usually are quite complicated topologically. Studying their topology proves to be an interesting and challenging problem.

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Jörg Schürman
University of Münster

Motivic characteristic classes for singular spaces

The talk gives an introduction to the recent theory of motivic characteristic classes for singular spaces using the language of Borel-Moore functors due to Levine-Morel. These are universal characteristic class transformations, which can be defined on the relative Grothendieck group of complex algebraic varieties. The Hirzebruch class transformation unifies the following transformations: The Chern class of MacPherson, the Todd class of Baum-Fulton-MacPherson and the L-class of Cappell-Shaneson. The motivic Chern class transformation is a refined K-theoretical version unifying corresponding transformations of Baum-Fulton-MacPherson and of Siegel-Sullivan. The motivic classes can also be defined for mixed Hodge modules leading to characteristic classes related to intersection cohomology.

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Stefan Schwede
University of Bonn

Triangulated categories: Enhancements, rigidity and exotic models

The notion of a triangulated category is a conceptual language used in several areas of pure mathematics. It has two historical origins, going back to the 1960s. In algebraic geometry, Verdier used triangulated categories as a convenient framework to describe duality phenomena.

Around the same time, Puppe introduced a very similar notion to extract the key formal properties of the stable homotopy category of algebraic topology.
It was apparent from the beginning though, that in many examples of interest the passage to the triangulated category loses information. Various concept of "models" or "enhancements" were proposed to capture the higher order information not seen by the triangulated category. In this talk I will survey recent results and illustrate by examples that almost anything can happen: interesting triangulated categories may have a unique model (rigidity), admit "exotic" models or may not have any enhancement at all.

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Andrzej Zuk
University of Paris

Automata groups

The class of automata groups contains several remarkable countable groups. Their study has led to the solution of a number of important problems in group theory. Its recent applications have extended to the fields of algebra, geometry, analysis and probability. We will present recent developments related to amenability and growth.

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Talks

Maia Averett
Mills College, Oakland

Completion of real Johnson-Wilson theory gives fixed points of Morava E-theory

This talk will summarize some recent work on a new family of cohomolology theories made accessible by Kitchloo and Wilson, the so-called real Johnson-Wilson theories $ER(n)$. We will relate the theories $ER(n)$ to homotopy fixed points of the Morava E-theories E_n under an action of a certain subgroup of the Morava stabilizer group. In doing so, we obtain a calculation of the coefficients of the homotopy fixed points of E_n for this subgroup and also see that after completion the $ER(n)$ are commutative S-algebras.

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Noe Barcenas
Westfälische Wilhelms-Universität Münster

Equivariant cohomotopy for Infinite Groups and the Segal Conjecture for Families

This is part of mi Ph.D. Project, supervised by Wolfgang Lueck. In this talk, I shall discuss the extension of equivariant cohomotopy in the framework of proper actions. I shall give a proof of the Segal conjecture for families of discrete subgroups in this context.

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Piotr Beben
National University of Singapore

Some stable and unstable homotopy theory of low rank cell complexes

Various decompositions of p-localized cell complexes have been given by Cohen, Neisendofer, Selick, and Wu. By giving even finer decompositions in certain low ranks, some interesting results regarding the stable and unstable homotopy theory of these cell complexes emerge. This is a joint work with Jie Wu.

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Wojciech Chacholski
KTH Royal Institute of Technology, Stockholm

Cofinality and homological algebra

The way we study R-modules is by investigating properties of various natural constructions: tensor and symmetric products, hom functors, etc. The properties we are most interested in are of homological nature: exactness, preservation of limits, commutation with other constructions. The aim of the talk is to show how these natural constructions can be studied globally. The idea is to assemble them over large subcategories of R-modules and study the derived functors of such an assembly. A convenient tool used by homotopy theorists to study such derived functors is cofinality. In the talk I will present how to use it to translate between homological algebra and cofinality properties of various categories. For example, I will illustrate how one can characterize the projective dimension of a module. In the same spirit I will give a reinterpretation of the Dold Kan correspondence. The talk is based on the doctoral thesis of Fredrik Nordstrom.

