Conference on Algebraic Topology

ABSTRACTS

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Matthew Ando
Elliptic cohomology in the last ten years

In the 1980's Witten discovered the remarkable connection between string theory and elliptic cohomology. Since then there has been great progress in the study of elliptic cohomology in stable homotopy theory and also in the mathematics of string theory. It is now possible to tell a story in stable homotopy theory which complements and significantly refines the physical results described by Witten. In addition, many things we don't know about elliptic cohomology seem to correspond closely to issues of current interest in the study of open strings.

Gregory Arone
Spaces of embeddings and the Fulton-McPherson compactification

We will discuss certain filtrations of the space of embeddings of manifolds. To describe the filtration we will need certain construction closely related to the Fulton-Mcpherson compactification. We will discuss the properties of the filtration and relate it to some classical constructions in geometric topology.

Grzegorz Banaszak
On the 2-adic cyclotomic elements in K-theory and etale cohomology of the ring of integers

I will define 2-adic cyclotomic elements in K-theory and etale cohomology of the ring $\mathbb{Z}[\frac{1}{2}]$. I will also show how to construct a map from 2-adic K-theory to 2-adic etale cohomology (using relative etale K-theory) to compare these cyclotomic elements. There are a few obstacles in these constructions when working with prime number 2, which do not appear for odd primes. I will discuss this during my talk. Applying results of Voevodsky, Rognes and Weibel I will explain how one can compute $K_{2n-1}(\mathbb{Z}) \otimes \mathbb{Z}_2$, for n odd, in terms of 2-adic cyclotomic elements.

Maria Basterra
Gamma Homology, Topological Andre-Quillen Homology and Stabilization

The study of obstructions to E-infinity ring structures on a spectrum has produced a number of apparently different constructions of homology in the category of E-infinity ring spectra. In this talk we report on joint work with Randy McCarthy in which we reconcile the above constructions when we work over the Eilemberg-Mac Lane spectrum corresponding to a commutative ring.

David Benson

Tate cohomology of finite groups, cohomology of differential graded algebras, and obstruction theory

(joint work with Henning Krause and Stefan Schwede)

After a brief introduction to Tate cohomology, I shall discuss the following question. Given a module X over Tate cohomology of a finite group G, how do we tell whether it is the Tate cohomology of a kG-module? I shall describe a single obstruction which determines whether it is a direct summand of the Tate cohomology of a module. The obstruction lies in Ext of the X with itself, in degree (3,1). This obstruction is the image of a universal predecessor, which lies in the Hochschild cohomology of the Tate cohomology of G, in the same degree.

The analogous question for differential graded algebras is this. Given a module over the cohomology of a DGA, how do we tell whether it is the cohomology of a module? The answer is essentially the same: there is a single obstruction to realizability as a direct summand of the cohomology of a module, and there is a universal predecessor in degree (3,1) Hochschild cohomology of the DGA.

The obstruction in each case can be viewed as a universal Massey triple product. In the language of A_∞-algebras, it is the element of Hochschild cohomology determined by the structure map m_3.

For an arbitrary triangulated category with direct sums, the obstruction in Ext in degree (3,1) can still be formulated, but it seems that more structure is necessary for the predecessor in Hochschild cohomology to work.

It is a pleasure to acknowledge that the research described in this talk is joint work with Henning Krause and Stefan Schwede.

Jon Berrick

Groups with infinite homology

In joint work with P H Kropholler, we consider the appropriate generalization of the fact that cyclic groups have an infinite group for the direct sum of all their (positive-dimensional) integral homology groups. An attractive feature of this research is the use it makes of Miller's theorem on the Sullivan Conjecture.

Stephen John Bigelow
Representations of braid groups

Braid groups have many equivalent definitions, and an even larger number of applications. The Jones representation of braid groups by matrices was originally defined using the Hecke algebra, and can be used to obtain the famous Jones polynomial of a knot or link.

