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In[1]:= md = 20; (* maximal degree *)
expansion = Normal[Series[u Exp[Sum[a[i] u^i, {i, 4, md + 5, 2}]], {u, 0, md + 5}]];
gg[x_] := expansion /. u -> x
FF[x_, y_] := g[x + y] / (g[x] * g[y])

In[5]:= Print["Blow-up relation"]
LHS = F[x1, b1 + b2] * F[x2 - x1, b2] + F[x2, b1 + b2] * F[x1 - x2, b1];
RHS = F[x1, b1] * F[x2, b2];
Rel0 = Numerator[Factor[LHS - RHS /. F -> FF]]
Print["if g'[0]=0, then g is an odd function"]
inic = {g[0] -> 0, g'[0] -> 1, g''[0] -> 0};
Factor[D[Rel0, {x2, 1}, {b1, 1}, {b2, 1}] /. {x2 -> 0, b1 -> 0, b2 -> 0, x1 -> x} /. inic] == 0
Print["relation, assuming that g is antisymmetric"]
Rel = Factor[Rel0 / g[x1 - x2] /. g[x2 - x1] -> -g[x1 - x2]]
Print["differential relation for odd function"]
inic = {g[0] -> 0, g'[0] -> 1, g''[0] -> 0, g'''[0] -> 0};
asym =
  {g[-x] -> -g[x], g'[-x] -> g'[x], g''[-x] -> -g''[x], g'''[-x] -> g'''[x], g''''[-x] -> -g''''[x]};
DR = Factor[D[Rel, {x2, 2}, {b1, 2}, {b2, 2}] /. {x2 -> 0, b1 -> 0, b2 -> 0, x1 -> x} /. inic /. asym]
Print["the sequence of relations, d[j] depends linearly on a[j+2]"]
Do[d[j] = Factor[Coefficient[DR /. g -> gg, x, j]], {j, 0, md, 2}]
Do[Print["d", j, "=", d[j]], {j, 0, 16, 2}]
Print["coefficients obey polynomial pattern -2(j-4)(1+j)(2+j)(3+j)"]
Table[-2 (j - 4) (1 + j) (2 + j) (3 + j) == Coefficient[d[j], a[j + 2]], {j, 6, md - 2, 2}]
Print["expressions for a[j], j>8"]
p = {};
Do[p = Union[p, Solve[0 == d[i] /. p, a[i + 2]]][[1]], {i, 6, md, 2}];
Column[Expand[p]]
Print["a simpler form of the equation"]
Expand[
  Solve[Factor[DR /. g -> Function[x, x E^(-2 v[x])] /. {v[0] -> 0, v'[0] -> 0, v''[0] -> 0} == 0, v''''[x]]]

Blow-up relation

Out[8]= -g[b1 + b2] * g[b1 + x1] * g[x1 - x2] * g[b2 + x2] * g[-x1 + x2] +
  g[b2] * g[x1] * g[b1 + x1 - x2] * g[b1 + b2 + x2] * g[-x1 + x2] +
  g[b1] * g[b1 + b2 + x1] * g[x1 - x2] * g[x2] * g[b2 - x1 + x2]

if g'[0]=0, then g is an odd function

Out[11]= g[x] (g[x] g'[-x] + g[-x] g'[x]) == 0

relation, assuming that g is antisymmetric

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Out[13]=

$$g[b1 + b2] \times g[b1 + x1] \times g[x1 - x2] \times g[b2 + x2] - \\ g[b2] \times g[x1] \times g[b1 + x1 - x2] \times g[b1 + b2 + x2] + g[b1] \times g[b1 + b2 + x1] \times g[x2] \times g[b2 - x1 + x2]$$

differential relation for odd function

Out[17]=

$$-2 \left(3 g''[x]^2 - 4 g'[x] g^{(3)}[x] + g[x] g^{(4)}[x] + g[x]^2 g^{(5)}[0] \right)$$

the sequence of relations, d[j] depends linearly on a[j+2]

$$d[0]=0$$

$$d[2]=0$$

$$d[4]=0$$

$$d[6]=-288 \left(6 a[4]^2 + 7 a[8] \right)$$

$$d[8]=-720 \left(12 a[4] \times a[6] + 11 a[10] \right)$$

$$d[10]=-144 \left(24 a[4]^3 + 75 a[6]^2 + 140 a[4] \times a[8] + 143 a[12] \right)$$

$$d[12]=-96 \left(216 a[4]^2 a[6] + 462 a[6] \times a[8] + 435 a[4] \times a[10] + 455 a[14] \right)$$

$$d[14]=-96 \left(36 a[4]^4 + 405 a[4] a[6]^2 + 414 a[4]^2 a[8] + 434 a[8]^2 + 840 a[6] \times a[10] + 825 a[4] \times a[12] + 850 a[16] \right)$$

$$d[16]=-48 \left(504 a[4]^3 a[6] + 450 a[6]^3 + 2880 a[4] \times a[6] \times a[8] + \right. \\ \left. 1482 a[4]^2 a[10] + 2934 a[8] \times a[10] + 2838 a[6] \times a[12] + 2912 a[4] \times a[14] + 2907 a[18] \right)$$

coefficients obey polynomial pattern $-2(j-4)(1+j)(2+j)(3+j)$

Out[22]=

{True, True, True, True, True, True, True}

expressions for a[j], j>8

Out[24]=

$$a[8] \rightarrow -\frac{6}{7} a[4]^2$$

$$a[10] \rightarrow -\frac{12}{11} a[4] \times a[6]$$

$$a[12] \rightarrow \frac{96 a[4]^3}{143} - \frac{75 a[6]^2}{143}$$

$$a[14] \rightarrow \frac{1440 a[4]^2 a[6]}{1001}$$

$$a[16] \rightarrow -\frac{144}{221} a[4]^4 + \frac{2700 a[4] a[6]^2}{2431}$$

$$a[18] \rightarrow -\frac{83520 a[4]^3 a[6]}{46189} + \frac{1500 a[6]^3}{4199}$$

$$a[20] \rightarrow \frac{13824 a[4]^5}{20995} - \frac{1047600 a[4]^2 a[6]^2}{508079}$$

$$a[22] \rightarrow \frac{26887680 a[4]^4 a[6]}{11685817} - \frac{13284000 a[4] a[6]^3}{11685817}$$

a simpler form of the equation

Out[26]=

$$\left\{ \left\{ v^{(4)}[x] \rightarrow \frac{12 v''[x]}{x^2} + 12 v''[x]^2 - 5 v^{(4)}[0] \right\} \right\}$$

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In[27]:= Print["Braid relation"]

LHS = F[z2 - z1, m3 - m2] * F[z3 - z2, m3 - m1] * F[z2 - z1, m2 - m1] +
      F[z1 - z2, h] * F[z3 - z1, m3 - m1] * F[z2 - z1, h];
RHS = F[z3 - z2, m2 - m1] * F[z2 - z1, m3 - m1] * F[z3 - z2, m3 - m2] +
      F[z2 - z3, h] * F[z3 - z1, m3 - m1] * F[z3 - z2, h];
Rel0 = Numerator[Factor[LHS - RHS /. F -> FF /. m1 -> 0 /. z1 -> 0]]
Print["if g'[0]=0, then g is an odd function"]

inic = {g[0] -> 0, g'[0] -> 1, g''[0] -> 0};
Factor[D[Rel0, {m2, 1}, {m3, 1}, {z2, 3}, {h, 1}] /. {m2 -> 0, m3 -> 0, z2 -> 0, h -> 0} /. inic] == 0
Print["reletion, assuming that g is antisymmetric"]

Rel = Factor[Rel0 / (g[z2] * g[z2 - z3]) /. {g[-z2] -> -g[z2], g[z3 - z2] -> -g[z2 - z3]}]
Print["differential relation for odd function"]

inic = {g[0] -> 0, g'[0] -> 1, g''[0] -> 0, g'''[0] -> 0};
asym =
  {g[-x] -> -g[x], g'[-x] -> g'[x], g''[-x] -> -g''[x], g'''[-x] -> g'''[x], g''''[-x] -> -g''''[x]};
DBr = Factor[
  D[Rel, {m2, 1}, {m3, 2}, {z2, 2}, {h, 4}] /. {m2 -> 0, m3 -> 0, z2 -> 0, z3 -> x, h -> 0} /. inic /. asym]
Print["new differential equation differs by an invertible factor"]
Factor[DBr / DR]

Braid relation

Out[30]=
g[m2] * g[-m2 + m3] * g[h - z2] * g[z2] * g[h + z2] * g[z2 - z3] * g[m3 + z3] g[-z2 + z3]^2 -
g[m2] * g[-m2 + m3] * g[-z2] g[z2]^2 g[h + z2 - z3] * g[m3 + z3] * g[-z2 + z3] * g[h - z2 + z3] +
g[h]^2 g[-z2] * g[m2 + z2] * g[-m2 + m3 + z2] * g[z2 - z3] * g[z3] * g[-z2 + z3] * g[m3 - z2 + z3] -
g[h]^2 g[-z2] * g[z2] * g[m3 + z2] * g[z2 - z3] * g[z3] * g[m2 - z2 + z3] * g[-m2 + m3 - z2 + z3]

if g'[0]=0, then g is an odd function

Out[33]=
6 g[z3]^2 (g[z3] g'[-z3] + g[-z3] g'[z3]) == 0

reletion, assuming that g is antisymmetric

Out[35]=
g[m2] * g[-m2 + m3] * g[h - z2] * g[h + z2] g[z2 - z3]^2 g[m3 + z3] -
g[m2] * g[-m2 + m3] g[z2]^2 g[h + z2 - z3] * g[m3 + z3] * g[h - z2 + z3] +
g[h]^2 g[m2 + z2] * g[-m2 + m3 + z2] * g[z2 - z3] * g[z3] * g[m3 - z2 + z3] +
g[h]^2 g[z2] * g[m3 + z2] * g[z3] * g[m2 - z2 + z3] * g[-m2 + m3 - z2 + z3]

differential relation for odd function

Out[39]=
-8 g'[x] (-3 g''[x]^2 + 4 g'[x] g^(3)[x] - g[x] g^(4)[x] - g[x]^2 g^(5)[0])

new differential equation differs by an invertible factor

Out[41]=
-4 g'[x]

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In[42]:= Print["Atiyah Flop"]
expansion = Normal[Series[u Exp[Sum[b[i] u ^ i, {i, 2, md + 14}]], {u, 0, md + 14}]];
gg[x_] := expansion /. u -> x
Print["the relation"]

SA1 = 1 / (g[t1 - t2] * g[s1 + t2] * g[s2 + t2]) + 1 / (g[s1 + t1] * g[s2 + t1] * g[-t1 + t2]);
SA2 = 1 / (g[-s1 + s2] * g[s1 + t1] * g[s1 + t2]) + 1 / (g[s1 - s2] * g[s2 + t1] * g[s2 + t2]);
RA = Numerator[Factor[SA1 - SA2]]
Print["differential equation assuming g(0)=0, g'(0)=1, g''(0)=0"]
inic = {g[0] -> 0, g'[0] -> 1, g''[0] -> 0};
DR = Factor[D[RA, {s1, 2}, {s2, 2}] /. {s1 -> 0, s2 -> 0, t1 -> 0, t2 -> t} /. inic];
Collect[DR, g[-t], Factor]
Print["the sequence of relations: d[j] depends linearly on b[j-1]"]
Do[d[j] = Factor[Coefficient[DR /. g -> gg, t, j]], {j, 0, md}]
Do[Print["d[" , j, "]=", d[j]], {j, 0, 10}]
Print["coefficient obey a polynomial pattern, 4(j-4)(j-1), j even "]
Table[Coefficient[d[j], b[j - 1]] == 4 (j - 4) (j - 1), {j, 6, md, 2}]
Print["coefficient obey a polynomial pattern, 4(j-4)j, j odd "]
Table[Coefficient[d[j], b[j - 1]] == 4 (j - 5) j, {j, 5, md, 2}]
Print["expressions for a[j] for j>4"]

p = {};
Do[p = Union[p, Solve[0 == d[i] /. p, b[i - 1]]][[1]], {i, 6, md}];
Column[Expand[p]]

