In[1]:=
```
(* działanie grupy Weyla *)
s1[f_] := f /. {L1 → L2, L2 → L1}
s2[f_] := f /. {L2 → L3, L3 → L2}

(* skręcone działanie grupy Weyla *)
rho = 2 L1 + L2;
r1[f_] := Expand[s1[f + rho] - rho /. L3 → -L1 - L2]
r2[f_] := Expand[s2[f + rho] - rho /. L3 → -L1 - L2]
```

In[6]:=
```
lambda = 3 L1 + L2;
```

In[7]:=
```
(* Orbita lambdy przy skręconym działaniu *)
lambda1 = r1[lambda]
lambda2 = r2[lambda]
lambda21 = r2[lambda1]
lambda12 = r1[lambda2]
lambda121 = r2[lambda12]
```

Out[7]= 4 L2

Out[8]= L1 - 3 L2

Out[9]= -5 L1 - 6 L2

Out[10]= -4 L1 + 2 L2

Out[11]= -7 L1 - 4 L2

In[12]:=
```
(* Rozwinięcie charakteru Un_=M0
   -pierwiastki proste w zapisie multiplikatywnym: α=t2/t1, β=t3/t2,
   trzeci pierwiastek ujemny: αβ=t3/t1 *)

maxdeg = 15;
Unminus = Expand[Normal[Series[1 / ((1 - h α) (1 - h β) (1 - h ^ 2 α β)), {h, 0, maxdeg}]]];
Do[Print[TraditionalForm[Coefficient[Unminus, h, i]]], {i, 0, maxdeg}];
Unminus = Unminus /. {h → 1, α → t2 / t1, β → t3 / t2} /. t3 → 1 / (t1 t2);
```

$$1$$

$$\alpha + \beta$$

$$\alpha^2 + 2\,\alpha\,\beta + \beta^2$$

$$\alpha^3 + 2\,\alpha^2\,\beta + 2\,\alpha\,\beta^2 + \beta^3$$

$$\alpha^4 + 2\,\alpha^3\,\beta + 3\,\alpha^2\,\beta^2 + 2\,\alpha\,\beta^3 + \beta^4$$

$$\alpha^5 + 2\,\alpha^4\,\beta + 3\,\alpha^3\,\beta^2 + 3\,\alpha^2\,\beta^3 + 2\,\alpha\,\beta^4 + \beta^5$$

$$\alpha^6 + 2\,\alpha^5\,\beta + 3\,\alpha^4\,\beta^2 + 4\,\alpha^3\,\beta^3 + 3\,\alpha^2\,\beta^4 + 2\,\alpha\,\beta^5 + \beta^6$$

$$\alpha^7 + 2\,\alpha^6\,\beta + 3\,\alpha^5\,\beta^2 + 4\,\alpha^4\,\beta^3 + 4\,\alpha^3\,\beta^4 + 3\,\alpha^2\,\beta^5 + 2\,\alpha\,\beta^6 + \beta^7$$

$$\alpha^8 + 2\,\alpha^7\,\beta + 3\,\alpha^6\,\beta^2 + 4\,\alpha^5\,\beta^3 + 5\,\alpha^4\,\beta^4 + 4\,\alpha^3\,\beta^5 + 3\,\alpha^2\,\beta^6 + 2\,\alpha\,\beta^7 + \beta^8$$

$$\alpha^9 + 2\,\alpha^8\,\beta + 3\,\alpha^7\,\beta^2 + 4\,\alpha^6\,\beta^3 + 5\,\alpha^5\,\beta^4 + 5\,\alpha^4\,\beta^5 + 4\,\alpha^3\,\beta^6 + 3\,\alpha^2\,\beta^7 + 2\,\alpha\,\beta^8 + \beta^9$$

$$\alpha^{10} + 2\,\alpha^9\,\beta + 3\,\alpha^8\,\beta^2 + 4\,\alpha^7\,\beta^3 + 5\,\alpha^6\,\beta^4 + 6\,\alpha^5\,\beta^5 + 5\,\alpha^4\,\beta^6 + 4\,\alpha^3\,\beta^7 + 3\,\alpha^2\,\beta^8 + 2\,\alpha\,\beta^9 + \beta^{10}$$