I will give two definitions of the braid group - one algebraic and one topological. I will then describe a new definition of the Jones representation from this more topological point of view. It is to be hoped that this will shed new light on the topological "meaning" of the invariants obtained from it.

Morten Brun
Title: K-theory and Hochschild-Witt homology

I will present a new homology theory for associative algebras that I call Hochschild-Witt homology. It is related to the de Rham-Witt complex in a way similar to the way Hochschild homology is related to the de Rham complex: There is a map from the de Rham-Witt complex to the complex of Hochschild-Witt homology groups euqipped with a boundary similar to Connes̓ boundary map. In the smooth case this map is an isomorphism. I will explain how Hochschild-Witt homology is related to topological cyclic homology and algebraic K-theory.

Christophe Eyral
Zariski-Lefschetz theory for singular varieties

We study the homotopy of a quasi-projective variety in a complex projective space following Lefschetz's method, that is, by considering its sections by the hyperplanes of a pencil. Specifically, we obtain a theorem of Lefschetz type which generalizes in a certain direction the best-known results due to Hamm, Lê, Goresky and MacPherson. This theorem is proved by induction on the dimension of the ambient projective space with the help of a theorem on pencils with generic axis which is the main result of the author's thesis. The latter compares the topology of the variety with that of its section by a generic hyperplane of the pencil, on the basis of the following comparisons: section by a generic hyperplane with section by the axis of the pencil; and sections by the exceptional hyperplanes with section by the axis. The effect of the singularities is measured by an invariant called "global rectified homotopical depth" (a global analogue of the notion of rectified homotopical depth of Grothendieck).

Wojciech Gajda

On cyclotomic numbers and the reduction map for the K-theory of the ingeres

We apply the recently proven compatibility of Beilinson and Soulé elements in K-theory to compute the density of rational primes p, for which the reduction map

$$K_{2n+1}(\mathbb{Z}) \to K_{2n+1}(\mathbb{F}_p)$$

is nontrivial. Here n is an even integer and $\mathbb{F}_p$ denotes the field of p elements. In the proof we use arithmetic of the cyclotomic numbers

$$u_k = \prod_{(a,l)=1;\, 1\leq a<l^k} (1-\xi_{l^k}^a)^{a^n},$$

where l is an odd prime, $k \geq 1$ and ξ_{l^k} is a primitive root of unity of order l^k. The density result on the redution map enables us to obtain an explicit description of an infinite family of elements in the Brauer group of the field $\mathbb{Q}(\xi_{l^k})$. We investigate divisibility of the number u_k in the group of certain S-units of the cyclotomic field $\mathbb{Q}(\xi_{l^k})$. It turns out that the divisibility of the number u_k is bounded from above by the l-part of the index of the subgroup of $K_{2n+1}(\mathbb{Z})$ generated by the nontorsion element which comes from Beilinson element. We also reformulate the Kummer-Vandiver conjecture on the class group of the cyclotomic field in terms of arithmetical properties of the numbers u_k.

Marek Golasiński

Polynomial and regular maps into 2-spheres

(joint work with Francisco Gomez Ruiz)

We make use of subtle algebraic methods to establish an injection

$$\mathcal{P}_{\mathbb{C}}(\mathbb{V}_{\mathbb{C}}, \mathbb{S}^2_{\mathbb{C}}) \longrightarrow \mathcal{R}_{\mathbb{R}}(\mathbb{V}, \mathbb{S}^2)$$

from the sets of complex-valued polynomial to real-valued regular maps provided that $\mathbb{V} \subseteq \mathbb{R}^n$ is a compact subset and its Zariski closure $\mathbb{V}_{\mathbb{C}}$ in the affine space $\mathbb{C}^n$ is irreducible. Then we list complex-valued polynomial maps $\mathbb{S}^2_{\mathbb{C}} \to \mathbb{S}^2_{\mathbb{C}}$ of any Brouwer degree and deduce that the injection $\mathcal{P}_{\mathbb{C}}(\mathbb{S}^2_{\mathbb{C}}, \mathbb{S}^2_{\mathbb{C}}) \to \mathcal{R}_{\mathbb{R}}(\mathbb{S}^2, \mathbb{S}^2)$ yields an isomorphism $[\mathbb{S}^2_{\mathbb{C}}, \mathbb{S}^2_{\mathbb{C}}] \xrightarrow{\cong} [\mathbb{S}^2, \mathbb{S}^2]$ of cyclic infinite homotopy groups.