Atiyah Flop

the relation

Out[48]=
g[s1 - s2] * g[-s1 + s2] * g[t1 - t2] * g[s1 + t2] * g[s2 + t2] +
g[s1 - s2] * g[-s1 + s2] * g[s1 + t1] * g[s2 + t1] * g[-t1 + t2] -
g[-s1 + s2] * g[s1 + t1] * g[t1 - t2] * g[s1 + t2] * g[-t1 + t2] -
g[s1 - s2] * g[s2 + t1] * g[t1 - t2] * g[s2 + t2] * g[-t1 + t2]

differential equation assuming g(0)=0, g'(0)=1, g''(0)=0

Out[52]=
8 g[t] + 4 g[-t] (2 g'[t]^2 - g[t] g''[t] - g[t]^2 g^(3)[0])

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the sequence of relations: $d[j]$ depends linearly on $b[j-1]$

$$d[0]=0$$

$$d[1]=0$$

$$d[2]=0$$

$$d[3]=0$$

$$d[4]=0$$

$$d[5]=0$$

$$d[6]=-8 \left(6 b[2] \times b[3] - 5 b[5] \right)$$

$$d[7]=-\frac{4}{3} \left(8 b[2]^3 + 27 b[3]^2 + 24 b[2] \times b[4] - 42 b[6] \right)$$

$$d[8]=-8 \left(18 b[2]^2 b[3] + 12 b[3] \times b[4] - 5 b[2] \times b[5] - 14 b[7] \right)$$

$$d[9]=-\frac{4}{3} \left(20 b[2]^4 + 117 b[2] b[3]^2 + 96 b[2]^2 b[4] + 36 b[4]^2 + 60 b[3] \times b[5] - 78 b[2] \times b[6] - 108 b[8] \right)$$

$$d[10]=-\frac{4}{3} \left(170 b[2]^3 b[3] + 27 b[3]^3 + 348 b[2] \times b[3] \times b[4] + \right. \\ \left. 45 b[2]^2 b[5] + 30 b[4] \times b[5] + 66 b[3] \times b[6] - 168 b[2] \times b[7] - 162 b[9] \right)$$

coefficient obey a polynomial pattern, $4(j-4)(j-1)$, j even

Out[57]=

{True, True, True, True, True, True, True, True}

coefficient obey a polynomial pattern, $4(j-4)j$, j odd

Out[59]=

{True, True, True, True, True, True, True, True}

expressions for $a[j]$ for $j > 4$

Out[61]=

$$b[5] \rightarrow \frac{6}{5} b[2] \times b[3]$$

$$b[6] \rightarrow \frac{4 b[2]^3}{21} + \frac{9 b[3]^2}{14} + \frac{4}{7} b[2] \times b[4]$$

$$b[7] \rightarrow \frac{6}{7} b[2]^2 b[3] + \frac{6}{7} b[3] \times b[4]$$

$$b[8] \rightarrow \frac{b[2]^4}{21} + \frac{9}{7} b[2] b[3]^2 + \frac{10}{21} b[2]^2 b[4] + \frac{b[4]^2}{3}$$

$$b[9] \rightarrow \frac{4}{7} b[2]^3 b[3] + \frac{3 b[3]^3}{7} + \frac{12}{7} b[2] \times b[3] \times b[4]$$

$$b[10] \rightarrow \frac{8 b[2]^5}{165} + \frac{18}{11} b[2]^2 b[3]^2 + \frac{64}{231} b[2]^3 b[4] + \frac{72}{77} b[3]^2 b[4] + \frac{136}{231} b[2] b[4]^2$$

$$b[11] \rightarrow \frac{32}{77} b[2]^4 b[3] + \frac{108}{77} b[2] b[3]^3 + \frac{152}{77} b[2]^2 b[3] \times b[4] + \frac{8}{11} b[3] b[4]^2$$

$$b[12] \rightarrow \frac{128 b[2]^6}{7007} + \frac{11448 b[2]^3 b[3]^2}{7007} + \frac{891 b[3]^4}{2548} + \frac{5048 b[2]^4 b[4]}{21021} + \frac{20232 b[2] b[3]^2 b[4]}{7007} + \frac{12080 b[2]^2 b[4]^2}{21021} + \frac{8 b[4]^3}{39}$$

$$b[13] \rightarrow \frac{304 b[2]^5 b[3]}{1001} + \frac{2538 b[2]^2 b[3]^3}{1001} + \frac{1888 b[2]^3 b[3] \cdot b[4]}{1001} + \frac{1026 b[3]^3 b[4]}{1001} + \frac{2032 b[2] \cdot b[3] b[4]^2}{1001}$$

$$b[14] \rightarrow \frac{32 b[2]^7}{1911} + \frac{954}{637} b[2]^4 b[3]^2 + \frac{972}{637} b[2] b[3]^4 + \frac{3232 b[2]^5 b[4]}{21021} + \frac{34380 b[2]^2 b[3]^2 b[4]}{7007} + \frac{11680 b[2]^3 b[4]^2}{21021} + \frac{90}{77} b[3]^2 b[4]^2 + \frac{480 b[2] b[4]^3}{1001}$$

$$b[15] \rightarrow$$

$$\frac{1528 b[2]^6 b[3]}{7007} + \frac{24048 b[2]^3 b[3]^3}{7007} + \frac{972 b[3]^5}{3185} + \frac{11912 b[2]^4 b[3] \cdot b[4]}{7007} + \frac{29808 b[2] b[3]^3 b[4]}{7007} + \frac{22376 b[2]^2 b[3] b[4]^2}{7007} + \frac{8}{13} b[3] b[4]^3$$

$$b[16] \rightarrow \frac{94 b[2]^8}{11781} + \frac{156}{119} b[2]^5 b[3]^2 + \frac{4698 b[2]^2 b[3]^4}{1309} + \frac{7624 b[2]^6 b[4]}{63063} + \frac{756312 b[2]^3 b[3]^2 b[4]}{119119} + \frac{132921 b[3]^4 b[4]}{119119} + \frac{171716 b[2]^4 b[4]^2}{357357} + \frac{541644 b[2] b[3]^2 b[4]^2}{119119} + \frac{744680 b[2]^2 b[4]^3}{1072071} + \frac{254 b[4]^4}{1989}$$

$$b[17] \rightarrow \frac{18672 b[2]^7 b[3]}{119119} + \frac{471780 b[2]^4 b[3]^3}{119119} + \frac{17982 b[2] b[3]^5}{10829} + \frac{176208 b[2]^5 b[3] \cdot b[4]}{119119} + \frac{103536 b[2]^2 b[3]^3 b[4]}{10829} + \frac{467088 b[2]^3 b[3] b[4]^2}{119119} + \frac{2196 b[3]^3 b[4]^2}{1309} + \frac{37584 b[2] \cdot b[3] b[4]^3}{17017}$$

$$b[18] \rightarrow \frac{905504 b[2]^9}{142585443} + \frac{17689080 b[2]^6 b[3]^2}{15842827} + \frac{97195788 b[2]^3 b[3]^4}{15842827} + \frac{46899 b[3]^6}{169442} + \frac{61952 b[2]^7 b[4]}{762489} + \frac{8662272 b[2]^4 b[3]^2 b[4]}{1218679} + \frac{92565828 b[2] b[3]^4 b[4]}{15842827} + \frac{6710208 b[2]^5 b[4]^2}{15842827} + \frac{153180600 b[2]^2 b[3]^2 b[4]^2}{15842827} + \frac{114748160 b[2]^3 b[4]^3}{142585443} + \frac{24720 b[3]^2 b[4]^3}{19019} + \frac{1192480 b[2] b[4]^4}{2909907}$$

$$b[19] \rightarrow \frac{767488 b[2]^8 b[3]}{6789783} + \frac{9422928 b[2]^5 b[3]^3}{2263261} + \frac{10884942 b[2]^2 b[3]^5}{2263261} + \frac{71200 b[2]^6 b[3] \cdot b[4]}{57057} + \frac{35580960 b[2]^3 b[3]^3 b[4]}{2263261} + \frac{2747358 b[3]^5 b[4]}{2263261} + \frac{9662560 b[2]^4 b[3] b[4]^2}{2263261} + \frac{18977040 b[2] b[3]^3 b[4]^2}{2263261} + \frac{29756960 b[2]^2 b[3] b[4]^3}{6789783} + \frac{6560 b[3] b[4]^4}{12597}$$

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In[62]:= Print["Grassmannian flop"]

S1[n_] := Sum[Product[1 / (g[x[i] - x[p]] * g[x[p] - x[i] + s]), {i, Complement[Range[n], {p}]}], {p, 1, n}]
S2[n_] := Sum[Product[1 / (g[x[p] - x[i]] * g[x[i] - x[p] + s]), {i, Complement[Range[n], {p}]}], {p, 1, n}]
R[n_] := Numerator[Factor[S1[n] - S2[n]]]
inic = {g[0] -> 0, g'[0] -> 1, g''[0] -> 0};

Print["differential equation for the nilpotent cone, n=3"]
DR3 = Factor[D[Factor[D[R[3] /. x[1] -> 0, {x[2], 5}] /. x[2] -> 0] /. inic, {x[3], 4}] /. x[3] -> 0 /. inic];
Collect[DR3 / 40, g[s], Factor]
Print["the sequence of relations, d[j] depends linearly on b[j-1]"]
Do[d[j] = Factor[SeriesCoefficient[DR3 /. g -> gg, {s, 0, j}]], {j, 0, md}]
Do[Print["d[" , j, "]=", d[j]], {j, 0, 16}]
Print["the polynomial pattern -240(j-6)(j-5)(j-2)(j+1)(j+2)"]
Table[Coefficient[d[j], b[j + 1]] == -240 (j - 6) (j - 5) (j - 2) (j + 1) (j + 2), {j, 7, md}]
Print["expressions for b[j] for j>7"]
p3 = {};
Do[p3 = Union[p3, Solve[0 == d[i] /. p3, b[i + 1]]], {i, 7, md}];
Column[Expand[p3]]

Grassmannian flop

differential equation for the nilpotent cone, n=3

Out[69]=
360 g'[s]^3 g''[s] - 60 g[s] g'[s] (3 g''[s]^2 + 2 g'[s]^2 g^(3)[0] + 4 g'[s] g^(3)[s]) +
60 g[s]^2 (g'[s] g''[s] g^(3)[0] + g''[s] g^(3)[s] + g'[s] g^(4)[s]) + 6 g[s]^3 (g'[s] g^(5)[0] - g^(5)[s]) + g[s]^4 (-5 g^(3)[0] g^(4)[0] + g^(6)[0])

the sequence of relations, d[j] depends linearly on b[j-1]

d[0]=0
d[1]=0
d[2]=0
d[3]=0
d[4]=0
d[5]=0
d[6]=0
d[7]=-57 600 (b[2]^4 + 2 b[2]^2 b[4] - b[4]^2 - 6 b[2] * b[6] + 3 b[8])
d[8]=-28 800 (12 b[2]^3 b[3] + 12 b[2] * b[3] * b[4] + 10 b[2]^2 b[5] - 10 b[4] * b[5] - 18 b[3] * b[6] - 42 b[2] * b[7] + 27 b[9])
d[9]=
-2880 (56 b[2]^5 + 270 b[2]^2 b[3]^2 + 400 b[2]^3 b[4] + 90 b[3]^2 b[4] + 40 b[2] b[4]^2 + 300 b[2] * b[3] * b[5] - 125 b[5]^2 -
300 b[2]^2 b[6] - 420 b[4] * b[6] - 630 b[3] * b[7] - 760 b[2] * b[8] + 770 b[10])

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$d[10]=$

$$-14\,400 \left(76\,b[2]^4\,b[3] + 54\,b[2]\,b[3]^3 + 344\,b[2]^2\,b[3] \times b[4] + 20\,b[3]\,b[4]^2 + 160\,b[2]^3\,b[5] + 45\,b[3]^2\,b[5] - 132\,b[2] \times b[3] \times b[6] - 150\,b[5] \times b[6] - 280\,b[2]^2\,b[7] - 224\,b[4] \times b[7] - 252\,b[3] \times b[8] - 180\,b[2] \times b[9] + 352\,b[11] \right)$$

