Problem list 6 — 14 November

1 Define the equivariant fundamental class not passing throug approximation of EG, but using the equivariant normal bundle on Y_{smooth} which gives rise to a bundle on $EG \times^G Y_{smooth}$.

2 Show that the localization functor

$$\Lambda - modules \quad \longrightarrow \quad K - modules$$

is exact.

3 See what goes wrong with the proof of localization theorem for $\mathbb{T}$ replaced by a nonabelian groups. In the torus case for any proper subgroup $K \subset \mathbb{T}$ the orbit $H^*_{\mathbb{T}}(\mathbb{T}/K)$ turned out to be a torsion $H^*_{\mathbb{T}}(pt)$–module. What happens for noncommutative groups?

4 Let $X = \mathbb{P}^n$ with the standard action of $\mathbb{T} = (\mathbb{C}^*)^{n+1}$. Show that the image

$$H^*_{\mathbb{T}}(\mathbb{P}^n) \hookrightarrow \bigoplus_{k=0}^{n} \Lambda = \Lambda^{n+1}$$

consists of such sequences $(f_0, f_1, \ldots, f_n) \in \mathbb{Q}[t_0, t_1, \ldots, t_n]^{n+1}$, such that $t_i - t_j$ divides $f_i - f_j$. (Of course not using the general theorem, but by a direct computation.)

5 Cohomology of Grassmannians →N.C.:

- Show that the Chern classes of the tautological bundle generate $H^*(Gras_k(\mathbb{C}^n))$. Find the relations.
- Let $E \to B$ be a complex vector bundle, $0 < k < rank(E)$. By $Gr_k(E)$ we denote the associated Grassmann bundle (we replace the fibers E_x by the Grassmannians of k-dimensional subspaces in E_x). Let γ be the tautological bundle over $Gr_k(E)$. Show that $H^*(Gr_k(E))$ is generated over $H^*(B)$ by the Chern classes $c_i(\gamma)$, $i = 1, 2, \ldots, k$.
- As a corollary compute $H^*_T(Gr_k(\mathbb{C}^n))$, where T is the maximal torus in $\mathrm{GL}_n(\mathbb{C})$.
- Compare the answer with Projective Bundle Theorem $H^*(\mathbb{P}(E)) \simeq H^*(B)[h]/(\textit{well known relation})$.