Problem list 8— 28 November

1 Let $L = \mathcal{O}(m)$, $m \geqslant 0$. Using Riemann-Roch theorem and the localization theorem compute $\chi(\mathbb{P}^n; L)$. Check if it agrees with the formula well known for algebraic geometers:

$$\chi(\mathbb{P}^n; L) = \dim H^0(\mathbb{P}^n; \mathcal{O}(m)) = \mathbb{C}[t_0, t_1, \dots, t_n]_{\deg = m} \,.$$

(The restriction of L at the standard fixed point $p_k \in \mathbb{P}^n$ is equal to $\mathbb{C}_{-m\, t_k}$, i.e. $\mathbb{C}$ with $\mathbb{T}$ acting with the weight $-m\, t_k$.)

2 Suppose $F \to X \to B$ is a fibration, and F is a sphere. Write how the $E_r^{p,q}$ table looks like for $r + 1 \leqslant \dim F$ and deduce the Gysin long exactsequence.

3 Suppose $F \to X \to B$ is a fibration, and B is a sphere. Write how the $E_r^{p,q}$ table looks like for $r \leqslant \dim B$ and deduce the resulting long exact sequence (called Wang sequence).

4 Write all the entries of the spectral sequence $E_2^{p,q} = H^p(B\mathbb{T}, H^q(X)) \Rightarrow H_{\mathbb{T}}^{p+q}(X)$ for $\mathbb{T} = S^1$ acting on $S^3 \subset \mathbb{C}^2$ as the scalar multiplication.

5 Write all the entries of the Serre spectral sequence of the fibration $E\mathbb{T} \times X \to E\mathbb{T} \times^{\mathbb{T}} X$ for $X = \mathbb{P}^1$ or S^3 (with the usual torus actions).

6 Show that if a a topological $\mathbb{T}$-space has no odd cohomology (i.e. $H^{odd}(X) = 0$), then X is equivariantly formal.