$$\alpha^{11} + 2\,\alpha^{10}\,\beta + 3\,\alpha^9\,\beta^2 + 4\,\alpha^8\,\beta^3 + 5\,\alpha^7\,\beta^4 + 6\,\alpha^6\,\beta^5 + 6\,\alpha^5\,\beta^6 + 5\,\alpha^4\,\beta^7 + 4\,\alpha^3\,\beta^8 + 3\,\alpha^2\,\beta^9 + 2\,\alpha\,\beta^{10} + \beta^{11}$$

$$\alpha^{12} + 2\,\alpha^{11}\,\beta + 3\,\alpha^{10}\,\beta^2 + 4\,\alpha^9\,\beta^3 + 5\,\alpha^8\,\beta^4 + 6\,\alpha^7\,\beta^5 + 7\,\alpha^6\,\beta^6 + 6\,\alpha^5\,\beta^7 + 5\,\alpha^4\,\beta^8 + 4\,\alpha^3\,\beta^9 + 3\,\alpha^2\,\beta^{10} + 2\,\alpha\,\beta^{11} + \beta^{12}$$

$$\alpha^{13} + 2\,\alpha^{12}\,\beta + 3\,\alpha^{11}\,\beta^2 + 4\,\alpha^{10}\,\beta^3 + 5\,\alpha^9\,\beta^4 + 6\,\alpha^8\,\beta^5 + 7\,\alpha^7\,\beta^6 + 7\,\alpha^6\,\beta^7 + 6\,\alpha^5\,\beta^8 + 5\,\alpha^4\,\beta^9 + 4\,\alpha^3\,\beta^{10} + 3\,\alpha^2\,\beta^{11} + 2\,\alpha\,\beta^{12} + \beta^{13}$$

$$\alpha^{14} + 2\,\alpha^{13}\,\beta + 3\,\alpha^{12}\,\beta^2 + 4\,\alpha^{11}\,\beta^3 + 5\,\alpha^{10}\,\beta^4 + 6\,\alpha^9\,\beta^5 + 7\,\alpha^8\,\beta^6 + 8\,\alpha^7\,\beta^7 + 7\,\alpha^6\,\beta^8 + 6\,\alpha^5\,\beta^9 + 5\,\alpha^4\,\beta^{10} + 4\,\alpha^3\,\beta^{11} + 3\,\alpha^2\,\beta^{12} + 2\,\alpha\,\beta^{13} + \beta^{14}$$

$$\alpha^{15} + 2\,\alpha^{14}\,\beta + 3\,\alpha^{13}\,\beta^2 + 4\,\alpha^{12}\,\beta^3 + 5\,\alpha^{11}\,\beta^4 + 6\,\alpha^{10}\,\beta^5 + 7\,\alpha^9\,\beta^6 + 8\,\alpha^8\,\beta^7 + 8\,\alpha^7\,\beta^8 + 7\,\alpha^6\,\beta^9 + 6\,\alpha^5\,\beta^{10} + 5\,\alpha^4\,\beta^{11} + 4\,\alpha^3\,\beta^{12} + 3\,\alpha^2\,\beta^{13} + 2\,\alpha\,\beta^{14} + \beta^{15}$$

In[15]:=
```
(* fukcja obliczająca współczynnik Mλ przy t^μ *)
coef[lambda_, mu_] := Coefficient[Coefficient[t1 ^ lambda[[1]] t2 ^ lambda[[2]] Unminus, t1, mu[[1]]], t2, mu[[2]]]
```

In[16]:=
```
(* dla lambda =3L1+L2 , mu=-L1-L2 *)
mu = {-1, -1};
{coef[{3, 1}, mu], coef[{0, 4}, mu], coef[{1, -3}, mu], coef[{-5, -6}, mu], coef[{-4, 2}, mu], coef[{-7, -4}, mu]}
```