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Natalia Dobrinskaya
Vrije Universiteit Amsterdam

Loops on polyhedral products: geometric models and homology

The construction of the polyhedral products (K-products) attracts a lot of interest during last years. This is a functor which associates to any simplicial complex K on m vertices and m based topological spaces the natural subspace of the cartesian product of these spaces. In my talk I plan to discuss the problem of studying the loops on this construction. We construct its geometric model as labelled configuration spaces which leads to the homology decompositions of the loop space homology. We also give the excplicit presentation of the loop space algebra of K-products for the certain class of simplicial complexes K in terms of higher commutator products and the generalizations of classical relations (generalized Jacobi rule, Leibnitz rule). I will also discuss the application in toric topology (joint work with Nigel Ray), and in the theory of subspace arrangements.

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Marek Golasiński
Nicolaus Copernicus University, Torun

Gottlieb groups: their applications and generalizations

Given a self-map $f : (X, x_0) \to (X, x_0)$ of a pointed space (X, x_0), the Nielson number $N(f)$ is a lower bound for the number of fixed points of f. One of the main tools to compute $N(f)$ is the Jiang subgroup $J(f, x_0) \subset \pi_1(X, x_0)$. The group $G_1(X, x_0) = J(\mathrm{id}_X, x_0)$ and its generalizations $G_n(X, x_0) \subset \pi_n(X, x_0)$ for $n \geq 1$ called the *Gottlieb* or *evaluation subgroups* have been first defined and then studied by D.H. Gottlieb and others. These subgroups are related to the problem of sectioning fibrations with fibre X.

(1) We take up a study of the Gottlieb groups $G_k(\mathbb{S}^n, s_0)$ and $G_k(\mathbb{F}P^n, p_0)$ of spheres $\mathbb{S}^n$ and projective spaces $\mathbb{F}P^n$ over $\mathbb{F} = \mathbb{R}$, $\mathbb{C}$, $\mathbb{H}$.

(2) The Gottlieb groups $G_n(\Sigma X)$ of the suspension ΣX are considered and an approach to the Gottlieb groups $G_n(M(A, m))$.

This is a joint work with Juno Mukai.

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Nikolay Gusevskii
Universidade Federal de Minas Gerais, Belo Horizonte

Representations of surface groups in complex hyperbolic space

We construct quasi-Fuchsian groups isomorphic to the fundamental group of a closed oriented surface of genus $g > 1$ acting on the complex hyperbolic plane H^2_C with limit set a wild knot. Also, we study the Teichmuller space $T(G)$ of faithful, discrete, type preserving representations of a Fuchsian group G without parabolic elements in the holomorphic isometry group $PU(2,1)$ of H^2_C. We show that $T(G)$ is not connected, and that the Toledo invariant does not distinguish different connected components of $T(G)$.

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Paul Igodt
K.U.Leuven, Belgium

An evolution of the Topological Spherical Space Form Problem

(This is for a short talk or just a poster, and reporting about on-going work at Kortrijk) Until the late 70's, a central question regarding finite group actions was the topological spherical space form problem. That is, when does a finite group act freely on a sphere $\mathbb{S}^n$? In 1978, using Swan's criteria, Madsen, Thomas and Wall settled the question by giving a complete algebraic characterization. One of the problems that has evolved is to classify groups that can at least act freely and properly discontinuously on $\mathbb{S}^n \times \mathbb{R}^k$. In 2001, Adem and Smith showed that a countable group G acts freely and properly discontinuously on some $\mathbb{S}^n \times \mathbb{R}^k$ if and only if G has periodic cohomology. In this talk, we will explore this cohomological periodicity, its implied conditions, and their relations to the Euclidean space form problem. This is a joint work with Dennis Dreesen and Nansen Potrosyan.

