

BOOK OF ABSTRACTS

Workshop
Transport phenomena in mathematical biology

INSTITUTE OF MATHEMATICS OF POLISH ACADEMY OF SCIENCES IN WARSAW
Banach Center, Warsaw, 22-24 January 2018

ORGANIZERS: TOMASZ DĘBIEC, PIOTR GWIAZDA, PIERRE-EMMANUEL JABIN,
BENOÎT PERTHAME AND AGNIESZKA ŚWIERCZEWSKA-GWIAZDA



Warsaw Center
of Mathematics
and Computer Science

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Schedule

Monday, January 22 MIMUW, Room 2180	Tuesday, January 23 IMPAN, Room 321	Wednesday, January 24 IMPAN, Room 321
8:50-9:20 Registration & coffee 9:20 Opening	8:30-9:00 Coffee	8:30-9:00 Coffee
9:30-10:15 JOSE A. CARRILLO <i>Splitting schemes & segregation in reaction-(cross-)diffusion systems</i>	9:00-9:45 IRENE GAMBA <i>The Cauchy problem for Kolmogorov-Vicsek type equation with Orientational Interactions. Analysis and numerics</i>	9:00-9:45 EWELINA ZATORSKA <i>Incompressible limit of the Navier-Stokes model with a growth term</i>
10:15-11:00 MARCO DI FRANCESCO <i>Deterministic particle approximations for local and nonlocal transport equations</i>	9:45-10:30 KAROLINA KROPIELNICKA <i>Structured population models in a space of measures; From analytical foundations to numerical results</i>	9:45-10:30 BENOÎT PERTHAME <i>Bacterial collective movement: models, patterns, pathways, scales</i>
11:00-11:30 Coffee break	10:30-11:00 Coffee break	10:30-11:00 Coffee break
11:30-12:15 CLAUDE BARDOS <i>Onsager Conjecture, the Kolmogorov 1/3 law and the 1984 Kato Criteria in bounded domains</i>	11:00-11:45 ANETA WRÓBLEWSKA-KAMIŃSKA <i>Hydrodynamic models of collective behavior with damping and nonlocal interactions</i>	11:00-11:45 ANNA MARCINIAK-CZOCHRA <i>Discrete and continuous transitions in models of cell differentiation</i>
12:15-13:00 PIERRE-EMMANUEL JABIN <i>Quantitative estimates for Advective Equation with Degenerate Anelastic Constraint</i>	11:45-12:30 PIOTR B. MUCHA <i>Cucker-Smale equation</i>	11:45-12:30 AGNIESZKA ŚWIERCZEWSKA-GWIAZDA <i>On ill-posedness of Euler system with non-local terms</i>

Monday, January 22 MIMUW, Room 2180	Tuesday, January 23 IMPAN, Room 321	Wednesday, January 24 IMPAN, Room 321
13:00-14:30 Lunch break	12:30-15:00 Lunch break	
<p>14:30-15:15</p> <p>PIERRE GABRIEL</p> <p><i>Coupling arguments for population dynamics</i></p>	<p>15:00-15:45</p> <p>ALEKSANDRA PUCHALSKA</p> <p><i>The graph structure impact on a singular limit of the generalized network transport</i></p>	
<p>15:15-15:45</p> <p>TOMASZ DEBIEC</p> <p><i>Generalized relative entropy inequality for measure-valued solutions of a structured population model</i></p>	<p>15:45-17:15</p> <p>Poster session</p>	
15:45-16:15 Coffee break	19:00 Conference dinner	
<p>16:15-16:45</p> <p>MARIE DOUMIC</p> <p><i>Long-time asymptotics of protein aggregation models</i></p>		

**Onsager Conjecture, the Kolmogorov 1/3 law
and the 1984 Kato Criteria in bounded domains**

Claude Bardos

Laboratoire J.-L. Lions, Paris

Abstract

In this talk I will recall the Onsager conjecture and compare it to the issue of anomalous energy dissipation and to the Kolmogorov 1/3 law. Then I will give a proof of the local conservation of energy under convenient hypothesis in a domain with boundary and show how this implies the absence of anomalous energy dissipation.

Eventually I will give several forms of a basic theorem of Kato in the presence of a Lipschitz solution of the Euler equations. Insisting that in such case the absence of anomalous energy dissipation is equivalent to the persistence of regularity in the zero viscosity limit.

A report on a joint work with E. Titi and E. Wiedemann.

