Research project
MEASURE ORIENTED APPROACH TO NONLINEAR PDE'S AND VARIATIONAL PROBLEMS
Description to the general public

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The project centers around two groups of problems in partial differential equations, with origins in physical models.

The first class of problems deals with finding the optimal shape of a heat-conducting body. The main difficulty in this topic is that due to the complicated geometry of possible shapes we need to apply new mathematical tools not only to find and study solutions, but to properly formulate models of heat conduct.

The second class of problems considers mathematical models describing the behaviour of strongly inhomogeneous and anisotropic materials under various physical actions. The main difficulty is caused by the fact that inhomogeneity and anisotropy of material are reflected in non-uniform ellipticity bounds for a differential operator appearing in a differential equation or in a system of dirrential equations consisting the mathematical model. As a result, in order to study well-posedness of these models, very general function spaces must be employed. The basic question is to bring results of mathematical analysis back to reality i.e. to show that solutions are regular enough to serve as description of real objects.

Classical differential geometry studies smooth manifolds, that is manifolds which can be described with functions that are smooth, in other words – differentiable (preferably infinitely many times). Unfortunately, such functions are not sufficient to describe and study real-life phenomena. To study the shape of a soap film spanned on a bent loop of wire, to predict the location of cracks forming in a squeezed material, or to model optimal shape of conductors contained in given area of space, we need functions, which are not differentiable, and might have no derivative at any point. In order to study such functions using the toolbox of analysis, the notion of weak derivative was introduced. From the point of view of partial differential equations and variational calculus, the weak derivatives are handy since they retain many properties of the usual derivative. However, at the end of the analysis, it is always important to know precisely what are properties of obtained solutions. Therefore it is important to investigate analytical properties of functions and mappings acting on objects of possibly complicated geometry.

The problems considered in the project are vividly researched, close to applications; we use wide array of tools from geometric analysis, measure theory, topology, calculus of variations and functional analysis.