Out[17]=
```
{3, 0, 1, 0, 0, 0}
```

In[18]:=
```
(* krotność w Vλ *)
%.{1, -1, -1, 1, 1, -1}
```

Out[18]=
```
2
```

In[19]:=
```
(* Schur function = charakter reprezentacji Vλ *)
tt = {t1, t2, t3};
S[lambda_] := Expand[Factor[Det[Table[tt[[i]]^(lambda + {2, 1, 0})[[j]], {i, 1, 3}, {j, 1, 3}]] / ((t1 - t2) (t1 - t3) (t2 - t3))]]
S[{3, 1, 0}]
```

Out[21]=

$$t1^3\, t2 + t1^2\, t2^2 + t1\, t2^3 + t1^3\, t3 + 2\, t1^2\, t2\, t3 + 2\, t1\, t2^2\, t3 + t2^3\, t3 + t1^2\, t3^2 + 2\, t1\, t2\, t3^2 + t2^2\, t3^2 + t1\, t3^3 + t2\, t3^3$$

In[22]:=
```
(* w zmiennych t1,t2 *)
```

In[23]:=
```
S[{3, 1, 0}] /. {t3 → 1 / (t1 t2)}
```

Out[23]=

$$\frac{1}{t1^2} + 2\, t1 + \frac{1}{t1^2\, t2^3} + \frac{1}{t2^2} + \frac{1}{t1^3\, t2^2} + \frac{2}{t1\, t2} + \frac{t1^2}{t2} + 2\, t2 + t1^3\, t2 + \frac{t2^2}{t1} + t1^2\, t2^2 + t1\, t2^3$$

In[24]:=
```
Coefficient[Coefficient[%, t1, -1], t2, -1]
```

Out[24]=
```
2
```

In[25]:=
```
(* ---------------------------------------------------------*)
(* dla lambda =3L1+L2 , mu=-3L1-L2 *)
mu = {-3, -1};
{coef[{3, 1}, mu], coef[{0, 4}, mu], coef[{1, -3}, mu], coef[{-5, -6}, mu], coef[{-4, 2}, mu], coef[{-7, -4}, mu]}
%.{1, -1, -1, 1, 1, -1}
```

Out[26]=

{0, 0, 0, 0, 0, 0}

Out[27]=

0

In[28]:=
```
S[{3, 1, 0}] /. {t3 → 1/(t1 t2)}
Coefficient[Coefficient[%, t1, -3], t2, -1]
```

Out[28]=

$$\frac{1}{t1^2} + 2\,t1 + \frac{1}{t1^2\,t2^3} + \frac{1}{t2^2} + \frac{1}{t1^3\,t2^2} + \frac{2}{t1\,t2} + \frac{t1^2}{t2} + 2\,t2 + t1^3\,t2 + \frac{t2^2}{t1} + t1^2\,t2^2 + t1\,t2^3$$

Out[29]=

0

In[30]:=
```
(* ---------------------------------------------------------*)
(* dla lambda =3L1+L2 , mu=-3L1-L2 *)
mu = {-7, -4};
{coef[{3, 1}, mu], coef[{0, 4}, mu], coef[{1, -3}, mu], coef[{-5, -6}, mu], coef[{-4, 2}, mu], coef[{-7, -4}, mu]}
%.{1, -1, -1, 1, 1, -1}
```

Out[31]=

{6, 3, 4, 1, 1, 1}

Out[32]=

0

In[33]:=
```
S[{3, 1, 0}] /. {t3 → 1/(t1 t2)}
Coefficient[Coefficient[%, t1, -7], t2, -4]
```

Out[33]=

$$\frac{1}{t1^2} + 2\, t1 + \frac{1}{t1^2\, t2^3} + \frac{1}{t2^2} + \frac{1}{t1^3\, t2^2} + \frac{2}{t1\, t2} + \frac{t1^2}{t2} + 2\, t2 + t1^3\, t2 + \frac{t2^2}{t1} + t1^2\, t2^2 + t1\, t2^3$$

Out[34]=

0