At the end we show that every non-zero even Brouwer degree of the spheres $\mathbb{S}^n$ and $\mathbb{S}^n_{\mathbb{C}}$ cannot be realized by a real-valued (resp. complex-valued) homogeneous polynomial map provided that n is even.

Joanna Kania-Bartoszynska

Knots, characters and integration

The quantum invariants of 3-manifolds are only defined at roots of unity. However, there is ample evidence that they exist as holomorphic functions on the unit disk, that diverge everywhere on the unit circle but at roots of unity. We take a step towards seeing that this holds in general, by showing that the Yang-Mills measure exists as a trace on the Kauffman bracket skein algebra of a closed surface F. The Kauffman bracket skein algebra of a 3-manifold is a complex vector space with isotopy classes of knots in the manifolds as its basis, divided by the relation corresponding to the Kauffman bracket knot invariant. This relation involves a complex parameter, t.

When the deformation parameter t is a generic point on the unit circle, then the measure does not converge. At roots of unity it coincides with Turaev's shadow world invariant. Finally, at $t = -1$, the Yang-Mills measure is the integration against the symplectic measure on the $SU(2)$ character variety of the fundamental group of the surface F.

An amazing consequence of all this is explicit formulas for integrating functions over the $SU(2)$ character variety of a surface with respect to the symplectic measure.

Piotr Krasoń

The support problem for K-theory of number fields

(joint work with G. Banaszak and W. Gajda)

The support problem was originally stated by Pál Erdös , who raised the following question in 1988:

Let $Supp(m)$ denote the set of prime divisors of the integer m. Let x and y be two natural numbers. Are the following two statements equivalent ?

(1) $Supp(x^n - 1) = Supp(y^n - 1)$ *for every* $n \in N$,
(2) $x = y$

This question, along with its extension to all number fields, and also its analogue for elliptic curves, were solved by Corrales-Rodrigáñez and Schoof in the paper [C-RS].
We consider the support problem for algebraic K-theory of a number field F. Let k_v denote the residue field for a prime v in F and r_v the corresponding reduction map for algebraic K-theory.
We prove the following theorem [BGK1].

Theorem 1. *Let P, Q be two nontorsion elements of the algebraic K-theory group $K_{2n+1}(F)$, where n is an even, positive integer. Assume that for almost every v and every integer m the following condition holds in the group $K_{2n+1}(k_v)$:*

$$m\, r_v(P) = 0 \qquad \textit{implies} \qquad m\, r_v(Q) = 0.$$

Then the elements P and Q of $K_{2n+1}(F)$ are linearly dependent over Z.

Theorem 1 implies the following result concerning the reduction maps

$$r'_v\colon\ H_{2n+1}(K(\mathcal{O}_F);\, Z) \to H_{2n+1}(Sl(k_v);\, Z)$$

defined on the integral homology of the K-theory spectrum $K(\mathcal{O}_F)$.

Theorem 2. *Let n be an even, positive integer. Let P', Q' be two nontorsion elements of the group $H_{2n+1}(K(\mathcal{O}_F);\, Z)$. Assume that for almost every prime ideal v and for every integer m the following condition holds in the group $H_{2n+1}(Sl(k_v);\, Z)$:*

$$mr'_v(P') = 0 \qquad \textit{implies} \qquad mr'_v(Q') = 0.$$

Then the cohomology classes P' and Q' are linearly dependent in $H_{2n+1}(K(\mathcal{O}_F);\, Z)$.

