Homework 3

Due date: December 18

1. Show that the following problem is complete for NL (nondeterministic logarithmic space): given an nondeterministic finite automaton, decide if it accepts some word of even length.

2. Show that AC^1 is closed under Kleene star, i.e. if a language

   $$L \subseteq \{\text{true, false}\}^*$$

   is in AC^1, then the same is true for L^*. (Remark: AC^0 is not closed under Kleene star, because the parity language "an even number of bits is false", which is not in AC^0, is the Kleene star of the language "exactly two bits are false", which is in AC^0.)

3. We define nondeterministic circuits similarly to normal Boolean circuits, but we can also use a "guess" gate. This gate takes two inputs and outputs one of the inputs nondeterministically. A nondeterministic circuit accepts an input if there exists a set of choices for the nondeterministic "guess" gates, under which the circuit output is true; otherwise the input is rejected. Show that if a language

   $$L \subseteq \{\text{true, false}\}^*$$

   is in NP, then it is recognised by a polynomial size, constant depth, family of nondeterministic circuits (i.e. a family of circuits as in AC^0, but which is additionally allowed to use "guess" gates).

   In other words, the n-th circuit has size polynomial in n and constant size, and if we feed it an input word of length n then: (a) if the input word belongs to L, then there is a set of nondeterministic choices for the "guess" gates in the n-th circuit that leads to a "true" output; and (b) if the input word does not belong to L, then every set of nondeterministic choices for the "guess" gates in the n-th circuit that leads to a "false" output.