$d[11]=-720 \left(256\,b[2]^6 + 4080\,b[2]^3\,b[3]^2 + 405\,b[3]^4 + 4480\,b[2]^4\,b[4] + 9840\,b[2]\,b[3]^2\,b[4] + \right.$

$$5760\,b[2]^2\,b[4]^2 + 13\,600\,b[2]^2\,b[3] \times b[5] + 800\,b[3] \times b[4] \times b[5] - 1000\,b[2]\,b[5]^2 + 1440\,b[2]^3\,b[6] - 1260\,b[3]^2\,b[6] - 6240\,b[2] \times b[4] \times b[6] - 3060\,b[6]^2 - 13\,440\,b[2] \times b[3] \times b[7] - 7000\,b[5] \times b[7] - 12\,480\,b[2]^2\,b[8] - 8640\,b[4] \times b[8] - 7560\,b[3] \times b[9] - 1680\,b[2] \times b[10] + 14\,040\,b[12] \left. \right)$$

$d[12]=-2880$

$$\left(464\,b[2]^5\,b[3] + 1350\,b[2]^2\,b[3]^3 + 5920\,b[2]^3\,b[3] \times b[4] + 1170\,b[3]^3\,b[4] + 4240\,b[2] \times b[3]\,b[4]^2 + 2180\,b[2]^4\,b[5] + 4800\,b[2]\,b[3]^2\,b[5] + 4360\,b[2]^2\,b[4] \times b[5] - 180\,b[4]^2\,b[5] - 125\,b[3]\,b[5]^2 + 2220\,b[2]^2\,b[3] \times b[6] - 1500\,b[3] \times b[4] \times b[6] - 2880\,b[2] \times b[5] \times b[6] - 1400\,b[2]^3\,b[7] - 1890\,b[3]^2\,b[7] - 5320\,b[2] \times b[4] \times b[7] - 2940\,b[6] \times b[7] - 7840\,b[2] \times b[3] \times b[8] - 3260\,b[5] \times b[8] - 5220\,b[2]^2\,b[9] - 3420\,b[4] \times b[9] - 2170\,b[3] \times b[10] + 1320\,b[2] \times b[11] + 6370\,b[13] \right)$$

$d[13]=$

$$-480 \left(128\,b[2]^7 + 8160\,b[2]^4\,b[3]^2 + 5265\,b[2]\,b[3]^4 + 9664\,b[2]^5\,b[4] + 69\,600\,b[2]^2\,b[3]^2\,b[4] + 31\,360\,b[2]^3\,b[4]^2 + 18\,480\,b[3]^2\,b[4]^2 + 12\,480\,b[2]\,b[4]^3 + 68\,880\,b[2]^3\,b[3] \times b[5] + 13\,500\,b[3]^3\,b[5] + 78\,480\,b[2] \times b[3] \times b[4] \times b[5] + 15\,900\,b[2]^2\,b[5]^2 - 3900\,b[4]\,b[5]^2 + 13\,680\,b[2]^4\,b[6] + 23\,940\,b[2]\,b[3]^2\,b[6] + 13\,920\,b[2]^2\,b[4] \times b[6] - 8400\,b[4]^2\,b[6] - 16\,920\,b[3] \times b[5] \times b[6] - 19\,620\,b[2]\,b[6]^2 - 21\,000\,b[2]^2\,b[3] \times b[7] - 34\,440\,b[3] \times b[4] \times b[7] - 47\,880\,b[2] \times b[5] \times b[7] - 14\,700\,b[7]^2 - 27\,840\,b[2]^3\,b[8] - 27\,720\,b[3]^2\,b[8] - 69\,120\,b[2] \times b[4] \times b[8] - 30\,240\,b[6] \times b[8] - 83\,160\,b[2] \times b[3] \times b[9] - 31\,320\,b[5] \times b[9] - 42\,840\,b[2]^2\,b[10] - 27\,720\,b[4] \times b[10] - 9240\,b[3] \times b[11] + 31\,320\,b[2] \times b[12] + 64\,680\,b[14] \right)$$

$d[14]=-240 \left(1152\,b[2]^6\,b[3] + 24\,480\,b[2]^3\,b[3]^3 + 2673\,b[3]^5 + 118\,080\,b[2]^4\,b[3] \times b[4] + 119\,520\,b[2]\,b[3]^3\,b[4] + \right.$

$$247\,680\,b[2]^2\,b[3]\,b[4]^2 + 37\,440\,b[3]\,b[4]^3 + 39\,488\,b[2]^5\,b[5] + 269\,160\,b[2]^2\,b[3]^2\,b[5] + 198\,400\,b[2]^3\,b[4] \times b[5] + 115\,320\,b[3]^2\,b[4] \times b[5] + 102\,720\,b[2]\,b[4]^2\,b[5] + 101\,400\,b[2] \times b[3]\,b[5]^2 - 6000\,b[5]^3 + 155\,520\,b[2]^3\,b[3] \times b[6] + 26\,460\,b[3]^3\,b[6] + 120\,960\,b[2] \times b[3] \times b[4] \times b[6] + 22\,800\,b[2]^2\,b[5] \times b[6] - 57\,360\,b[4] \times b[5] \times b[6] - 40\,500\,b[3]\,b[6]^2 + 9360\,b[2]^4\,b[7] - 22\,680\,b[2]\,b[3]^2\,b[7] - 61\,920\,b[2]^2\,b[4] \times b[7] - 49\,680\,b[4]^2\,b[7] - 101\,640\,b[3] \times b[5] \times b[7] - 162\,000\,b[2] \times b[6] \times b[7] - 175\,680\,b[2]^2\,b[3] \times b[8] - 155\,520\,b[3] \times b[4] \times b[8] - 194\,880\,b[2] \times b[5] \times b[8] - 92\,880\,b[7] \times b[8] - 112\,320\,b[2]^3\,b[9] - 102\,060\,b[3]^2\,b[9] - 241\,920\,b[2] \times b[4] \times b[9] - 93\,960\,b[6] \times b[9] - 246\,960\,b[2] \times b[3] \times b[10] - 89\,040\,b[5] \times b[10] - 95\,040\,b[2]^2\,b[11] - 63\,360\,b[4] \times b[11] + 9720\,b[3] \times b[12] + 149\,760\,b[2] \times b[13] + 207\,360\,b[15] \left. \right)$$

d[15]=

$$\begin{aligned}
& 480 \left(256 b[2]^8 + 192 b[2]^5 b[3]^2 - 9720 b[2]^2 b[3]^4 - 7936 b[2]^6 b[4] - 141600 b[2]^3 b[3]^2 b[4] - 18900 b[3]^4 b[4] - \right. \\
& 55360 b[2]^4 b[4]^2 - 160320 b[2] b[3]^2 b[4]^2 - 63360 b[2]^2 b[4]^3 - 6720 b[4]^4 - 121920 b[2]^4 b[3] \times b[5] - \\
& 115380 b[2] b[3]^3 b[5] - 394080 b[2]^2 b[3] \times b[4] \times b[5] - 79200 b[3] b[4]^2 b[5] - \\
& 68200 b[2]^3 b[5]^2 - 38775 b[3]^2 b[5]^2 - 63000 b[2] \times b[4] b[5]^2 - 25344 b[2]^5 b[6] - \\
& 161640 b[2]^2 b[3]^2 b[6] - 106560 b[2]^3 b[4] \times b[6] - 54360 b[3]^2 b[4] \times b[6] - 38400 b[2] b[4]^2 b[6] - \\
& 68760 b[2] \times b[3] \times b[5] \times b[6] + 23250 b[5]^2 b[6] + 11520 b[2]^2 b[6]^2 + 29520 b[4] b[6]^2 - \\
& 44880 b[2]^3 b[3] \times b[7] - 1890 b[3]^3 b[7] + 29040 b[2] \times b[3] \times b[4] \times b[7] + 48000 b[2]^2 b[5] \times b[7] + \\
& 71280 b[4] \times b[5] \times b[7] + 86220 b[3] \times b[6] \times b[7] + 68880 b[2] b[7]^2 + 16480 b[2]^4 b[8] + \\
& 84720 b[2] b[3]^2 b[8] + 116160 b[2]^2 b[4] \times b[8] + 52320 b[4]^2 b[8] + 107040 b[3] \times b[5] \times b[8] + \\
& 146400 b[2] \times b[6] \times b[8] + 34560 b[8]^2 + 190080 b[2]^2 b[3] \times b[9] + 140400 b[3] \times b[4] \times b[9] + \\
& 167400 b[2] \times b[5] \times b[9] + 69120 b[7] \times b[9] + 91120 b[2]^3 b[10] + 79590 b[3]^2 b[10] + \\
& 184080 b[2] \times b[4] \times b[10] + 67020 b[6] \times b[10] + 158400 b[2] \times b[3] \times b[11] + 56760 b[5] \times b[11] + \\
& 36720 b[2]^2 b[12] + 28080 b[4] \times b[12] - 35880 b[3] \times b[13] - 147840 b[2] \times b[14] - 159120 b[16] \Big)
\end{aligned}$$

$$\begin{aligned}
d[16]= & 240 \left(6656 b[2]^7 b[3] + 11840 b[2]^4 b[3]^3 - 7452 b[2] b[3]^5 - 104192 b[2]^5 b[3] \times b[4] - 332000 b[2]^2 b[3]^3 b[4] - \right. \\
& 538880 b[2]^3 b[3] b[4]^2 - 135680 b[3]^3 b[4]^2 - 330240 b[2] \times b[3] b[4]^3 - 38272 b[2]^6 b[5] - \\
& 593760 b[2]^3 b[3]^2 b[5] - 73035 b[3]^4 b[5] - 365760 b[2]^4 b[4] \times b[5] - 1025280 b[2] b[3]^2 b[4] \times b[5] - \\
& 549120 b[2]^2 b[4]^2 b[5] - 72000 b[4]^3 b[5] - 550200 b[2]^2 b[3] b[5]^2 - 204600 b[3] \times b[4] b[5]^2 - \\
& 48000 b[2] b[5]^3 - 325440 b[2]^4 b[3] \times b[6] - 291360 b[2] b[3]^3 b[6] - 903360 b[2]^2 b[3] \times b[4] \times b[6] - \\
& 148800 b[3] b[4]^2 b[6] - 276480 b[2]^3 b[5] \times b[6] - 136980 b[3]^2 b[5] \times b[6] - 169920 b[2] \times b[4] \times b[5] \times b[6] - \\
& 9360 b[2] \times b[3] b[6]^2 + 86220 b[5] b[6]^2 - 45248 b[2]^5 b[7] - 243960 b[2]^2 b[3]^2 b[7] - \\
& 131200 b[2]^3 b[4] \times b[7] - 25320 b[3]^2 b[4] \times b[7] + 31680 b[2] b[4]^2 b[7] + 96480 b[2] \times b[3] \times b[5] \times b[7] + \\
& 100500 b[5]^2 b[7] + 207600 b[2]^2 b[6] \times b[7] + 230160 b[4] \times b[6] \times b[7] + 147840 b[3] b[7]^2 + \\
& 86400 b[2]^3 b[3] \times b[8] + 47760 b[3]^3 b[8] + 420480 b[2] \times b[3] \times b[4] \times b[8] + 312960 b[2]^2 b[5] \times b[8] + \\
& 281280 b[4] \times b[5] \times b[8] + 318240 b[3] \times b[6] \times b[8] + 446400 b[2] \times b[7] \times b[8] + 103440 b[2]^4 b[9] + \\
& 403920 b[2] b[3]^2 b[9] + 494880 b[2]^2 b[4] \times b[9] + 184560 b[4]^2 b[9] + 376920 b[3] \times b[5] \times b[9] + \\
& 476640 b[2] \times b[6] \times b[9] + 198000 b[8] \times b[9] + 648720 b[2]^2 b[3] \times b[10] + 441360 b[3] \times b[4] \times b[10] + \\
& 512160 b[2] \times b[5] \times b[10] + 194040 b[7] \times b[10] + 255200 b[2]^3 b[11] + 219120 b[3]^2 b[11] + \\
& 501600 b[2] \times b[4] \times b[11] + 176880 b[6] \times b[11] + 345600 b[2] \times b[3] \times b[12] + 128520 b[5] \times b[12] - \\
& 3120 b[2]^2 b[13] + 21840 b[4] \times b[13] - 184800 b[3] \times b[14] - 523440 b[2] \times b[15] - 471240 b[17] \Big)
\end{aligned}$$

the polynomial pattern -240(j-6)(j-5)(j-2)(j+1)(j+2)