Further generalizations of the support problem of Erdös to the context of l-adic representations of the absolute Galois group of a number field will also be indicated [BGK2].

[BGK1] G. Banaszak, W. Gajda, P. Krasoń *A support problem for K-theory of number fields* C. R. Acad. Sci. Paris Sér. 1 Math. 331 no. 3 2000 pp. 185 - 190

[BGK2] G. Banaszak, W. Gajda, P. Krasoń *A support problem for the intermediate jacobians of l-adic representations* (in preparation)

[C-RS]C.Corralez-Rodrigáñez, R.Schoof *Support problem and its elliptic analogue* Journal of Number Theory 64 1997 pp. 276-290

Nicholas Kuhn

Periodic homology of infinite loop spaces

Consider the Goodwillie tower of the functor that assigns to a spectrum X, the suspension spectrum of its 0th space. This tower has rth fiber equal to the rth extended power of X. Our theorem is that, after localizing at any Morava K-theory $K(n)$ $(n > 0)$, the tower naturally splits (for ALL spectra X). This has implications for computing the E-homology or cohomology of infinite loopspaces, where E is Bousfield equivalent to $K(n)$. It also suggests some general conjectures regarding how Goodwillie calculus and $K(n)$-localization interact.

Markus Linckelmann

On spaces and spectra associated with blocks of finite groups

Wim Malfait

Bieberbach groups with polycyclic-by-finite (outer) automorphism group

(joint work with Andrzej Szczepański)

Let M be a flat Riemannian manifold of dimension n and assume that $E = \pi_1(M)$ denotes its fundamental group. Then E is a torsion–free group fitting into an extension $\mathbb{Z}^n \hookrightarrow E \twoheadrightarrow F$ where $\mathbb{Z}^n$ is maximal abelian in E and F is a finite group. We refer to F as the holonomy group of $M = E\backslash\mathbb{R}^n$ and E is said to be a Bieberbach group. The holonomy group F acts on $\mathbb{Z}^n$ by conjugation in E, defining a faithful representation $T : F \to \mathrm{Gl}(n, \mathbb{Z})$. It is known that $\mathrm{Out}(E)$ is finite if and only if all $\mathbb{Q}$-irreducible components in the $\mathbb{Q}$-decomposition of the holonomy representation T are $\mathbb{R}$-irreducible and of multiplicity one. Essentially, the proof reduces to analyzing when the normalizer of $T(F)$ in $\mathrm{Gl}(n, \mathbb{Z})$ is finite.

Motivated by Tits' alternative, we develop a necessary and sufficient condition to decide whether the normalizer of a finite group of integral matrices is polycyclic-by-finite or is containing a non-abelian free group. If $T : F \to \mathrm{Gl}(n, \mathbb{Z})$ is a faithful representation of a finite group F, then the normalizer of $T(F)$ in $\mathrm{Gl}(n, \mathbb{Z})$ is polycyclic-by-finite if and only if all components in the $\mathbb{Q}$-decomposition of T occur with multiplicity one and the Schur index over $\mathbb{Q}$ of each $\mathbb{Q}$-irreducible component which is $\mathbb{R}$-reducible is equal to one. This result translates straightforwardly into a criterion for the (outer) automorphism group of a Bieberbach group to be polycyclic-by-finite (or alternatively, to contain a non-abelian free group).

Wacław Marzantowicz
Homotopical dynamics

A natural number is called the homotopy minimal period of a map f if it is a minimal period of every map homotopic to f. The talk gives a survey of recent results describimg the sets of homotopy minimal periods of a map on compact nil and completely solvable manifold. The approach is based on new facts on the homotopy fixed point theory as the Nielsen number, the periodic Nielsen number, the Lefschetz number, connections between them i.e. Anosov theorem, and some combinatorial and algebraic number theory arguments.

