Oral Exam – The List of Questions

During the examination, three questions will be selected at random from the following list.

1) Weierstrass preparation lemma
2) Hartogs theorem
3) Properties of the local ring of analytic functions $\mathcal{O}_{\mathbb{C}^n,0}$.
4) GAGA
5) Classification of complex compact curves.
6) Examples of complex compact manifolds.
7) All about $\mathbb{P}^n$ (Hodge structure in cohomology, line bundles over $\mathbb{P}^n$, tangent bundle).
8) Laplasian for compact Riemannian manifolds.
9) Hodge decomposition of C^∞ forms for compact Riemannian manifolds.
10) Heat equation and harmonic forms.
11) Differential forms on complex manifolds, differentials ∂ and $\bar{\partial}$.
12) Dolbeault cohomology, relation with de Rham cohomology, Hodge filtration
13) Definition(s) of Kähler structure (three different ones).
14) Kähler identities, what do they imply?
15) Hard Lefschetz theorem.
16) The action of $\mathfrak{sl}_2(\mathbb{Z})$ on differential forms and on cohomology of Kähler manifolds, corollaries.
17) Hodge diamond and dualities.
18) Signature of Kähler manifolds.
19) Fubini-Study form on complex projective space, relation with curvature of $\mathcal{O}_{\mathbb{P}^n}(1)$.
20) Connection and curvarure of C^∞-bundles.
21) Connection and curvature for holomorphic vector bundles (concordance with structures).
22) Chern classes of complex vector bundles - construction through curvature form.
23) Hirzebruch-Riemann-Roch. Examples of computations.
24) Positive line bundles (Kodaira theorems: vanishing, embedding).
25) Weak Lefschetz (hyperplane) theorem.
26) TBA