Homework 4

Due date: January 29

1. Consider the following variant of the class BPP: the algorithm runs in polynomial time, but there is a stronger requirement on the error probability: the probability that the algorithm gives the wrong answer on an input of length n is at most

$$\frac{1}{2^{2^n}},$$

   i.e. it is doubly exponentially small. Prove that this variant is equivalent to deterministic polynomial time.

2. For a language $L \subseteq \Sigma^*$, define $L_3 \subseteq (\Sigma^*)^*$ to be the set of lists of words where at least three list items belong to L (there can be other list items that are not in L). We can view L_3 as a language, by representing a list of words as a word where consecutive list elements are separated by a fresh separator symbol. Show that if L is in the class RP, then the same is true for L_3.

3. Consider the following variant of the Orthogonal Vectors problem, call it *Almost Orthogonal Vectors*: we are given two sets of vectors

$$A, B \subseteq \{0, 1\}^d,$$

   both of size n, and we want to know if there exist vectors $a \in A$ and $b \in B$ such that the inner product of a and b is ≤ 1 (as opposed to exactly 0 in the original Orthogonal Vectors problem). The *Almost Orthogonal Vectors Conjecture* says that we cannot solve this variant in time

$$n^{(2-\varepsilon)} \cdot poly(d) \qquad \text{for some fixed } \varepsilon > 0.$$

   Prove that the Almost Orthogonal Vectors Conjecture is equivalent to the usual Orthogonal Vectors Conjecture.

4. Find an FPT reduction from the parameterized problem:

   - given an undirected graph, and a parameter k, decide if there exists a set U of k vertices such that every vertex $v \notin U$ is adjacent to some vertex in U.

   to the parameterized problem

   - given a family of sets $\mathcal{F} \subseteq \{1, \ldots, n\}$, and a parameter k, decide if there exists a choice of k sets in the family $\mathcal{F}$ whose union is the entire set $\{1, \ldots, n\}$.