Mikiya Masuda
Torus manifolds and multi-fans
(joint work with Akio Hattori)

The theory of toric varieties is a bridge between algebraic geometry and combinatorics. The foundation of the theory was laid down at the beginning of 1970's by Demazure, Miyake-Oda and Mumford etc. Roughly speaking, the theory consists of the following two correspondences:

(1) there is a one-to-one correspondence between toric varieties and fans,
(2) an equivariant ample line bundle over a compact non-singular toric variety corresponds to a lattice convex polytope through a moment map.

Through the correspondence (1), algebro-geometric properties of a toric variety can be described in terms of the associated fan, and through (2), results on convex polytopes can be obtained by applying results on algebraic geometry. For instance, the existence of the Ehrhart polynomial concerning the number of lattice points in a lattice convex polytope can be proved by applying Hirzebruch-Riemann-Roch Theorem to an ample line bundle corresponding to the lattice convex polytope.

In this talk we report that the theory of toric varieties can be developed from a topological point of view using equivariant cohomology.

This viewpoint leads us to treat a wider object than what is treated in the theory of toric varieties. A *torus manifold* in the title is a topological counterpart to a toric variety, and a *multi-fan*, which is a generalization of a fan, is a combinatorial object associated with a torus manifold. Topological properties of a torus manifold can be described in terms of the associated multi-fan.

The correspondence (2) can also be generalized to our topological setting. An equivariant complex line bundle over a torus manifold corresponds to what we call a lattice *multi-polytope* which is a generalization of a lattice convex polytope. It turns out that results on convex polytopes such as Ehrhart polynomial can be generalizedTetsu Nishimoto to multi-polytopes.

References

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Mamoru Mimura

On the cellular decomposition of the exceptional Lie group G_2

We give a cellular decomposition of G_2, the compact connected exceptional Lie group of rank 2.

Jack Morava

New analogs of the braid groups

The space of unordered configurations of distinct points in the plane is aspherical, with Artin's braid group as its fundamental group. Remarkably enough, the space of ordered configurations of distinct points on the real projective line, modulo projective equivalence, has a natural compactification (as a space of equivalence classes of trees) which is also (by a theorem of Davis, Januszkiewicz, and Scott) aspherical.

The classical braid groups are ubiquitous in modern mathematics, with applications from the theory of operads to the study of the Galois group of the rationals. The (rather mysterious) fundamental groups of these new configuration spaces are not braid groups, but they have many similar formal properties. This talk will be an introduction to their study, and their (possible) applications.

Juno Mukai

The 22-nd and below homotopy groups of the n-th rotation group

(joint work with Y. Hirato and H. Kachi)

We determine the group strucures of the homotopy groups $\pi_k(R_n)$ of the n-th rotation group R_n for $k = 21$ and 22 by use of the fibration $R_{n+1}/R_n = S^n$.

Michele Mulazzani

Strongly-cyclic branched coverings of $(1,1)$-knots and the mapping class group of the twice punctured torus

The connections between strongly-cyclic branched coverings of $(1,1)$-knots and cyclic presentations of groups will be explained. We show that every n-fold strongly-cyclic branched covering of a $(1,1)$-knot admits a geometric cyclic presentation for the fundamental group. Moreover, the conditions on the existence and uniqueness of such coverings will be illustrated, by using the presentation of $(1,1)$-knots by elements of the mapping class group of the twice punctured torus.

Amnon Neeman

Thomason's localisation theorem in K-theory and its applications to the the non-commutative rings that arise in surgery.

We all know how to localise a commutative ring with respect to a set of elements. And the existence of a long exact sequence in K-theory is also very classical.

But in surgery the rings that naturally come up are decidedly not commutative. Cohen and Schofield defined what localisation should mean for them, and produced a six-term exact sequence in K-theory. If the localisation is flat over the ring R, then Vogell showed the six-term exact sequence may be extended indefinitely.

Ranicki asked me what Thomaosn's localisation theorem teaches us about this problem. It turns out that it allows us to elucidate and improve on the previous work. We will discuss some of the results.

