312. If a harmonic function is regular in a closed disk its absolute value at the center does not exceed the arithmetic mean of its absolute values on the boundary. Under what conditions is there equality?

313. A harmonic function is supposed to be regular and single-valued in a domain $\mathfrak{D}$. Then it attains its maximum and its minimum on the boundary and only on the boundary unless it is a constant.

314. A harmonic function that is regular in the domain $\mathfrak{D}$ and vanishes at all the boundary points of $\mathfrak{D}$ is identically zero.

316. A harmonic function is assumed to be single-valued in the domain $\mathfrak{D}$ and regular with the exception of finitely many points at which it becomes $-\infty$ (i.e. the function converges to $-\infty$ as z approaches such a point). Then it assumes its maximum on the boundary of $\mathfrak{D}$.

319. Let $u_1(x, y), u_2(x, y), \ldots, u_n(x, y)$, $z = x + iy$, be regular harmonic functions in a domain $\mathfrak{D}$. The continuous function

$$|u_1(x, y)| + |u_2(x, y)| + \cdots + |u_n(x, y)|$$

assumes its maximum on the boundary of $\mathfrak{D}$.

320. Consider a regular harmonic function in the disk $|z| < R$. We denote by $A(r)$ its maximum on the circle $|z| = r$, $r < R$. When $0 < r_1 < r_2 < r_3 < R$ we have

$$A(r_2) \leqq \frac{\log r_2 - \log r_1}{\log r_3 - \log r_1} A(r_3) + \frac{\log r_3 - \log r_2}{\log r_3 - \log r_1} A(r_1),$$

i.e. $A(r)$ is a convex function of $\log r$.