Out[74]=

{True, True, True, True, True, True, True, True, True, True, True, True, True, True, True}

expressions for b[j] for j>7

Out[76]=

$$\begin{aligned}
b[8] & \rightarrow -\frac{1}{3} b[2]^4 - \frac{2}{3} b[2]^2 b[4] + \frac{b[4]^2}{3} + 2 b[2] \times b[6] \\
b[9] & \rightarrow -\frac{4}{9} b[2]^3 b[3] - \frac{4}{9} b[2] \times b[3] \times b[4] - \frac{10}{27} b[2]^2 b[5] + \frac{10}{27} b[4] \times b[5] + \frac{2}{3} b[3] \times b[6] + \frac{14}{9} b[2] \times b[7] \\
b[10] & \rightarrow -\frac{464 b[2]^5}{1155} - \frac{27}{77} b[2]^2 b[3]^2 - \frac{272}{231} b[2]^3 b[4] - \frac{9}{77} b[3]^2 b[4] + \\
& \frac{64}{231} b[2] b[4]^2 - \frac{30}{77} b[2] \times b[3] \times b[5] + \frac{25 b[5]^2}{154} + \frac{26}{11} b[2]^2 b[6] + \frac{6}{11} b[4] \times b[6] + \frac{9}{11} b[3] \times b[7] \\
b[11] & \rightarrow -\frac{15}{22} b[2]^4 b[3] - \frac{27}{176} b[2] b[3]^3 - \frac{37}{22} b[2]^2 b[3] \times b[4] + \frac{2}{11} b[3] b[4]^2 - \frac{85}{132} b[2]^3 b[5] - \\
& \frac{45}{352} b[3]^2 b[5] + \frac{25}{132} b[2] \times b[4] \times b[5] + \frac{189}{88} b[2] \times b[3] \times b[6] + \frac{75}{176} b[5] \times b[6] + \frac{35}{22} b[2]^2 b[7] + \frac{7}{11} b[4] \times b[7]
\end{aligned}$$

$$\begin{aligned}
b[12] \rightarrow & -\frac{1400 b[2]^6}{3861} - \frac{736 b[2]^3 b[3]^2}{1287} - \frac{3 b[3]^4}{104} - \frac{4856 b[2]^4 b[4]}{3861} - \frac{1228 b[2] b[3]^2 b[4]}{1287} - \\
& \frac{632 b[2]^2 b[4]^2}{1287} + \frac{8 b[4]^3}{39} - \frac{4690 b[2]^2 b[3] \cdot b[5]}{3861} + \frac{50}{351} b[3] \times b[4] \times b[5] + \frac{350 b[2] b[5]^2}{3861} + \frac{280}{143} b[2]^3 b[6] + \\
& \frac{35}{78} b[3]^2 b[6] + \frac{2240 b[2] \cdot b[4] \cdot b[6]}{1287} + \frac{17 b[6]^2}{78} + \frac{812}{429} b[2] \times b[3] \times b[7] + \frac{175}{351} b[5] \times b[7] \\
b[13] \rightarrow & -\frac{5906 b[2]^5 b[3]}{7007} - \frac{8397 b[2]^2 b[3]^3}{28028} - \frac{16854 b[2]^3 b[3] \cdot b[4]}{7007} - \frac{1566 b[3]^3 b[4]}{7007} - \frac{3064 b[2] \cdot b[3] \cdot b[4]^2}{7007} - \\
& \frac{435}{637} b[2]^4 b[5] - \frac{6885 b[2] b[3]^2 b[5]}{8008} - \frac{1835 b[2]^2 b[4] \cdot b[5]}{1911} + \frac{760 b[4]^2 b[5]}{1911} + \frac{75 b[3] b[5]^2}{1001} + \frac{465}{154} b[2]^2 b[3] \times b[6] + \\
& \frac{60}{77} b[3] \times b[4] \times b[6] + \frac{505}{364} b[2] \times b[5] \times b[6] + \frac{106}{91} b[2]^3 b[7] + \frac{576 b[3]^2 b[7]}{1001} + \frac{20}{13} b[2] \times b[4] \times b[7] + \frac{6}{13} b[6] \times b[7] \\
b[14] \rightarrow & -\frac{4960 b[2]^7}{21021} - \frac{12517 b[2]^4 b[3]^2}{14014} - \frac{1431 b[2] b[3]^4}{16016} - \frac{23872 b[2]^5 b[4]}{21021} - \frac{27175 b[2]^2 b[3]^2 b[4]}{14014} - \frac{7968 b[2]^3 b[4]^2}{7007} - \\
& \frac{90}{539} b[3]^2 b[4]^2 + \frac{1280 b[2] b[4]^3}{7007} - \frac{11605 b[2]^3 b[3] \cdot b[5]}{7644} - \frac{3915 b[3]^3 b[5]}{17248} - \frac{97645 b[2] \cdot b[3] \cdot b[4] \cdot b[5]}{84084} - \\
& \frac{7600 b[2]^2 b[5]^2}{21021} + \frac{500 b[4] b[5]^2}{1617} + \frac{1112 b[2]^4 b[6]}{1001} + \frac{11481 b[2] b[3]^2 b[6]}{8008} + \frac{2144 b[2]^2 b[4] \cdot b[6]}{1001} + \frac{40}{77} b[4]^2 b[6] + \\
& \frac{795 b[3] \cdot b[5] \cdot b[6]}{1232} + \frac{162}{143} b[2] b[6]^2 + \frac{4359 b[2]^2 b[3] \cdot b[7]}{2002} + \frac{75}{77} b[3] \times b[4] \times b[7] + \frac{3760 b[2] \cdot b[5] \cdot b[7]}{3003} + \frac{5 b[7]^2}{22} \\
b[15] \rightarrow & -\frac{190955 b[2]^6 b[3]}{275184} - \frac{426757 b[2]^3 b[3]^3}{733824} - \frac{3 b[3]^5}{260} - \frac{750955 b[2]^4 b[3] \cdot b[4]}{275184} - \frac{284387 b[2] b[3]^3 b[4]}{366912} - \\
& \frac{232849 b[2]^2 b[3] b[4]^2}{137592} + \frac{3}{26} b[3] b[4]^3 - \frac{948295 b[2]^5 b[5]}{1397088} - \frac{10209685 b[2]^2 b[3]^2 b[5]}{6918912} - \frac{2426575 b[2]^3 b[4] \cdot b[5]}{1397088} - \\
& \frac{232165 b[3]^2 b[4] \cdot b[5]}{494208} + \frac{97525 b[2] b[4]^2 b[5]}{698544} - \frac{450875 b[2] \cdot b[3] b[5]^2}{864864} + \frac{625 b[5]^3}{6336} + \frac{137909 b[2]^3 b[3] \cdot b[6]}{52416} + \\
& \frac{7}{39} b[3]^3 b[6] + \frac{56467 b[2] \cdot b[3] \cdot b[4] \cdot b[6]}{26208} + \frac{160565 b[2]^2 b[5] \cdot b[6]}{88704} + \frac{5125 b[4] \cdot b[5] \cdot b[6]}{6336} + \frac{19}{39} b[3] b[6]^2 + \frac{1621 b[2]^4 b[7]}{3024} + \\
& \frac{3917 b[2] b[3]^2 b[7]}{2912} + \frac{40}{27} b[2]^2 b[4] \times b[7] + \frac{7}{12} b[4]^2 b[7] + \frac{12635 b[3] \cdot b[5] \cdot b[7]}{15444} + \frac{295}{144} b[2] \times b[6] \times b[7] \\
b[16] \rightarrow & -\frac{1438 b[2]^8}{13923} - \frac{9743 b[2]^5 b[3]^2}{10829} - \frac{21249 b[2]^2 b[3]^4}{86632} - \frac{1400 b[2]^6 b[4]}{1989} - \frac{7199 b[2]^3 b[3]^2 b[4]}{2499} - \frac{5715 b[3]^4 b[4]}{43316} - \\
& \frac{6236 b[2]^4 b[4]^2}{4641} - \frac{30944 b[2] b[3]^2 b[4]^2}{32487} - \frac{5000 b[2]^2 b[4]^3}{13923} + \frac{254 b[4]^4}{1989} - \frac{13175 b[2]^4 b[3] \cdot b[5]}{7644} - \frac{2655 b[2] b[3]^3 b[5]}{3808} - \\
& \frac{1179335 b[2]^2 b[3] \cdot b[4] \cdot b[5]}{389844} + \frac{5225 b[3] b[4]^2 b[5]}{97461} - \frac{5125 b[2]^3 b[5]^2}{8568} - \frac{22275 b[3]^2 b[5]^2}{99008} - \frac{275 b[2] \cdot b[4] b[5]^2}{12376} + \\
& \frac{2840 b[2]^5 b[6]}{51051} + \frac{143529 b[2]^2 b[3]^2 b[6]}{68068} + \frac{2720 b[2]^3 b[4] \cdot b[6]}{3003} + \frac{6359 b[3]^2 b[4] \cdot b[6]}{17017} + \frac{26968 b[2] b[4]^2 b[6]}{17017} + \\
& \frac{470615 b[2] \cdot b[3] \cdot b[5] \cdot b[6]}{272272} + \frac{199575 b[5]^2 b[6]}{544544} + \frac{6745 b[2]^2 b[6]^2}{2431} + \frac{1103 b[4] b[6]^2}{2431} + \frac{7363 b[2]^3 b[3] \cdot b[7]}{4641} + \frac{414 b[3]^3 b[7]}{1547} + \\
& \frac{1342}{663} b[2] \times b[3] \times b[4] \times b[7] + \frac{10315 b[2]^2 b[5] \cdot b[7]}{7956} + \frac{1225 b[4] \cdot b[5] \cdot b[7]}{1326} + \frac{2606 b[3] \cdot b[6] \cdot b[7]}{2431} + \frac{35}{39} b[2] b[7]^2 \\
b[17] \rightarrow & -\frac{113551 b[2]^7 b[3]}{282744} - \frac{74711 b[2]^4 b[3]^3}{107712} - \frac{633 b[2] b[3]^5}{10472} - \frac{85673 b[2]^5 b[3] \cdot b[4]}{40392} - \\
& \frac{632767 b[2]^2 b[3]^3 b[4]}{376992} - \frac{2782739 b[2]^3 b[3] b[4]^2}{989604} - \frac{2052 b[3]^3 b[4]^2}{9163} - \frac{7045 b[2] \cdot b[3] b[4]^3}{27489} - \frac{70670095 b[2]^6 b[5]}{154378224} - \\
& \frac{833939815 b[2]^3 b[3]^2 b[5]}{411675264} - \frac{254025 b[3]^4 b[5]}{1905904} - \frac{6223255 b[2]^4 b[4] \cdot b[5]}{3150576} - \frac{405926665 b[2] b[3]^2 b[4] \cdot b[5]}{205837632} - \\
& \frac{83902475 b[2]^2 b[4]^2 b[5]}{77189112} + \frac{340360 b[4]^3 b[5]}{1072071} - \frac{62173625 b[2]^2 b[3] b[5]^2}{51459408} - \frac{69500 b[3] \cdot b[4] b[5]^2}{1072071} - \\
& \frac{352375 b[2] b[5]^3}{34306272} + \frac{339223 b[2]^4 b[3] \cdot b[6]}{376992} + \frac{20873 b[2] b[3]^3 b[6]}{31416} + \frac{359129 b[2]^2 b[3] \cdot b[4] \cdot b[6]}{188496} + \frac{52}{77} b[3] b[4]^2 b[6] + \\
& \frac{11054765 b[2]^3 b[5] \cdot b[6]}{9801792} + \frac{71345 b[3]^2 b[5] \cdot b[6]}{272272} + \frac{11159635 b[2] \cdot b[4] \cdot b[5] \cdot b[6]}{4900896} + \frac{5713 b[2] \cdot b[3] b[6]^2}{2244} + \frac{7825 b[5] b[6]^2}{19448} - \\
& \frac{55189 b[2]^5 b[7]}{282744} + \frac{32959 b[2]^2 b[3]^2 b[7]}{20944} + \frac{8458 b[2]^3 b[4] \cdot b[7]}{35343} + \frac{72}{119} b[3]^2 b[4] \times b[7] + \frac{11191 b[2] b[4]^2 b[7]}{7854} + \\
& \frac{217895 b[2] \cdot b[3] \cdot b[5] \cdot b[7]}{131274} + \frac{63725 b[5]^2 b[7]}{153153} + \frac{60689 b[2]^2 b[6] \cdot b[7]}{13464} + \frac{182}{187} b[4] \times b[6] \times b[7] + \frac{105}{187} b[3] b[7]^2
\end{aligned}$$