Taras E. Panov

Cobordism invariants of manifolds with torus actions

We propose an approach for calculating characteristic numbers and Hirzebruch genera (e.g. signature or Todd genus) of manifolds with torus actions in terms of combinatorics of the quotient. The case of particular interest is smooth toric varieties and (quasi)toric manifold. The latter were introduced by M.Davis and T.Januszkievicz in 1991 as a topological generalisation of toric varieties. As it was recently proved by V.Buchstaber and N.Ray, stably almost complex structures on (quasi)toric manifolds are determined by the combinatorial notion of omniorientation for quotient polytopes, and every complex cobordism class contains a (quasi)toric manifold. This open the way to combinatorial (polytopal) description of generator set for complex cobordism and justifies the importance of combinatorial calculation of cobordism invariants for (quasi)toric manifolds.

Some results are joint with Victor Buchstaber and Nigel Ray.

Reference:

V.M.Buchstaber and T.E.Panov. *Torus actions, combinatorial topology and homology algebra*, Russian Math. Surveys **55** (2000), no. 5., 825-921.
Available at http://arXiv.org/abs/math.AT/0010073

Krzysztof Pawałowski

Finite Oliver groups and Smith equivalence of representations

(joint work with Ronald Solomon)

In 1960, P.A. Smith asked the following question: If a finite group G acts smoothly on a sphere with exactly two fixed points x and y, is it true that the representations of G on the tangent spaces at x and y are isomorphic? During the talk, we shall present recent progress in answering the Smith isomorphism question. In particular, we give the negative answer to the question in the case where G is a finite Oliver group of odd order, or G is a finite Oliver group with a cyclic quotient of order pq for two distinct odd primes p and q, or G is a finite nonsolvable (and thus Oliver) gap group with two or more real conjugacy classes represented by elements of G which are not of prime power order.

Teimuraz Pirashvili

On functor homology

Homological algebra in the category of functors from finite pointed sets to abelian groups plays an important role in algebra and topology. We will explain the main recent applications and particularly the applications to the homology theory of commutative algebras.

Volker Puppe

Group actions and codes

A $\mathbb{Z}/2\mathbb{Z}$-action with "maximal number of isolated fixed points" (i.e. with as many isolated fixed points as the total dimension of the cohomology with $\mathbb{Z}/2\mathbb{Z}$ coefficients) on a closed 3-manifold determines a self-dual code. In turn this codes determines the cohomology algebra and the equivariant cohomology of the manifold. Hence, from results on binary self-dual codes one gets information about the cohomology type of 3-manifolds which admit involutions with maximal number of fixed points. In particular, "most" cohomology types of closed 3-manifolds do not admit such involutions. Generalizations of the above result are possible in several directions, e.g., one gets that "most" cohomology types (over $\mathbb{Z}/2\mathbb{Z}$) of closed 3-manifolds do not admit a non-trivial involution.

Nigel Ray

Real K-Theory of Bott Towers

(joint work with Yusuf Civan)

Bott towers are sequences of toric varieties that were investigated by Grossberg and Karshon in the mid 90s. The aim of the talk is to define such towers in the topological setting of Davis and Januszkiewicz, and to discuss their homotopy structure in terms of Thom complexes of the defining line bundles. Our treatment involves a fruitful interplay beween algebra, geometry and combinatorics, and leads on to computations of the real and complex K theory of the towers. Our viewpoint breathes geometric life into recent Adams spectral sequence calculations of Bahri and Benderskey.

Michał Sadowski

On the number of connected components of finite group actions on tori

Jolanta Słomińska

Equivariant homotopy decomposition of CW-complexes

We will give a general framework for studying homotopy and homology decompositions which are associated to the equivariant structure of a G-CW complex X, where G is a Lie group. We will also show the connections between certain equivariant homotopy decompositions and the facts that, for a compact Lie group G with a non-trivial p-subgroup the orbit space $|A_p(G)|/G$ is contractible and the canonical map $EG \times_G |A_p(G)| \to BG$ is a mod p isomorphism. Here $|A_p(G)|$ is the "geometrical realization" of the G-poset $A_p(G)$ of all non-trivial elementary abelian p-subgroups of G.

