

In[1]:= (* x_y -GENUS OF A SURFACE OF DEGREE d IN P^3 *)

(* Expansion of $td_y(L)$ *)

Series $\left[x \left(1 + y E^x\right) / \left(1 - E^x\right), \{x, 0, 2\}\right]$

$$\text{Out[1]} = (1 + y) + \frac{1}{2} (1 - y) x + \frac{1}{12} (1 + y) x^2 + O[x]^3$$

$$\text{In[2]} := f[x_] := (1 + y) + \frac{1}{2} (1 - y) x + \frac{1}{12} (1 + y) x^2$$

In[3]:= (* Integral of $i_*(td_y(X))$ over P^3 *)

chiy = Collect[SeriesCoefficient[d h f[h]^4 / (1 + y) / f[d h], {h, 0, 3}], y, Factor]

$$\text{Out[3]} = \frac{1}{6} d (11 - 6 d + d^2) - \frac{1}{3} d (7 - 6 d + 2 d^2) y + \frac{1}{6} d (11 - 6 d + d^2) y^2$$

In[4]:= (* $d=1 \Rightarrow X=P^2$ *)

chiy /. d -> 1

$$\text{Out[4]} = 1 - y + y^2$$

In[5]:= (* $d=2 \Rightarrow X=P^1 \times P^1$ *)

chiy /. d -> 2

$$\text{Out[5]} = 1 - 2 y + y^2$$

In[6]:= (* $d=3 \Rightarrow$ cubic surface, $K_X=0(-1)$, isomorphic to a blow-up of P^2 at 6 points *)

chiy /. d -> 3

$$\text{Out[6]} = 1 - 7 y + y^2$$

In[7]:= (* $d=4$, $K3$ surface $K_X=0$ *)

chiy /. d -> 4

$$\text{Out[7]} = 2 - 20 y + 2 y^2$$

In[8]:= (* higher degrees, K_X positive *)

Do[Print["d=", k, " $x_y =$ ", chiy /. d -> k], {k, 5, 12}]

$$d=5 \quad x_y = 5 - 45 y + 5 y^2$$

$$d=6 \quad x_y = 11 - 86 y + 11 y^2$$

$$d=7 \quad x_y = 21 - 147 y + 21 y^2$$

$$d=8 \quad x_y = 36 - 232 y + 36 y^2$$

$$d=9 \quad x_y = 57 - 345 y + 57 y^2$$

$$d=10 \quad x_y = 85 - 490 y + 85 y^2$$

$$d=11 \quad x_y = 121 - 671 y + 121 y^2$$

$$d=12 \quad x_y = 166 - 892 y + 166 y^2$$