$$\begin{aligned}
b[18] \rightarrow & \frac{118\,400\,b[2]^9}{6\,043\,653} - \frac{21\,150\,025\,b[2]^6\,b[3]^2}{32\,232\,816} - \frac{30\,389\,959\,b[2]^3\,b[3]^4}{85\,954\,176} - \frac{b[3]^6}{152} - \frac{734\,336\,b[2]^7\,b[4]}{6\,043\,653} - \frac{89\,009\,513\,b[2]^4\,b[3]^2\,b[4]}{32\,232\,816} - \\
& \frac{23\,047\,649\,b[2]\,b[3]^4\,b[4]}{42\,977\,088} - \frac{5\,464\,960\,b[2]^5\,b[4]^2}{6\,043\,653} - \frac{38\,927\,003\,b[2]^2\,b[3]^2\,b[4]^2}{16\,116\,408} - \frac{310\,144\,b[2]^3\,b[4]^3}{287\,793} - \frac{15}{154}\,b[3]^2\,b[4]^3 + \\
& \frac{37\,120\,b[2]\,b[4]^4}{223\,839} - \frac{35\,955\,775\,b[2]^5\,b[3]\cdot b[5]}{26\,544\,672} - \frac{4\,857\,761\,045\,b[2]^2\,b[3]^3\,b[5]}{3\,610\,075\,392} - \frac{6\,041\,736\,005\,b[2]^3\,b[3]\cdot b[4]\cdot b[5]}{1\,353\,778\,272} - \\
& \frac{52\,960\,195\,b[3]^3\,b[4]\cdot b[5]}{106\,178\,688} - \frac{767\,760\,905\,b[2]\cdot b[3]\,b[4]^2\,b[5]}{676\,889\,136} - \frac{58\,481\,825\,b[2]^4\,b[5]^2}{84\,611\,142} - \frac{57\,158\,275\,b[2]\,b[3]^2\,b[5]^2}{64\,465\,632} - \\
& \frac{83\,177\,375\,b[2]^2\,b[4]\,b[5]^2}{84\,611\,142} + \frac{41\,750\,b[4]^2\,b[5]^2}{130\,977} - \frac{57\,875\,b[3]\,b[5]^3}{2\,528\,064} - \frac{3\,509\,416\,b[2]^6\,b[6]}{3\,741\,309} + \frac{104\,829\,931\,b[2]^3\,b[3]^2\,b[6]}{79\,814\,592} + \\
& \frac{541\,b[3]^4\,b[6]}{8892} - \frac{130\,616\,b[2]^4\,b[4]\cdot b[6]}{73\,359} + \frac{35\,064\,701\,b[2]\,b[3]^2\,b[4]\cdot b[6]}{39\,907\,296} + \frac{614\,936\,b[2]^2\,b[4]^2\,b[6]}{340\,119} + \frac{1160\,b[4]^3\,b[6]}{2717} + \\
& \frac{5\,754\,997\,265\,b[2]^2\,b[3]\cdot b[5]\cdot b[6]}{3\,352\,212\,864} + \frac{86\,625\,895\,b[3]\cdot b[4]\cdot b[5]\cdot b[6]}{98\,594\,496} + \frac{207\,368\,825\,b[2]\,b[5]^2\,b[6]}{209\,513\,304} + \frac{297\,556\,b[2]^3\,b[6]^2}{65\,637} + \\
& \frac{2233\,b[3]^2\,b[6]^2}{4446} + \frac{2\,834\,636\,b[2]\cdot b[4]\,b[6]^2}{1\,247\,103} + \frac{307\,b[6]^3}{2223} + \frac{581\,395\,b[2]^4\,b[3]\cdot b[7]}{4\,604\,688} + \frac{227\,213\,b[2]\,b[3]^3\,b[7]}{341\,088} + \\
& \frac{35\,045\,b[2]^2\,b[3]\cdot b[4]\cdot b[7]}{30\,294} + \frac{735}{836}\,b[3]\,b[4]^2\,b[7] + \frac{13\,308\,385\,b[2]^3\,b[5]\cdot b[7]}{24\,174\,612} + \frac{1\,806\,565\,b[3]^2\,b[5]\cdot b[7]}{3\,792\,096} + \\
& \frac{7\,149\,625\,b[2]\cdot b[4]\cdot b[5]\cdot b[7]}{3\,453\,516} + \frac{96\,360\,985\,b[2]\cdot b[3]\cdot b[6]\cdot b[7]}{19\,953\,648} + \frac{3\,046\,055\,b[5]\cdot b[6]\cdot b[7]}{3\,521\,232} + \frac{20\,153\,b[2]^2\,b[7]^2}{11\,286} + \frac{105}{209}\,b[4]\,b[7]^2 \\
b[19] \rightarrow & -\frac{20\,269\,b[2]^8\,b[3]}{1\,953\,504} - \frac{463\,154\,569\,b[2]^5\,b[3]^3}{744\,936\,192} - \frac{1\,025\,097\,b[2]^2\,b[3]^5}{8\,712\,704} - \frac{2\,597\,537\,b[2]^6\,b[3]\cdot b[4]}{3\,627\,936} - \frac{11\,962\,967\,b[2]^3\,b[3]^3\,b[4]}{5\,819\,814} - \\
& \frac{8811\,b[3]^5\,b[4]}{117\,572} - \frac{171\,001\,205\,b[2]^4\,b[3]\,b[4]^2}{69\,837\,768} - \frac{64\,270\,649\,b[2]\,b[3]^3\,b[4]^2}{62\,078\,016} - \frac{38\,025\,109\,b[2]^2\,b[3]\,b[4]^3}{23\,279\,256} + \frac{205\,b[3]\,b[4]^4}{1938} - \\
& \frac{13\,172\,905\,b[2]^7\,b[5]}{98\,594\,496} - \frac{37\,092\,505\,b[2]^4\,b[3]^2\,b[5]}{20\,224\,512} - \frac{160\,326\,765\,b[2]\,b[3]^4\,b[5]}{331\,082\,752} - \frac{135\,569\,215\,b[2]^5\,b[4]\cdot b[5]}{98\,594\,496} - \\
& \frac{30\,795\,055\,b[2]^2\,b[3]^2\,b[4]\cdot b[5]}{7\,255\,872} - \frac{83\,015\,b[2]^3\,b[4]^2\,b[5]}{33\,858} - \frac{167\,871\,545\,b[3]^2\,b[4]^2\,b[5]}{372\,468\,096} + \frac{2\,243\,125\,b[2]\,b[4]^3\,b[5]}{8\,216\,208} - \\
& \frac{86\,140\,325\,b[2]^3\,b[3]\,b[5]^2}{50\,791\,104} - \frac{898\,425\,b[3]^3\,b[5]^2}{3\,762\,304} - \frac{112\,218\,875\,b[2]\cdot b[3]\cdot b[4]\,b[5]^2}{93\,117\,024} - \frac{638\,125\,b[2]^2\,b[5]^3}{2\,434\,432} + \\
& \frac{194\,375\,b[4]\,b[5]^3}{1\,217\,216} - \frac{4\,427\,889\,943\,b[2]^5\,b[3]\cdot b[6]}{2\,607\,276\,672} + \frac{610\,380\,217\,b[2]^2\,b[3]^3\,b[6]}{869\,092\,224} - \frac{2\,938\,648\,b[2]^3\,b[3]\cdot b[4]\cdot b[6]}{1\,198\,197} + \\
& \frac{100\,189\,b[3]^3\,b[4]\cdot b[6]}{2\,263\,261} + \frac{432\,537\,529\,b[2]\cdot b[3]\,b[4]^2\,b[6]}{217\,273\,056} - \frac{16\,110\,635\,b[2]^4\,b[5]\cdot b[6]}{27\,885\,312} + \frac{4\,352\,905\,b[2]\,b[3]^2\,b[5]\cdot b[6]}{7\,524\,608} + \\
& \frac{89\,757\,145\,b[2]^2\,b[4]\cdot b[5]\cdot b[6]}{38\,342\,304} + \frac{1\,765\,375\,b[4]^2\,b[5]\cdot b[6]}{1\,966\,272} + \frac{7\,559\,925\,b[3]\,b[5]^2\,b[6]}{20\,692\,672} + \frac{824\,065\,279\,b[2]^2\,b[3]\,b[6]^2}{124\,156\,032} + \\
& \frac{286\,927\,b[3]\cdot b[4]\,b[6]^2}{323\,323} + \frac{9\,616\,195\,b[2]\cdot b[5]\,b[6]^2}{4\,868\,864} - \frac{1\,932\,055\,b[2]^6\,b[7]}{2\,347\,488} + \frac{868\,107\,b[2]^3\,b[3]^2\,b[7]}{1\,478\,048} + \\
& \frac{3429\,b[3]^4\,b[7]}{33\,592} - \frac{2\,166\,161\,b[2]^4\,b[4]\cdot b[7]}{1\,173\,744} + \frac{698\,875\,b[2]\,b[3]^2\,b[4]\cdot b[7]}{739\,024} + \frac{233\,917\,b[2]^2\,b[4]^2\,b[7]}{195\,624} + \frac{1435\,b[4]^3\,b[7]}{2964} + \\
& \frac{527\,455\,b[2]^2\,b[3]\cdot b[5]\cdot b[7]}{453\,492} + \frac{516\,355\,b[3]\cdot b[4]\cdot b[5]\cdot b[7]}{415\,701} + \frac{87\,925\,b[2]\,b[5]^2\,b[7]}{97\,812} + \frac{18\,528\,157\,b[2]^3\,b[6]\cdot b[7]}{2\,738\,736} + \\
& \frac{766\,797\,b[3]^2\,b[6]\cdot b[7]}{646\,646} + \frac{277\,561\,b[2]\cdot b[4]\cdot b[6]\cdot b[7]}{65\,208} + \frac{213}{494}\,b[6]^2\,b[7] + \frac{6027\,b[2]\cdot b[3]\,b[7]^2}{2717} + \frac{35}{78}\,b[5]\,b[7]^2
\end{aligned}$$