Christophe Soulé

Varieties on the field with one element

Tits, and later Manin, suggested the existence of an algebraic geometry over "the field with one element". I will describe their arguments in favor of such a theory. Then I will propose a provisional definition of these varieties. Those speculations will also lead us to a motivic interpretation of the image of Adams' J-homomorphism.

Peter Symonds

Complexes of permutation modules for profinite groups

An important tool when studying the cohomology of many groups is the existence of a contractible complex on which the group acts with finite stabilisers. We construct an algebraic analogue which also works for profinite groups and develop some of its properties.

András Szűcs

The Pontrjagin-Thom construction for singular maps and its applications to elimination of singularities

Given a stable map of a closed surface in the plane the number of cusps has the same pairity as the Euler characteristics of the surface. If the surface has even Euler characteristics then it has a stable map in the plane without cusps. These are well - known old theorems by Whitney, Levine and Eliashberg. Our aim is to generalize these statements to higher dimensions and arbitrary stable singularities.

Arnold and his coauthers posed the following question (in [AVGL]):

Question 1: Suppose that a stable map has vanishing Thom polynomials for a given singularity type η. Is then the map homotopic to a map having no singular points of type η ? (Vanishing of the Thom polynomials is clearly necessary for the elimination of η points.)

We shall consider the following similar question:

Question 2. Let η be a maximal singularity of a stable $f : N^n \to P^{n+k}$. Under what conditions is the map cobordant to an η-free map g ? By cobordism we mean such a stable map in the cylinder $P^{n+k} \times [0, 1]$ of a compact $n + 1$ manifold with boundary, which has no other singularities than the map f has.

We give necessary and sufficient conditions for the positive answer. They differ only by a finite factor.

1. *Necessary conditions:* At this time we do not need vanishing of the Thom polynomials of η for the map f, but the Gysin map must annihilate it. Let us suppose that the manifold P is stably paralellizable. Then the higher Thom polynomials of η (expressing the images of the characteritic classes of the submanifold formed by the η-singular

points in the cohomologies of the source of f) also must be mapped into zero by the Gysin map $f_!$.

2. *Sufficient conditions:* We shall show that up to a finite factor these necessary conditions are also sufficient. That is the following theorem holds.

Theorem. Let $f : N^n \to P^{n+k}$ be a stable map, where P^{n+k} is stably paralellizable, let η be a maximal singularity of f and suppose that the Gysin map $f_!$ anihilites the Thom polynomial of the η singularity, and also all the higher Thom polynomials. Then there is a non-zero integer L such that $L \cdot f$ is cobordant to an η-free map by a cobordism having no other singularities than f has.

The proof relies on the construction of the universal τ-map $f_\tau : Y_\tau \to X_\tau$. Here τ is a set of multisingularities and the universal τ map is universal among the maps having only such multisingularities, that belong to τ. The map f_τ is universal in the same sense as the inclusion of the Grassmann manifold $BO(k)$ in the Thom space $MO(k)$ is universal among the (codimension k) embeddings.

Considering the cohomologies of the source space Y_τ of f_τ one can find the Thom polynomials mentioned above, see [R].

Investigating the homotopy type of the target space – combined with a so called stabilization trick – one can prove the sufficient condition formulated in the theorem. (The special case, when $\eta = \Sigma^{1,0}$ and so η-free means *non-singular*, has been considered in [Sz1] and [Sz2].)

Finally we give examples showing that the integer L can not be omitted, that is the map f itself may not be cobordant to an η-free map. For example if $\eta = A_{2r+1}$, N^n is oriented, and $k = 0$, then the obstruction to the elimination of η-points lies in the stable homotopy groups of the target manifold P.

