Examples of problems for written exam AD 2026

Problem 1 Suppose $f : \mathbb{C} \to \mathbb{C}$ is holomorphic. Express the differential of the function $|f(z)|$ in terms of $\frac{d}{dz}$, dz and $d\bar{z}$.

Problem 2 Let $f(x,y) := 2x^2 - x^4 - 2y^2 - 2x^2y^2 - y^4$. Find the associated Weierstrass polynomial $g \in \mathcal{O}_{(\mathbb{C},0)}[x]$, such that x is the distinguished variable.

Problem 3 Suppose $U \subset \mathbb{C}^n$ is an open subset, $n > 1$. Let $f : U \to \mathbb{C}$ be a holomorphic function. Show that if the zero set $Z(f)$ is compact, then it is empty.

Problem 4 Let $X_k \subset \mathbb{C}^n$, $k \in \mathbb{N}$ be a family of analytic sets. Prove that $\bigcap_{k\in\mathbb{N}} X_k$ is an analytic set.

Problem 5 Consider $\mathbb{R}^3$ with the standard scalar product inducing Riemannian metric and with the standard orientation. Let (x,y,z) be coordinates. Compute $d^*(x^2y\,dx + xz\,dy)$.

Problem 6 Let $\mathbb{C} \subset \mathbb{P}^1$ be equipped with the metric induced from the Fubini-Study metric. Compute the Hodge star of the forms dz and $d\bar{z}$.

Problem 7 Let $V = \mathbb{C}^2$ considered as a real vector space. The space $(\Lambda^2 V^*) \otimes_{\mathbb{R}} \mathbb{C}$ contains the subspaces

$$W_1 = \Lambda^{1,1}V^* \qquad \text{and} \qquad W_2 = \Lambda^2 V^* \,.$$

Find a basis of the intersection $W_1 \cap W_2$ (of course over $\mathbb{R}$ because W_2 is a real subspace).

Problem 8 Let V be a real vector space and let $I \in End(V)$ be an almost complex structure. Construct a map of complex vector spaces

$$(V, I) \longrightarrow (V \otimes \mathbb{C}, 1 \otimes i)\,.$$

Problem 9 Let $U \subset \mathbb{C}^n$ and $V \subset \mathbb{C}^m$. Let $\varphi : U \to V$ be a holomorphic map. Check that

$$\varphi^*(\alpha) \in A^{p,q}(U) \qquad \text{for} \qquad \alpha \in A^{p,q}(V)\,.$$

Problem 10 Let $X \subset \mathbb{P}^n$ be a hypersurface of degree d. Compute the volume of X with respect to the Fubini-Study metric.

Problem 11 Let $X = \mathbb{C}^*$. Let α be a 1–form of the type $(0,1)$. Does there exist a 0–form β, such that $d\beta = \alpha$?

Problem 12 For which $n \in \mathbb{N}$ does the product of spheres $S^2 \times S^n$ admit a structure of a Kähler manifold?

Problem 13 Let $V = \mathbb{C}^2$ with the standard volume form. Define a hermitian form Φ on $\Lambda^{p,q}V^*$, $p+q=2$ given by the identity

$$\alpha \wedge \bar{\beta} = \Phi(\alpha, \beta) vol\,.$$

What is the signature of that form?

Problem 14 Let $X = \mathbb{C}^2/A$, where A is generated by the vectors

$$\alpha_1 = (1,0), \quad \alpha_2 = (0,1), \quad \alpha_3 = (1+i, i), \quad \alpha_4 = (2\pi i, e+i)\,.$$

Define a real submanifold $Y \subset X$

$$\widetilde{Y} = lin_{\mathbb{R}}\{\alpha_3, \alpha_4\}\,, \qquad Y = \widetilde{Y}/(\widetilde{Y} \cap A)\,.$$

Compute $\int_Y dz_1 \wedge d\bar{z}_2$.

Problem 15 List Hodge numbers of the smooth quadrics of dimensions 1, 2, 3 and 4.

Problem 16 Suppose that M is a compact Kähler manifold of dimension 3 (over $\mathbb{C}$). Is it possible, that the dimensions of the cohomology groups are

$$1, 2, 2, 0, 2, 2, 1.$$

What about

$$1, 1, 2, 2, 2, 1, 1?$$

Problem 17 Suppose M is a compact Kähler manifold. Let $\alpha \in \Omega^p(M)$ be a global holomorphic form. Show that $\partial\alpha = 0$.

Problem 18 Let X be a connected compact Kähler surface. Suppose that its cohomology is generated by the fundamental classes of complex submanifolds. Find a relation between Euler characteristic and the signature.

Problem 19 Let X be the blow-up of $\mathbb{P}^3$ at a point. Compute its Hodge numbers.

Problem 20 Let X be a smooth hypersurface of degree 4 in $\mathbb{P}^4$. Compute its Euler characteristic,
— or (applying Weak Lefschetz) compute the dimensions of $H^k(X)$,
— or (applying Weak Lefschetz and HRR) compute all Hodge numbers.

Problem 21 Let X be quintic (a hypersurface of degree 5) in $\mathbb{P}^4$. Compute $\chi(X, \mathcal{O}_X)$.

Problem 22 Let E be a vector bundle of rank n. Show that $c_1(E) = c_1(\Lambda^n E)$.

Problem 23 Let E, F be smooth vector bundles with connections ∇_E, ∇_F. Construct a connection on the bundle $\mathrm{Hom}(E, F)$.

Problem 24 Let $0 \to E_1 \to E_2 \to E_3 \to 0$ be a short exact sequence of vector bundles over a smooth manifold. Suppose that on E_2 there is a connection preserving E_1. It induces a connection on $E_3 = E_2/E_1$. Show that the equality of total Chern classes $c(E_2) = c(E_1)c(E_3)$ holds not only in cohomology, but also on the level of differential forms defined by the connections.

Problem 25 TBA