Exam

1 Answer: YES/NO + explanation

1.1 Let G be a compact connected Lie group. Is it true that any abelian subgroup of G is contained in a maximal torus?

1.2 Let V be any representation of $SL_2(\mathbb{C})$, and let d_i denote the dimension of the space of the weight i (with respect to some choice of the maximal torus). Is it possible that $d_5 = 2$, $d_6 = 3$?

1.3 Consider the group $PSL_3(\mathbb{C}) = GL_3(\mathbb{C})/center$. Does the irreducible representation of $sl_3(\mathbb{C})$ corresponding to the partition (5,4,0) come form a representations of $PSL_3(\mathbb{C})$?

1.4 Is $N_{SL_2(\mathbb{C})}(\mathbb{C}^*)$ a semidirect product of $\mathbb{C}^*$ and $\mathbb{Z}_2$?

2 Short answer

2.1 Give an example of three nonisomorphic Lie algebras of the dimension three.

2.2 Decompose $gl_2(\mathbb{C}) = End(\mathbb{C}^2)$ into an orthogonal sum of real vector spaces on which the Killing form is either positive definite or negative definite or zero.

2.3 What is the center of $Spin_8(\mathbb{C})$?

3 Explain the solution.

3.1 Consider the representation of $SL_3(\mathbb{C})$: $\bigwedge^2 \mathbb{C}^3 \otimes Sym^2\mathbb{C}^3$. Decompose it into irreducible components.

3.2 Compute the character for the representation of $SL_5(\mathbb{C})$ which is associated to the partition (8,6,4,2,0). (Present the character as a polynomial.) What is the dimension of this representation?

3.3 Let $G = Sp(2)$ with the natural representation in $\mathbb{C}^4$. Consider the representation $\bigwedge^2(\bigwedge^2 \mathbb{C}^4)$. Find at least two nonproportional vectors, which are the highest vectors in subrepresentations.