In[1]:=
```
A = {{2, -1, 1, 2}, {1, 2, 3, 1}, {0, 0, 2, -1}, {0, 0, 1, 2}};
MatrixForm[A]
```

Out[2]//MatrixForm=

$$\begin{pmatrix} 2 & -1 & 1 & 2 \\ 1 & 2 & 3 & 1 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

In[3]:=
```
chi = Det [A - t IdentityMatrix[4]]
Factor[chi]
Solve[chi == 0, t]
```

Out[3]= $25 - 40\,t + 26\,t^2 - 8\,t^3 + t^4$

Out[4]= $\left(5 - 4\,t + t^2\right)^2$

Out[5]= $\{\{t \to 2 - i\}, \{t \to 2 - i\}, \{t \to 2 + i\}, \{t \to 2 + i\}\}$

In[6]:=
```
(* wartosc wlasna 2+i *)
B = A - (2 + i) IdentityMatrix[4]; MatrixForm[B]
```

Out[6]//MatrixForm=

$$\begin{pmatrix} -i & -1 & 1 & 2 \\ 1 & -i & 3 & 1 \\ 0 & 0 & -i & -1 \\ 0 & 0 & 1 & -i \end{pmatrix}$$

In[7]:=
```
K1 = NullSpace[B]
K2 = NullSpace[B.B]
```

Out[7]= $\{\{i, 1, 0, 0\}\}$

Out[8]= $\{\{-5\,i, 0, 2\,i, 2\}, \{i, 1, 0, 0\}\}$

In[9]:=
```
w2 = K2[[1]]
w1 = B.w2
```

Out[9]= $\{-5\,i, 0, 2\,i, 2\}$

Out[10]= $\{-1 + 2\,i, 2 + i, 0, 0\}$

In[11]:=
```
(* lancuszek dla sprzezonej wartosci wlasnej 2-i *)
v2 = Conjugate[w2]
v1 = Conjugate[w1]
```

Out[11]= $\{5\,i, 0, -2\,i, 2\}$

Out[12]= $\{-1 - 2\,i, 2 - i, 0, 0\}$

In[13]:=
```
(* Macierz przej cia z bazy Jordana do standardowej *)
X = Transpose[{v1, v2, w1, w2}]; MatrixForm[X]
```

Out[13]//MatrixForm=

$$\begin{pmatrix} -1 - 2\,i & 5\,i & -1 + 2\,i & -5\,i \\ 2 - i & 0 & 2 + i & 0 \\ 0 & -2\,i & 0 & 2\,i \\ 0 & 2 & 0 & 2 \end{pmatrix}$$

In[14]:=
```
MatrixForm[Inverse[X].A.X]
```

Out[14]//MatrixForm=

$$\begin{pmatrix} 2-i & 1 & 0 & 0 \\ 0 & 2-i & 0 & 0 \\ 0 & 0 & 2+i & 1 \\ 0 & 0 & 0 & 2+i \end{pmatrix}$$

In[15]:=
```
(* baza rzeczywista *)
a1 = (v1 + w1) / 2
a2 = (v2 + w2) / 2
b1 = (v1 - w1) / (2 I)
b2 = (v2 - w2) / (2 I)
```

Out[15]= {-1, 2, 0, 0}

Out[16]= {0, 0, 0, 2}

Out[17]= {-2, -1, 0, 0}

Out[18]= {5, 0, -2, 0}

```
(* Rzeczywista postac Jordana *)
Y = Transpose[{a1, b1, a2, b2}]; MatrixForm[Y]
MatrixForm[Inverse[Y].A.Y]
```

Out[19]//MatrixForm=

$$\begin{pmatrix} -1 & -2 & 0 & 5 \\ 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 2 & 0 \end{pmatrix}$$

Out[20]//MatrixForm=

$$\begin{pmatrix} 2 & -1 & 1 & 0 \\ 1 & 2 & 0 & 1 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$