$$\begin{aligned}
b[20] \rightarrow & \frac{164\,758\,432\,b[2]^{10}}{1\,568\,439\,873} - \frac{45\,898\,597\,b[2]^7\,b[3]^2}{448\,125\,678} - \frac{3\,244\,860\,053\,b[2]^4\,b[3]^4}{8\,365\,012\,656} - \frac{411\,309\,b[2]\,b[3]^6}{18\,106\,088} + \frac{2\,194\,195\,616\,b[2]^8\,b[4]}{4\,705\,319\,619} - \\
& \frac{1\,411\,434\,769\,b[2]^5\,b[3]^2\,b[4]}{1\,045\,626\,582} - \frac{3\,860\,253\,091\,b[2]^2\,b[3]^4\,b[4]}{4\,182\,506\,328} + \frac{91\,065\,280\,b[2]^6\,b[4]^2}{1\,568\,439\,873} - \frac{4\,437\,728\,849\,b[2]^3\,b[3]^2\,b[4]^2}{1\,568\,439\,873} - \\
& \frac{203\,655\,b[3]^4\,b[4]^2}{1\,079\,078} - \frac{841\,464\,128\,b[2]^4\,b[4]^3}{672\,188\,517} - \frac{79\,365\,796\,b[2]\,b[3]^2\,b[4]^3}{82\,549\,467} - \frac{453\,544\,544\,b[2]^2\,b[4]^4}{1\,568\,439\,873} + \frac{96\,b[4]^5}{1105} - \\
& \frac{39\,388\,402\,615\,b[2]^6\,b[3]\cdot b[5]}{75\,285\,113\,904} - \frac{22\,944\,488\,485\,b[2]^3\,b[3]^3\,b[5]}{15\,443\,100\,288} - \frac{990\,585\,b[3]^5\,b[5]}{13\,341\,328} - \frac{7\,137\,177\,505\,b[2]^4\,b[3]\cdot b[4]\cdot b[5]}{1\,930\,387\,536} - \\
& \frac{200\,027\,104\,015\,b[2]\,b[3]^3\,b[4]\cdot b[5]}{100\,380\,151\,872} - \frac{52\,979\,609\,495\,b[2]^2\,b[3]\,b[4]^2\,b[5]}{12\,547\,518\,984} + \frac{6\,628\,225\,b[3]\,b[4]^3\,b[5]}{45\,026\,982} - \frac{77\,506\,538\,275\,b[2]^5\,b[5]^2}{150\,570\,227\,808} - \\
& \frac{55\,012\,853\,575\,b[2]^2\,b[3]^2\,b[5]^2}{30\,886\,200\,576} - \frac{17\,077\,427\,275\,b[2]^3\,b[4]\,b[5]^2}{8\,857\,072\,224} - \frac{5\,197\,929\,575\,b[3]^2\,b[4]\,b[5]^2}{10\,566\,331\,776} + \frac{177\,663\,625\,b[2]\,b[4]^2\,b[5]^2}{1\,320\,791\,472} - \\
& \frac{2\,608\,810\,375\,b[2]\cdot b[3]\,b[5]^3}{7\,170\,010\,848} + \frac{540\,625\,b[5]^4}{14\,798\,784} - \frac{120\,311\,680\,b[2]^7\,b[6]}{74\,687\,613} - \frac{766\,497\,131\,b[2]^4\,b[3]^2\,b[6]}{597\,500\,904} + \\
& \frac{2\,452\,711\,b[2]\,b[3]^4\,b[6]}{18\,106\,088} - \frac{366\,756\,800\,b[2]^5\,b[4]\cdot b[6]}{74\,687\,613} - \frac{14\,425\,121\,b[2]^2\,b[3]^2\,b[4]\cdot b[6]}{9\,053\,044} - \frac{105\,772\,928\,b[2]^3\,b[4]^2\,b[6]}{224\,062\,839} + \\
& \frac{1\,429\,814\,b[3]^2\,b[4]^2\,b[6]}{3\,930\,927} + \frac{336\,517\,184\,b[2]\,b[4]^3\,b[6]}{224\,062\,839} - \frac{1\,729\,539\,095\,b[2]^3\,b[3]\cdot b[5]\cdot b[6]}{1\,593\,335\,744} - \frac{50\,905\,b[3]^3\,b[5]\cdot b[6]}{1\,905\,904} + \\
& \frac{5\,378\,556\,325\,b[2]\cdot b[3]\cdot b[4]\cdot b[5]\cdot b[6]}{2\,390\,003\,616} + \frac{264\,636\,175\,b[2]^2\,b[5]^2\,b[6]}{245\,128\,576} + \frac{173\,144\,525\,b[4]\,b[5]^2\,b[6]}{251\,579\,328} + \frac{1\,025\,210\,b[2]^4\,b[6]^2}{187\,187} + \\
& \frac{3\,846\,083\,b[2]\,b[3]^2\,b[6]^2}{1\,293\,292} + \frac{58\,805\,884\,b[2]^2\,b[4]\,b[6]^2}{10\,669\,659} + \frac{43\,682\,b[4]^2\,b[6]^2}{80\,223} + \frac{101\,755\,b[3]\cdot b[5]\,b[6]^2}{136\,136} + \frac{43\,926\,b[2]\,b[6]^3}{46\,189} - \\
& \frac{61\,350\,859\,b[2]^5\,b[3]\cdot b[7]}{34\,471\,206} + \frac{6\,674\,799\,b[2]^2\,b[3]^3\,b[7]}{14\,226\,212} - \frac{4\,969\,892\,b[2]^3\,b[3]\cdot b[4]\cdot b[7]}{1\,684\,683} + \frac{35\,820\,b[3]^3\,b[4]\cdot b[7]}{187\,187} + \\
& \frac{143\,473\,678\,b[2]\cdot b[3]\,b[4]^2\,b[7]}{74\,687\,613} - \frac{264\,292\,855\,b[2]^4\,b[5]\cdot b[7]}{325\,909\,584} + \frac{4\,995\,345\,125\,b[2]\,b[3]^2\,b[5]\cdot b[7]}{7\,170\,010\,848} + \frac{92\,721\,605\,b[2]^2\,b[4]\cdot b[5]\cdot b[7]}{61\,108\,047} + \\
& \frac{72\,175\,b[4]^2\,b[5]\cdot b[7]}{68\,796} + \frac{1\,471\,475\,b[3]\,b[5]^2\,b[7]}{2\,774\,772} + \frac{240\,502\,211\,b[2]^2\,b[3]\cdot b[6]\cdot b[7]}{21\,339\,318} + \frac{386\,822\,b[3]\cdot b[4]\cdot b[6]\cdot b[7]}{187\,187} + \\
& \frac{8\,235\,325\,b[2]\cdot b[5]\cdot b[6]\cdot b[7]}{2\,217\,072} + \frac{2\,391\,892\,b[2]^3\,b[7]^2}{969\,969} + \frac{124\,704\,b[3]^2\,b[7]^2}{187\,187} + \frac{268\,240\,b[2]\cdot b[4]\,b[7]^2}{138\,567} + \frac{1065\,b[6]\,b[7]^2}{2431}
\end{aligned}$$

$$\begin{aligned}
b[21] \rightarrow & \frac{3474431369b[2]^9b[3]}{9964206252} - \frac{1332068071b[2]^6b[3]^3}{8857072224} - \frac{2938073713b[2]^3b[3]^5}{17714144448} - \frac{b[3]^7}{532} + \\
& \frac{1422899771b[2]^7b[3]b[4]}{1107134028} - \frac{8921737463b[2]^4b[3]^3b[4]}{6642804168} - \frac{3640585b[2]b[3]^5b[4]}{15484392} - \frac{13886090b[2]^5b[3]b[4]^2}{63873117} - \\
& \frac{1061889091b[2]^2b[3]^3b[4]^2}{603891288} - \frac{6864161377b[2]^3b[3]b[4]^3}{2491051563} - \frac{1404b[3]^3b[4]^3}{5929} - \frac{906812b[2]b[3]b[4]^4}{4393389} + \\
& \frac{1672131775b[2]^8b[5]}{7641722088} - \frac{341028768055b[2]^5b[3]^2b[5]}{387180585792} - \frac{1336455324565b[2]^2b[3]^4b[5]}{1806842733696} - \frac{5734321445b[2]^6b[4]b[5]}{53492054616} - \\
& \frac{1005810403795b[2]^3b[3]^2b[4]b[5]}{225855341712} - \frac{87153265555b[3]^4b[4]b[5]}{225855341712} - \frac{3718830845b[2]^4b[4]^2b[5]}{1485890406} - \\
& \frac{38887121225b[2]b[3]^2b[4]^2b[5]}{13827878064} - \frac{15170700785b[2]^2b[4]^3b[5]}{13373013654} + \frac{2500640b[4]^4b[5]}{9172163} - \frac{83526840125b[2]^4b[3]b[5]^2}{59785237512} - \\
& \frac{102115428925b[2]b[3]^3b[5]^2}{112927670856} - \frac{1857987954575b[2]^2b[3]b[4]b[5]^2}{508174518852} + \frac{209450050b[3]b[4]^2b[5]^2}{6686506827} - \\
& \frac{52862102375b[2]^3b[5]^3}{106984109232} - \frac{1787297375b[3]^2b[5]^3}{11582325216} + \frac{747744625b[2]b[4]b[5]^3}{53492054616} - \frac{2769044783b[2]^6b[3]b[6]}{632648016} - \\
& \frac{1594497875b[2]^3b[3]^3b[6]}{2530592064} + \frac{2b[3]^5b[6]}{2223} - \frac{11406961b[2]^4b[3]b[4]b[6]}{1065064} - \frac{317884507b[2]b[3]^3b[4]b[6]}{421765344} + \\
& \frac{20863177b[2]^2b[3]b[4]^2b[6]}{79081002} + \frac{142997b[3]b[4]^3b[6]}{209209} - \frac{27508728055b[2]^5b[5]b[6]}{10188962784} - \frac{340150106255b[2]^2b[3]^2b[5]b[6]}{258120390528} - \\
& \frac{379621835b[2]^3b[4]b[5]b[6]}{299675376} + \frac{33515547595b[3]^2b[4]b[5]b[6]}{129060195264} + \frac{1272842885b[2]b[4]^2b[5]b[6]}{424540116} + \\
& \frac{29463485575b[2]b[3]b[5]^2b[6]}{32265048816} + \frac{1597375b[5]^3b[6]}{7916832} + \frac{5142635b[2]^3b[3]b[6]^2}{453024} + \frac{889b[3]^3b[6]^2}{2223} + \frac{1645129b[2]b[3]b[4]b[6]^2}{331056} + \\
& \frac{670505125b[2]^2b[5]b[6]^2}{138625344} + \frac{407698975b[4]b[5]b[6]^2}{485188704} + \frac{788b[3]b[6]^3}{2223} - \frac{29545379b[2]^7b[7]}{24972948} - \frac{110901325b[2]^4b[3]^2b[7]}{67783716} + \\
& \frac{485081b[2]b[3]^4b[7]}{3195192} - \frac{148556645b[2]^5b[4]b[7]}{37459422} - \frac{11758198b[2]^2b[3]^2b[4]b[7]}{6243237} - \frac{19824439b[2]^3b[4]^2b[7]}{18729711} + \\
& \frac{67770b[3]^2b[4]^2b[7]}{112651} + \frac{45646b[2]b[4]^3b[7]}{33033} - \frac{241312765b[2]^3b[3]b[5]b[7]}{157134978} + \frac{274195135b[3]^3b[5]b[7]}{2481926832} + \\
& \frac{9233429675b[2]b[3]b[4]b[5]b[7]}{4033131102} + \frac{1402963175b[2]^2b[5]^2b[7]}{1910430522} + \frac{784953275b[4]b[5]^2b[7]}{955215261} + \frac{696001b[2]^4b[6]b[7]}{92664} + \\
& \frac{537831467b[2]b[3]^2b[6]b[7]}{90378288} + \frac{390710b[2]^2b[4]b[6]b[7]}{42471} + \frac{6107b[4]^2b[6]b[7]}{5082} + \frac{4068282545b[3]b[5]b[6]b[7]}{2304646344} + \\
& \frac{916739b[2]b[6]^2b[7]}{339768} + \frac{2921351b[2]^2b[3]b[7]^2}{627627} + \frac{18498b[3]b[4]b[7]^2}{16093} + \frac{1512100b[2]b[5]b[7]^2}{891891} + \frac{53b[7]^3}{363}
\end{aligned}$$