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Rainer Vogt

Semi-model structures on categories of algebras over a CAT-operad, applications to iterated loop spaces.

In the study of the homotopy theory of algebras over an operad one usually encounters substantial coherence problems. An elegant way to deal with those is Quillen's notion of a closed model category structure. The existence of suitable model category structures on categories of algebras is often far from clear. We show that the category of algebras over an operad in CAT, the category of small categories, has a semi-model structure with the correct weak equivalences. The notion of a semi-model structure is slightly weaker than the one of a model structure, but it is rich enough to do homotopy theory and to deal with derived equivalences.

As application we show the equivalence of the homotopy categories of n-fold monoidal categories and of $\mathcal{C}_n$-spaces. In particular, we recover Thomason's equivalence of the homotopy categories of symmetric monoidal categories and connective spectra without appeal to categorical coherence theory.

Reinhard Waldmueller

A Bieberbach group with trivial outer automorphism group

I found an example of a Bieberbach group with no outer automorphisms and would like to present it. It is the fundamental group of a flat manifold without nontrivial affinities and the first known example of such a group/manifold.

Andrzej Weber

Equivariant cohomology of algebraic varieties

I will prove that the equivariant cohomology of any complete algebraic variety acted by an algebraic group G is a free module over $H^*(BG)$. The proof uses reduction of the base field to a finite characteristic. The result is known for projective varieties. In this case it can be obtained by symplectic methods. I will compare both proofs.

Also, for singular varieties an analogous theorem holds. The ordinary cohomology is replaced by the intersection homology.

Jarosław Włodarczyk
Algebraic Morse Theory and factorization of birational maps

We develop a Morse-like theory for complex algebraic varieties. In this theory the Morse function f is replaced by a K^*-action. The critical points of the Morse function correspond to fixed points of the action. The homotopy type changes when we pass through the critical points. Analogously, in the algebraic setting "passing through" the fixed points of the K^*-action induces some simple birational transformations called blowups, blowdowns and flips. They are analogous to spherical modifications.

In classical Morse Theory by means of the Morse function we can decompose the manifold into elementary pieces -"handles". In the algebraic Morse Theory we decompose a birational map between two smooth complex algebraic varieties into a sequence of blowups and blowdowns with smooth centers. This provides an affirmative solution to a long standing factorization conjecture of birational maps.

Ergün Yalçın
On Nilpotent ideals in the cohomology ring of a finite group
(joint work with Jonathan Pakianathan)

In this talk, I will present some results on the nilpotency degree of ideals in the cohomology ring of a finite group. These results are obtained by studying fixed point free actions of the group on suitable spaces. The ideals we study are the kernels of restriction maps to certain collections of proper subgroups. In particular, we consider Mui's essential cohomology conjecture (which states that the essential cohomology ideal has nilpotence degree 2), and show that it is equivalent to a conjecture about group actions on connected graphs.

Andreas Zastrow
The higher homology of subsets of surfaces does not behave anomalously

The talk will be on methods for how to prove that all higher singular homology groups of all subsets of surfaces vanish, with a 2^{nd}-countable topology of the surface being the only assumption and H_2 of closed surfaces being the only exception. The result should be understood as a result in the algebraic topology of non-tame spaces, and this is an area where many results for tame spaces do not hold. In particular, a wild two-dimensional complex with a non-trivial H_3 is known. On the other hand, the non-anomalous behaviour of higher homotopy groups and higher (singular) homology groups of subsets of the plane are former results of the author. While the extension from the plane to surfaces is not difficult in the homotopy case, it requires some work in the homology case, because homology groups are not invariant with respect to the covering space construction. The talk will quickly sketch the relevant methods of the author's former results and then focus on the surgery techniques of wild maps that are the main tool for proving the new result as stated in the title.

Alexei V. Zhubr

Some results on "cancellation" for connected sums of $2q$-manifolds with $S^q \times S^q$