

# EKT&EC — Problem List 1: to be solved 04.03

**Problem 1** Let  $G = (S^1)^n$  act on  $\mathbb{P}(\mathbb{C}^n)$  coordinate-wise. Find a presentation of  $\mathbb{P}(\mathbb{C}^n)$  as a  $G$ -CW-complex.

**Problem 2** Consider the Hopf fibration

$$\mathbb{C}^2 \supset S^3 \xrightarrow{p} S^2.$$

We identify  $S^2 = \mathbb{C}\mathbb{P}^1$ . We have a decomposition

$$\mathbb{C}\mathbb{P}^1 = D_0 \cup D_1, \quad D_N = \{[z_0 : z_1] : z_1/z_0 \leq 1\}, \quad D_S = \{[z_0 : z_1] : z_0/z_1 \leq 1\}.$$

and

$$p^{-1}(D_N) \simeq S^1 \times D_N, \quad p^{-1}(D_S) \simeq S^1 \times D_S.$$

The boundaries of both sets are homeomorphic to  $S^1 \times S^1$ . Write explicitly in coordinates the gluing map.

**Problem 3** Hopf (continued): We have a cell decomposition of

$$\mathbb{C}\mathbb{P}^1 = \{[0 : 1]\} \sqcup \{z_0 \neq 0\} = \{\infty\} \sqcup \mathbb{C}.$$

This determines  $S^1$ -CW-decomposition of  $S^3$  as

$$S^1 \sqcup (S^1 \times \mathbb{C}),$$

Identify  $\mathbb{C}$  with the open disk  $\mathring{D}$  and find the attaching map from  $S^1 \times \partial D \rightarrow S^1$ .

**Problem 4** Let  $N \subset G$  be a normal subgroup. Suppose  $X$  is a  $G$  space on which  $N$  acts freely. Find a bijection

$$\text{Vect}_G^n(X) \simeq \text{Vect}_{G/N}^n(X/N).$$

**Problem 5** Let  $H \subset G$ . Find a  $G$ -equivariant homeomorphism

$$G \overset{H}{\times} G/H \simeq G/H \times G/H.$$

On  $G \overset{H}{\times} G/H$  the group  $G$  acts on the first factor, and on  $G/H \times G/H$  acts diagonally.