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In[77]:= Print["differential equation for the nilpotent cone, n=4"]
DR4 = Factor[D[R[4] /. x[1] -> 0, {x[2], 5}] /. x[2] -> 0 /. inic];
DR4 = Factor[D[DR4, {x[3], 5}] /. x[3] -> 0 /. inic];
DR4 = Collect[D[DR4, {x[4], 5}] /. x[4] -> 0 /. inic, g[s], Factor];
Collect[DR4/144000, g[s]]
Print["the sequence of relations, d[j] depends linearly on b[j-3]"]
Do[d[j] = Factor[SeriesCoefficient[DR4 /. g -> gg, {s, 0, j}]], {j, 0, md}]
Do[Print["d", j, "=", d[j]], {j, 0, 16}]
Print["polynomial pattern 144000(j-12)(j-11)(j-10)(j-3)(j-2)(7j-43)"]
Table[Coefficient[d[j], b[j-3]] == 144000 (j-12)(j-11)(j-10)(j-3)(j-2)(7j-43), {j, 13, md}]
Print["expressions for b[j] j>9"]
p4 = {}; Do[p4 = Union[p4, Solve[0 == d[i] /. p4, b[i-3]]][[1]], {i, 13, md}];
Column[Expand[p4]]

```

differential equation for the nilpotent cone, n=4

Out[81]=

$$\begin{aligned}
& 5040 g[s]^4 g'[s]^4 g''[s] - 420 g[s]^5 g'[s]^2 (9 g''[s]^2 + 4 g'[s]^2 g^{(3)}[0] + 8 g'[s] g^{(3)}[s]) + \\
& 30 g[s]^6 (9 g''[s]^3 + 36 g'[s]^2 g''[s] g^{(3)}[0] + 60 g'[s] g''[s] g^{(3)}[s] - 2 g'[s]^3 g^{(4)}[0] + 30 g'[s]^2 g^{(4)}[s]) - \\
& 5 g[s]^7 (12 g''[s]^2 g^{(3)}[0] - 20 g'[s]^2 g^{(3)}[0]^2 - 8 g'[s] g^{(3)}[0] g^{(3)}[s] + \\
& 16 g^{(3)}[s]^2 - 3 g'[s] g''[s] g^{(4)}[0] + 33 g''[s] g^{(4)}[s] - 12 g'[s]^2 g^{(5)}[0] + 24 g'[s] g^{(5)}[s]) - \\
& g[s]^8 (20 g''[s] g^{(3)}[0]^2 + 55 g'[s] g^{(3)}[0] g^{(4)}[0] - 5 g^{(3)}[s] g^{(4)}[0] + 20 g^{(3)}[0] g^{(4)}[s] + 12 g''[s] g^{(5)}[0] - \\
& 13 g'[s] g^{(6)}[0] - 7 g^{(6)}[s]) + g[s]^9 (10 g^{(3)}[0]^3 - 5 g^{(4)}[0]^2 - 11 g^{(3)}[0] g^{(5)}[0] + g^{(7)}[0])
\end{aligned}$$

the sequence of relations, $d[j]$ depends linearly on $b[j-3]$

$$d[0]=0$$

$$d[1]=0$$

$$d[2]=0$$

$$d[3]=0$$

$$d[4]=0$$

$$d[5]=0$$

$$d[6]=0$$

$$d[7]=0$$

$$d[8]=0$$

$$d[9]=0$$

$$d[10]=0$$

$$d[11]=0$$

$$d[12]=0$$

$$d[13]=$$

$$-27\,648\,000 (128 b[2]^5 + 160 b[2]^3 b[4] - 240 b[2] b[4]^2 - 780 b[2]^2 b[6] + 240 b[4] \times b[6] + 600 b[2] \times b[8] - 165 b[10])$$

$$d[14]=-69\,120\,000$$

$$\begin{aligned}
& (384 b[2]^4 b[3] + 288 b[2]^2 b[3] \times b[4] - 144 b[3] b[4]^2 + 160 b[2]^3 b[5] - 480 b[2] \times b[4] \times b[5] - 936 b[2] \times b[3] \times b[6] + \\
& 240 b[5] \times b[6] - 1092 b[2]^2 b[7] + 336 b[4] \times b[7] + 360 b[3] \times b[8] + 1080 b[2] \times b[9] - 363 b[11])
\end{aligned}$$

$$d[15]=-4\,608\,000$$

$$\begin{aligned}
& (5888 b[2]^6 + 17\,280 b[2]^3 b[3]^2 + 20\,160 b[2]^4 b[4] + 6480 b[2] b[3]^2 b[4] - 10\,080 b[2]^2 b[4]^2 - 2640 b[4]^3 + \\
& 10\,800 b[2]^2 b[3] \times b[5] - 10\,800 b[3] \times b[4] \times b[5] - 9000 b[2] b[5]^2 - 38\,280 b[2]^3 b[6] - 10\,530 b[3]^2 b[6] - \\
& 18\,360 b[2] \times b[4] \times b[6] + 6525 b[6]^2 - 49\,140 b[2] \times b[3] \times b[7] + 12\,600 b[5] \times b[7] - \\
& 6480 b[2]^2 b[8] + 19\,080 b[4] \times b[8] + 24\,300 b[3] \times b[9] + 37\,260 b[2] \times b[10] - 18\,135 b[12])
\end{aligned}$$

$$d[16]=-6\,912\,000$$

$$\begin{aligned}
& (33\,024 b[2]^5 b[3] + 17\,280 b[2]^2 b[3]^3 + 77\,760 b[2]^3 b[3] \times b[4] + 2160 b[3]^3 b[4] - 15\,840 b[2] \times b[3] b[4]^2 + \\
& 27\,200 b[2]^4 b[5] + 10\,800 b[2] b[3]^2 b[5] - 38\,400 b[2]^2 b[4] \times b[5] - 10\,800 b[4]^2 b[5] - 9000 b[3] b[5]^2 - \\
& 100\,800 b[2]^2 b[3] \times b[6] - 22\,680 b[3] \times b[4] \times b[6] - 15\,000 b[2] \times b[5] \times b[6] - 94\,920 b[2]^3 b[7] - \\
& 24\,570 b[3]^2 b[7] - 26\,040 b[2] \times b[4] \times b[7] + 22\,050 b[6] \times b[7] - 23\,760 b[2] \times b[3] \times b[8] + 25\,800 b[5] \times b[8] + \\
& 46\,080 b[2]^2 b[9] + 37\,080 b[4] \times b[9] + 40\,230 b[3] \times b[10] + 37\,620 b[2] \times b[11] - 31\,395 b[13])
\end{aligned}$$

polynomial pattern 144000(j-12)(j-11)(j-10)(j-3)(j-2)(7j-43)

Out[86]=

{True, True, True, True, True, True, True, True}

expressions for b[j] j>9

Out[89]=

$$\begin{aligned}
b[10] &\rightarrow \frac{128 b[2]^5}{165} + \frac{32}{33} b[2]^3 b[4] - \frac{16}{11} b[2] b[4]^2 - \frac{52}{11} b[2]^2 b[6] + \frac{16}{11} b[4] \times b[6] + \frac{40}{11} b[2] \times b[8] \\
b[11] &\rightarrow \frac{128}{121} b[2]^4 b[3] + \frac{96}{121} b[2]^2 b[3] \times b[4] - \frac{48}{121} b[3] b[4]^2 + \frac{160}{363} b[2]^3 b[5] - \frac{160}{121} b[2] \times b[4] \times b[5] - \\
&\quad \frac{312}{121} b[2] \times b[3] \times b[6] + \frac{80}{121} b[5] \times b[6] - \frac{364}{121} b[2]^2 b[7] + \frac{112}{121} b[4] \times b[7] + \frac{120}{121} b[3] \times b[8] + \frac{360}{121} b[2] \times b[9] \\
b[12] &\rightarrow \frac{5888 b[2]^6}{3069} + \frac{384}{403} b[2]^3 b[3]^2 + \frac{13760 b[2]^4 b[4]}{4433} + \frac{144}{403} b[2] b[3]^2 b[4] - \frac{15712 b[2]^2 b[4]^2}{4433} - \frac{176 b[4]^3}{1209} + \\
&\quad \frac{240}{403} b[2]^2 b[3] \times b[5] - \frac{240}{403} b[3] \times b[4] \times b[5] - \frac{200}{403} b[2] b[5]^2 - \frac{157240 b[2]^3 b[6]}{13299} - \frac{18}{31} b[3]^2 b[6] + \frac{8760 b[2] \times b[4] \times b[6]}{4433} + \\
&\quad \frac{145 b[6]^2}{403} - \frac{84}{31} b[2] \times b[3] \times b[7] + \frac{280}{403} b[5] \times b[7] + \frac{31536 b[2]^2 b[8]}{4433} + \frac{424}{403} b[4] \times b[8] + \frac{540}{403} b[3] \times b[9] \\
b[13] &\rightarrow \frac{76288 b[2]^5 b[3]}{23023} + \frac{1152 b[2]^2 b[3]^3}{2093} + \frac{15360 b[2]^3 b[3] \times b[4]}{3289} + \frac{144 b[3]^3 b[4]}{2093} - \\
&\quad \frac{5952 b[2] \times b[3] b[4]^2}{2093} + \frac{13760 b[2]^4 b[5]}{9867} + \frac{720 b[2] b[3]^2 b[5]}{2093} - \frac{64640 b[2]^2 b[4] \times b[5]}{23023} - \frac{720 b[4]^2 b[5]}{2093} - \frac{600 b[3] b[5]^2}{2093} - \\
&\quad \frac{284520 b[2]^2 b[3] \times b[6]}{23023} + \frac{26280 b[3] \times b[4] \times b[6]}{23023} + \frac{7240 b[2] \times b[5] \times b[6]}{23023} - \frac{21800 b[2]^3 b[7]}{3289} - \frac{18}{23} b[3]^2 b[7] + \\
&\quad \frac{40}{143} b[2] \times b[4] \times b[7] + \frac{210}{299} b[6] \times b[7] + \frac{10656 b[2] \times b[3] \times b[8]}{2093} + \frac{1720 b[5] \times b[8]}{2093} + \frac{115872 b[2]^2 b[9]}{23023} + \frac{2472 b[4] \times b[9]}{2093} \\
b[14] &\rightarrow \frac{176128 b[2]^7}{61845} + \frac{65588352 b[2]^4 b[3]^2}{20635615} + \frac{864 b[2] b[3]^4}{4655} + \frac{10333184 b[2]^5 b[4]}{1768767} + \frac{1431648 b[2]^2 b[3]^2 b[4]}{421135} - \\
&\quad \frac{211544576 b[2]^3 b[4]^2}{61906845} - \frac{27504 b[3]^2 b[4]^2}{51205} - \frac{44093824 b[2] b[4]^3}{20635615} + \frac{12215648 b[2]^3 b[3] \times b[5]}{4127123} + \\
&\quad \frac{72}{931} b[3]^3 b[5] - \frac{12318816 b[2] \times b[3] \times b[4] \times b[5]}{4127123} - \frac{373440 b[2]^2 b[5]^2}{375193} - \frac{40}{133} b[4] b[5]^2 - \frac{929664 b[2]^4 b[6]}{53599} - \\
&\quad \frac{1090872 b[2] b[3]^2 b[6]}{226765} - \frac{11356288 b[2]^2 b[4] \times b[6]}{2947945} + \frac{10352 b[4]^2 b[6]}{7315} + \frac{2172 b[3] \times b[5] \times b[6]}{10241} + \frac{384 b[2] b[6]^2}{2015} - \\
&\quad \frac{316188 b[2]^2 b[3] \times b[7]}{32395} + \frac{276 b[3] \times b[4] \times b[7]}{1463} + \frac{1228 b[2] \times b[5] \times b[7]}{7657} + \frac{13 b[7]^2}{38} + \frac{368896 b[2]^3 b[8]}{38285} + \\
&\quad \frac{33912 b[3]^2 b[8]}{51205} + \frac{14709696 b[2] \times b[4] \times b[8]}{2947945} + \frac{72}{95} b[6] \times b[8] + \frac{102838968 b[2] \times b[3] \times b[9]}{20635615} + \frac{876}{931} b[5] \times b[9] \\
b[15] &\rightarrow \frac{28752320 b[2]^6 b[3]}{4556783} + \frac{11347536 b[2]^3 b[3]^3}{5385289} + \frac{81 b[3]^5}{2905} + \frac{676755776 b[2]^4 b[3] \times b[4]}{59238179} + \frac{7577658 b[2] b[3]^3 b[4]}{5385289} - \\