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Sören Illman
University of Helsinki

Equivariant Alexander-Spanier cohomology for actions of the p-adic groups A_p

Let G be any compact abelian group, for example, $G = A_p$, p a prime. We describe the construction of a G-equivariant Alexander-Spanier type cohomology theory. This equivariant cohomology theory satisfies appropriate equivariant analogues of all seven Eilenberg-Steenrod axioms. A key notion, which enables us to handle the case where G is a non-Lie group, is that of a balanced G-equivariant cochain. Interestingly enough it is not clear whether the strong excision property (mapping excision) holds, in the case where G is a non-Lie group.

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Iryna Iurchuk
Institute of Mathematics of NAS of Ukraine

On topological equivalence of some classes of continuous functions

We will be interested in topological equivalence of two classes of continuous functions defined on either the union of two circles or the 2-dimensional disk with a finite number of "holes". Necessary and sufficient conditions for functions of every class to be equivalent will be obtained in terms of their invariants which are contained "all information about them".

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Rafał Lutowski
University of Gdańsk

On Symmetry of Flat Manifolds

We give an example of a Bieberbach group Γ for which $\mathrm{Out}(\Gamma)$ is a cyclic group of order n. We also calculate the outer automorphism

group of a direct product of n copies of a Bieberbach group with trivial center, for $n \in \mathbb{N}$. As a corollary we get, that every symmetric group can be realized as an outer automorphism group of some Bieberbach group.

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Snigdhayan Mahanta
IHES

KK-equivalences and T^{2n}-duality type isomorphisms.

KK-theory is a bivariant theory that can be used to turn the category of separable C^*-algebras into an additive (in fact triangulated) category. T-duality is a duality in string theory that relates type IIA and IIB theories. One of the features of topological (homological) T-duality is an isomorphism at the level of K-theories, when applied an even number of times. In this talk I will explain how an isomorphism in KK-theory can be used to deduce a T^{2n}-duality type isomorphism between certain differential graded categories localized along K-theory spectra.

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Swarnali Majumder, Prabhakar G. Vaidya
National Institute of Advanced Studies, India

A Preliminary Investigation of the Feasibility of using SVD and Algebraic Topology to study Dynamics on a Manifold

While fitting equations to data, we often find that the data lies on a manifold of dimension M which has been embedded in R^N where N is greater than M. We have recently shown that the attempts to develop equations in the larger dimension leads to foliations which might be unstable. An alternative is to develop dynamics in M dimension itself. We have found that both the SVD, (Singular Value Decomposition), in its linear and its recent nonlinear version; and Algebraic Topology are useful in arriving at a solution. We illustrate the ideas by covering some data sets generated by a four dimensional differential equation by overlapping patches. We use SVD to establish M and to get a parametric representation for each patch. We then transform the patches to fit in the R^M framework. The consistency conditions in the network yield Algebraic Topological information. As a simple example, we show that one set of our data lies on a Mobius Strip. This is a joint work with Prabhakar G. Vaidya.

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Sergiy Maksymenko
Institute of Mathematics of NAS of Ukraine

Path components of certain diffeomorphism groups with respect to distinct Whitney topologies

Let $f : R^2 \to R$ be a homogeneous polynomial and $S(f) = \{h \in Diff(R^2) : f \circ h = f\}$ be the stabilizer of f with respect to the right action of the group of diffeomorphisms of R^2. For every $r \geq 0$ let also $S_{id}(f)^r$ be the identity path component of $S(f)$ with respect to the weak C^r Whitney topology. Theorem [1]. $S_{id}(f)^\infty = \cdots = S_{id}(f)^1$. Moreover, $S_{id}(f)^1 \neq S_{id}(f)^0$ if and only if f is a product of at least two distinct irreducible over R quadratic forms. This local result is then applied to study stabilizers of smooth functions on surfaces.