&\quad \frac{41588304 b[2]^2 b[3] b[4]^2}{8462597} - \frac{1914072 b[3] b[4]^3}{2575573} + \frac{1237480 b[2]^5 b[5]}{440979} + \frac{14878170 b[2]^2 b[3]^2 b[5]}{5385289} - \\
&\quad \frac{537140 b[2]^3 b[4] \times b[5]}{440979} - \frac{191925 b[3]^2 b[4] \times b[5]}{234143} - \frac{52760 b[2] b[4]^2 b[5]}{13363} - \frac{976725 b[2] \times b[3] b[5]^2}{769327} - \frac{625 b[5]^3}{6972} - \\
&\quad \frac{1636381521 b[2]^3 b[3] \times b[6]}{59238179} - \frac{29277 b[3]^3 b[6]}{36022} - \frac{64196061 b[2] \times b[3] \times b[4] \times b[6]}{59238179} - \frac{135985 b[2]^2 b[5] \times b[6]}{26726} + \\
&\quad \frac{3065 b[4] \times b[5] \times b[6]}{1826} + \frac{4788 b[3] b[6]^2}{33449} - \frac{182999 b[2]^4 b[7]}{20999} - \frac{1265085 b[2] b[3]^2 b[7]}{236716} - \frac{4050}{913} b[2]^2 b[4] \times b[7] + \\
&\quad \frac{665}{913} b[4]^2 b[7] + \frac{32235 b[3] \times b[5] \times b[7]}{267592} + \frac{492 b[2] \times b[6] \times b[7]}{1909} + \frac{712490148 b[2]^2 b[3] \times b[8]}{59238179} + \frac{4103928 b[3] \times b[4] \times b[8]}{2575573} + \\
&\quad \frac{591510 b[2] \times b[5] \times b[8]}{146993} + \frac{59}{83} b[7] \times b[8] + \frac{127360 b[2]^3 b[9]}{20999} + \frac{1117989 b[3]^2 b[9]}{936572} + \frac{654552 b[2] \times b[4] \times b[9]}{146993} + \frac{69}{83} b[6] \times b[9]
\end{aligned}$$

$$\begin{aligned}
b[16] \rightarrow & \frac{29\,192\,704\,b[2]^8}{9\,462\,285} + \frac{617\,327\,232\,b[2]^5\,b[3]^2}{88\,665\,115} + \frac{240\,192\,b[2]^2\,b[3]^4}{260\,015} + \frac{16\,020\,352\,b[2]^6\,b[4]}{1\,892\,457} + \frac{12\,367\,651\,496\,b[2]^3\,b[3]^2\,b[4]}{1\,152\,646\,495} + \\
& \frac{99}{391}\,b[3]^4\,b[4] - \frac{2\,392\,448\,b[2]^4\,b[4]^2}{41\,003\,235} - \frac{1\,992\,731\,016\,b[2]\,b[3]^2\,b[4]^2}{1\,152\,646\,495} - \frac{230\,688\,736\,b[2]^2\,b[4]^3}{41\,003\,235} - \frac{12\,712\,b[4]^4}{61\,659} + \\
& \frac{351\,443\,284\,b[2]^4\,b[3]\cdot b[5]}{53\,199\,069} + \frac{68\,181\,b[2]\,b[3]^3\,b[5]}{52\,003} - \frac{80\,335\,634\,b[2]^2\,b[3]\cdot b[4]\cdot b[5]}{32\,932\,757} - \frac{21\,932\,250\,b[3]\,b[4]^2\,b[5]}{12\,133\,121} - \\
& \frac{282\,130\,b[2]^3\,b[5]^2}{771\,001} - \frac{2325\,b[3]^2\,b[5]^2}{5474} - \frac{83\,453\,620\,b[2]\cdot b[4]\cdot b[5]^2}{30\,069\,039} - \frac{125\,010\,128\,b[2]^5\,b[6]}{6\,939\,009} - \frac{2\,960\,115\,489\,b[2]^2\,b[3]^2\,b[6]}{177\,330\,230} - \\
& \frac{223\,481\,744\,b[2]^3\,b[4]\cdot b[6]}{14\,549\,535} - \frac{34\,563\,b[3]^2\,b[4]\cdot b[6]}{169\,694} + \frac{30\,392\,368\,b[2]\,b[4]^2\,b[6]}{21\,477\,885} - \frac{1\,875\,019\,b[2]\cdot b[3]\cdot b[5]\cdot b[6]}{572\,033} + \\
& \frac{325}{748}\,b[5]^2\,b[6] - \frac{376\,327\,b[2]^2\,b[6]^2}{86\,955} + \frac{327\,785\,b[4]\,b[6]^2}{226\,083} - \frac{1\,366\,045\,313\,b[2]^3\,b[3]\cdot b[7]}{75\,998\,670} - \frac{3489\,b[3]^3\,b[7]}{3128} - \\
& \frac{992\,505\,b[2]\cdot b[3]\cdot b[4]\cdot b[7]}{220\,286} - \frac{4\,982\,651\,b[2]^2\,b[5]\cdot b[7]}{1\,321\,716} + \frac{1\,057\,085\,b[4]\cdot b[5]\cdot b[7]}{904\,332} + \frac{82}{391}\,b[3] \times b[6] \times b[7] + \frac{147\,b[2]\,b[7]^2}{1292} + \\
& \frac{3\,520\,288\,b[2]^4\,b[8]}{350\,455} + \frac{11\,982\,276\,b[2]\,b[3]^2\,b[8]}{2\,860\,165} + \frac{52\,329\,712\,b[2]^2\,b[4]\cdot b[8]}{4\,555\,915} + \frac{6056\,b[4]^2\,b[8]}{6851} + \frac{39\,360\,b[3]\cdot b[5]\cdot b[8]}{30\,107} + \\
& \frac{63\,144\,b[2]\cdot b[6]\cdot b[8]}{17\,765} + \frac{6\,b[8]^2}{17} + \frac{49\,990\,734\,b[2]^2\,b[3]\cdot b[9]}{5\,215\,595} + \frac{144\,843\,b[3]\cdot b[4]\cdot b[9]}{64\,883} + \frac{90\,886\,b[2]\cdot b[5]\cdot b[9]}{24\,871} + \frac{103}{136}\,b[7] \times b[9] \\
b[17] \rightarrow & \frac{54\,726\,297\,216\,128\,b[2]^7\,b[3]}{6\,424\,585\,567\,785} + \frac{531\,531\,136\,992\,b[2]^4\,b[3]^3}{101\,977\,548\,695} + \frac{21\,989\,718\,b[2]\,b[3]^5}{91\,016\,555} + \\
& \frac{777\,148\,108\,672\,b[2]^5\,b[3]\cdot b[4]}{38\,936\,882\,229} + \frac{4\,307\,592\,554\,964\,b[2]^2\,b[3]^3\,b[4]}{713\,842\,840\,865} - \frac{3\,576\,106\,416\,992\,b[2]^3\,b[3]\,b[4]^2}{2\,141\,528\,522\,595} - \frac{21\,953\,808\,b[3]^3\,b[4]^2}{277\,436\,005} - \\
& \frac{10\,143\,853\,725\,904\,b[2]\cdot b[3]\,b[4]^3}{2\,141\,528\,522\,595} + \frac{936\,858\,624\,560\,b[2]^6\,b[5]}{202\,881\,649\,509} + \frac{2\,027\,664\,466\,604\,b[2]^3\,b[3]^2\,b[5]}{265\,141\,626\,607} + \frac{8064\,b[3]^4\,b[5]}{31\,331} + \\
& \frac{3\,267\,991\,442\,840\,b[2]^4\,b[4]\cdot b[5]}{879\,153\,814\,539} - \frac{1\,629\,257\,583\,606\,b[2]\,b[3]^2\,b[4]\cdot b[5]}{1\,855\,991\,386\,249} - \frac{2\,693\,407\,989\,200\,b[2]^2\,b[4]^2\,b[5]}{293\,051\,271\,513} - \\
& \frac{667\,574\,560\,b[4]^3\,b[5]}{962\,928\,603} - \frac{130\,485\,129\,090\,b[2]^2\,b[3]\,b[5]^2}{168\,726\,489\,659} - \frac{3\,324\,979\,080\,b[3]\cdot b[4]\,b[5]^2}{2\,032\,849\,273} - \frac{1\,809\,158\,875\,b[2]\,b[5]^3}{2\,316\,610\,842} - \\
& \frac{2\,445\,394\,715\,174\,b[2]^4\,b[3]\cdot b[6]}{61\,186\,529\,217} - \frac{150\,198\,497\,793\,b[2]\,b[3]^3\,b[6]}{31\,036\,645\,255} - \frac{1\,001\,527\,184\,522\,b[2]^2\,b[3]\cdot b[4]\cdot b[6]}{64\,894\,803\,715} + \\
& \frac{339\,289\,648\,b[3]\,b[4]^2\,b[6]}{277\,436\,005} - \frac{13\,406\,549\,318\,065\,b[2]^3\,b[5]\cdot b[6]}{879\,153\,814\,539} - \frac{9\,661\,416\,b[3]^2\,b[5]\cdot b[6]}{10\,683\,871} + \frac{28\,528\,744\,235\,b[2]\cdot b[4]\cdot b[5]\cdot b[6]}{26\,641\,024\,683} - \\
& \frac{617\,949\,128\,b[2]\cdot b[3]\,b[6]^2}{233\,358\,235} + \frac{16\,686\,140\,b[5]\,b[6]^2}{21\,930\,051} - \frac{2\,476\,703\,470\,b[2]^5\,b[7]}{311\,646\,159} - \frac{175\,547\,114\,481\,b[2]^2\,b[3]^2\,b[7]}{11\,997\,358\,670} - \\
& \frac{3\,278\,545\,628\,b[2]^3\,b[4]\cdot b[7]}{311\,646\,159} - \frac{11\,692\,932\,b[3]^2\,b[4]\cdot b[7]}{7\,926\,743} - \frac{4\,887\,370\,b[2]\,b[4]^2\,b[7]}{4\,516\,611} - \frac{1\,272\,479\,859\,b[2]\cdot b[3]\cdot b[5]\cdot b[7]}{322\,458\,652} + \\
& \frac{315\,700\,b[5]^2\,b[7]}{664\,547} - \frac{727\,725\,872\,b[2]^2\,b[6]\cdot b[7]}{103\,882\,053} + \frac{2\,884\,000\,b[4]\cdot b[6]\cdot b[7]}{1\,251\,591} + \frac{3087\,b[3]\,b[7]^2}{31\,331} + \frac{13\,231\,848\,142\,168\,b[2]^3\,b[3]\cdot b[8]}{713\,842\,840\,865} + \\
& \frac{435\,144\,b[3]^3\,b[8]}{1\,723\,205} + \frac{6\,291\,407\,614\,704\,b[2]\cdot b[3]\cdot b[4]\cdot b[8]}{713\,842\,840\,865} + \frac{84\,368\,598\,220\,b[2]^2\,b[5]\cdot b[8]}{8\,880\,341\,561} + \frac{157\,559\,600\,b[4]\cdot b[5]\cdot b[8]}{106\,992\,067} + \\
& \frac{120\,888\,b[3]\cdot b[6]\cdot b[8]}{101\,365} + \frac{4\,799\,050\,b[2]\cdot b[7]\cdot b[8]}{1\,505\,537} + \frac{1\,391\,576\,960\,b[2]^4\,b[9]}{242\,391\,457} + \frac{302\,458\,580\,469\,b[2]\,b[3]^2\,b[9]}{62\,073\,290\,510} + \\
& \frac{2\,096\,426\,736\,b[2]^2\,b[4]\cdot b[9]}{242\,391\,457} + \frac{278\,472\,b[4]^2\,b[9]}{265\,489} + \frac{23\,542\,086\,b[3]\cdot b[5]\cdot b[9]}{12\,626\,393} + \frac{4\,800\,894\,b[2]\cdot b[6]\cdot b[9]}{1\,505\,537} + \frac{1196\,b[8]\cdot b[9]}{1649}
\end{aligned}$$

```
In[93]:= Print["combined relations"]
```

```
diff = Factor[(Table[b[i], {i, 10, 13}]/. p3) - Table[b[i], {i, 10, 13}]/. p4 /. p3];
Solve[diff == {0, 0, 0, 0}, Table[b[i], {i, 5, 7}]]
```

```
combined relations
```

```
Out[95]=
```

$$\left\{ \left\{ b[5] \rightarrow \frac{6}{5}\,b[2] \times b[3], \, b[6] \rightarrow \frac{1}{42}\,(8\,b[2]^3 + 27\,b[3]^2 + 24\,b[2] \times b[4]), \, b[7] \rightarrow \frac{6}{7}\,(b[2]^2\,b[3] + b[3] \times b[4]) \right\} \right\}$$