References
1. S. Maksymenko, Connected components of partition preserving diffeomorphisms, to appear in Methods of Functional Analysis and Topology, http://lanl.arxiv.org/pdf/0806.0159

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Sergey A. Melikhov
Steklov Math Institute, Moscow

Steenrod homotopy and classifying spaces of p-adic integers

The open-ended "Research Problem B_2" from Isbell's "Uniform spaces" (1964) asks, essentially, whether his uniform structure for finite-dimensional polyhedra can be extended to all polyhedra so that each separable metrizable complete uniform (SMCU) space is an inverse limit of uniform polyhedra, and every uniform polyhedron is an ANE for the category of SMCU's. Our solution to this problem involves a rethink of foundations of PL topology. Based on it we define a Steenrod homotopy category of SMCU's, which on compacta coincides with the strong shape category and on uniform polyhedra with the usual (non-uniform!) homotopy category. Let G be the group of p-adic integers. Using "complex K-theory in Steenrod homotopy" we prove Theorem. Every ANE for the category of free G-SMCU's is non-compact. E.V. Shchepin and I independently verify that compact 3-manifolds with free action of G satisfy the hypothesis. Corollary: the 3-dimensional free Hilbert-Smith Conjecture.

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Krzysztof Pawałowski
Adam Mickiewicz University, Poznań

Smooth actions of finite nonsolvable groups on spheres

We prove that except for $G = \mathrm{Aut}(A_6)$, the automorphism group of the alternating group A_6, any finite nonsolvable group G with $r_G \geq 2$ has a smooth action on a sphere with exactly k fixed points for a

given integer $k \geq 2$, such that the representations of G at any two different fixed points are not isomorphic to each other. Here, r_G is the number of real conjugacy classes of elements of G not of prime power order. Thus, for $k = 2$, we obtain a negative answer to the corresponding Smith question posed in 1960. It turns out that for $k \geq 3$, the algebraic condition that $r_G \geq 2$ is both necessary and sufficient for the existence of such actions of finite nonsolvable groups G not isomorphic to $\mathrm{Aut}(A_6)$. This is a joint work with Toshio Sumi.

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Adam Przeździecki
University of Warsaw of Life Sciences

A faithful and "almost" full functor from the category of graphs to the category of groups

The talk will describe a functor $F : \mathcal{G}raphs \to \mathcal{G}roups$ which is faithful and "almost" full in the sense that it induces bijections $F_{X,Y} : \mathrm{Hom}_{\mathcal{G}raphs}(X, Y) \cup \{*\} \to \mathrm{Rep}(FX, FY)$ for every X and Y. Here $\mathrm{Rep}(A, B) = \mathrm{Hom}(A, B)/B$. One obvious application of F is the possibility of embedding, up to constant maps, almost any point-set category into the homotopy category. Other, less obvious, applications that will be presented are related to set theoretic problems involving localizations, that is idempotent functors.

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José L. Rodríguez
University of Almería

Cellular covers of cotorsion-free modules

In this paper we improve recent results dealing with cellular covers of R-modules. Cellular covers come up in the context of homotopical localization of topological spaces. Recall that a homomorphism of R-modules $\pi : G \to H$ is a *cellular cover* over H if π induces an isomorphism $\pi_* : Hom_R(G, G) \cong Hom_R(G, H)$, given by $\pi_*(\varphi) =$

$\pi\varphi$. On the one hand, we show that every cotorsion-free R-module of rank $\kappa < Cont$ is realizable as the kernel of some cellular cover $G \to H$ where the rank of G is $\kappa + 1$ (or 3, if $\kappa = 1$). The proof is based on Corner's classical idea of how to construct torsion-free abelian groups with prescribed countable endomorphism rings. This complements results by Buckner–Dugas. On the other hand, we prove that every cotorsion-free R-module satisfying some rigid conditions admits arbitrarily large cellular covers. This improves results by Fuchs-Göbel and Farjoun-Göbel-Segev-Shelah. This is a joint work with Rüdiger Göbel and Lutz Strüngmann.

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Albert Ruiz and Antonio Viruel
Universitat Autonoma de Barcelona

Massey products and classifying spaces of 2-groups

Given a finite group G, it is well known that the cohomology algebra $H^*(G;\mathbb{F}_p)$ does not determine the isomorphism type of G, not even when the Steenrod action is taken into account. Therefore, a cohomological characterization of finite groups needs to consider secondary operations. In this talk we shall show that if G is a maximal class 2-group, then BG (the classifying space of G), and therefore G itself, is completely determined by the cohomology algebra $H^*(G;\mathbb{F}_p)$ and its iterated Massey products. Actually:

Theorem:
Let X be a 2-complete space and G be a maximal class 2-group such that there exists an abstract algebra isomorphism $\psi : H^*(G;\mathbb{F}_2) \to H^*(X;\mathbb{F}_2)$ which is compatible with iterated Massey products. Then $X \simeq BG$.

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Shoham Shamir
University of Sheffield

Complete intersections in rational homotopy theory

In commutative algebra, complete intersection rings are the next best thing after regular rings. The quotient of a polynomial ring by a regular ideal is a prime example of a complete intersection ring. Gulliksen showed that a local Noetherian ring is complete intersection if and only if its homology has polynomial growth. Benson and Greenlees recently characterized local complete intersection rings by the existence of a certain structure on their derived category. These definitions have obvious adaptations for rational spaces. For simply connected rational spaces these adapted definitions are shown to be equivalent, yielding a structural characterization of complete intersection rational spaces using spherical fibrations. This is joint work with John Greenlees and Kathryn Hess.

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Jan Spaliński
University of Warsaw of Technology

A discrete model of O(2) homotopy theory

In this talk I will show how a triple consisting of a Z/2-set, a dihedral set and appropriate compatibility data serves as a model for equivariant homotopy theory of O(2)-spaces (keeping tract of both the finite subgroups and the subgroups S^1 and O(2)). This work is built on a result of A. Blumberg who has shown that a triple consisting of a simplicial set, a cyclic set and appropriate compatibility data serves a model for equivariant homotopy theory of spaces with a circle action (keeping track of both the finite subgroups of the circle itself).

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Victor Turchin
Kansas State University

Hodge decomposition in the homology of long knots

It will be described a natural splitting in the rational homology and homotopy of the spaces of long knots $Emb(R^1, R^N)$. This decomposition arises from the cabling maps in the same way as a natural decomposition in the homology of loop spaces arises from power maps. The generating function for the Euler characteristics of the terms of this splitting will be presented. Based on this generating function one can show that both the homology and homotopy ranks of the spaces in question grow at least exponentially. There are two more motivations to study this decomposition. First, it is related to the study of the homology of higher dimensional knots $Emb(R^k, R^N)$. Second, it is deeply related to the question whether Vassiliev invariants can distinguish knots from their inverses.

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Min Yan
Hong Kong University of Science and Technology

Replacement of Fixed Sets for Compact Group Action

If M and N are equivariantly homotopy equivalent G-manifolds, then the fixed sets M^G and N^G are also homotopy equivalent. The replacement problem asks the converse question: If F is homotopy equivalent to the fixed set M^G, is $F = N^G$ for a G-manifold equivariantly homotopy equivalent to M? The answer to the question ranges from always possible to the rigidity. We present a general theorem and study some special cases.

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Andreas Zastrow
University of Gdańsk

The non-Hausdorffness of Milnor-Thurston homology groups

The basic idea of Milnor-Thurston Homology-Theory (also called: measure-homology-theory) is that via certain measures that can be put on the set of all singular simplices, infinite chains in homology theory can be described that should generate the ordinary singular homology groups. Since its original use in Thurston's work (who also credited Milnor) it has been described by Gromov and Ratcliffe, and some papers appeared which discussed the well-definedness of this homology-theory, and whether it always generates isomorphic or (with respect to the Gromov norm even: isometric) groups as singular theory, ('yes' for triangulable spaces, but 'no' in general). Recently a paper by Berlanga appeared which topologized these homology groups and proved that for triangulable spaces the first homology groups are always Hausdorff. This talk will be about to answer in negative the question from this paper, whether this also holds in general, ie. also for non-triangulable spaces.

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