Splitting schemes & segregation in reaction-(cross-)diffusion systems

Jose A. Carrillo

Imperial College London

Abstract

One of the most fascinating phenomena observed in reaction-diffusion systems is the emergence of segregated solutions, population densities with disjoint supports. We analyse such a reaction cross-diffusion system. In order to prove existence of weak solutions for a wide class of initial data without restriction about their supports or their positivity, we propose a variational splitting scheme combining ODEs with methods from optimal transport. In addition, this approach allows us to prove conservation of segregation for initially segregated data even in the presence of vacuum.

**Generalized relative entropy inequality
for measure-valued solutions of a structured population model**

Tomasz Dębiec

University of Warsaw

Abstract

We study the long-time asymptotics of the linear growth-fragmentation equation in a general form with measure initial data. We formulate a generalized relative entropy inequality in this context and show how it can be used to show convergence of a solution to a steady profile multiplied by an exponential function of time.

The talk is based on a joint work with M. Doumic, P. Gwiazda and E. Wiedemann.

Long-time asymptotics of protein aggregation models

Marie Doumic

INRIA

Abstract

Mathematical modelling of protein polymerisation is a challenging topic, with wide applications, from actin filaments in myocytes (muscle tissues) to the so-called amyloid diseases (e.g. Alzheimer's, Parkinson's or Creutzfeldt-Jakob's diseases). In this talk, we will focus on nucleation-polymerisation-fragmentation processes taken in an homogeneous spatial environment, so that the equations are structured by the aggregates size, without a space variable. We shall give an overview of recent results and focus on the long-time asymptotics, to investigate under which assumptions a steady behaviour can emerge. The results we will present are partly joint work with A. Armiento, J. Calvo, S. Eugène, M. Escobedo, P. Moireau, B. Perthame, H. Rezaei, P. Robert, M. Tournus and W.F. Xue.

Deterministic particle approximations for local and nonlocal transport equations.

Marco Di Francesco

University of L'Aquila

Abstract

The derivation of first-order nonlinear transport PDEs as many particle limits of interacting particle systems subject only to deterministic forces is crucial in the context of social-biological sciences and real world applications. This talk will collect recent results on the rigorous derivation of entropy solutions to scalar conservation laws (arising e.g. in traffic flow) as many particle limit of follow-the-leader type systems, including extensions to the case with Dirichlet boundary conditions and to the Hughes model for pedestrian movements (the results involve S. Fagioli, M. D. Rosini, G. Russo). I will then describe a recent extension of this approach to entropy solutions for nonlocal transport equations with a nonlinear mobility modelling prevention of overcrowding for high densities (in collaboration with S. Fagioli and E. Radici).

Coupling arguments for population dynamics

Pierre Gabriel

Laboratoire de Mathématiques de Versailles

Abstract

We present quantitative estimates for non-conservative semigroups set on the space of measures. These estimates allow to get the asymptotic behaviour of measure solutions to linear population dynamics PDEs. They provide a generalisation of the Perron or Floquet decomposition in the time-inhomogeneous setting, with an explicit speed of convergence in total variation distance. The method relies on a probabilistic approach to obtain Doeblin contraction for a family of conservative auxiliary semigroups. The results will be illustrated on the renewal equation.

The talk is based on a joint work with V. Bansaye and B. Cloez.

The Cauchy problem for Kolmogorov-Vicsek type equation with Orientational Interactions. Analysis and numerics.

Irene Gamba

The University of Texas at Austin

Abstract

We study the global existence and uniqueness of weak solutions to kinetic Kolmogorov–Vicsek models describing non-local, non-linear, Fokker–Planck type equations for dynamics of individuals with orientational interactions moving with a velocity on the unit sphere. Our analysis covers both the space homogeneous and inhomogeneous cases to obtain L^p estimates and compactness properties taking advantage of the orientational interaction property, meaning that the velocity space is a compact manifold. We also present a numerical approach for this kinetic model consisting in a spectral representation linked with a discrete constrained optimization to compute the orientational interactions. The numerical scheme is tested for this kinetic model at different scales and compared the micro and macro descriptions of the Kolmogorov-Vicsek model. We observe that the kinetic model captures key features such as vortex formation and traveling waves.

This is work in collaboration with Moon-Jin Kang, and also with Jeff Haack and Sebastien Motsch.

Quantitative estimates for Advective Equation with Degenerate Anelastic Constraint

Pierre-Emmanuel Jabin

University of Maryland

Abstract

In this work with D. Bresch, we are interested in quantitative estimates for advective equations with an anelastic constraint in presence of vacuum. More precisely, we derive a stability estimate and obtain the existence of renormalized solutions. The method itself introduces weights which solve a dual equation and allow to propagate appropriately weighted norms on the initial solution. In a second time, a control on where those weights may vanish allow to deduce global and precise quantitative regularity estimates.

**Structured population models in a space of measures;
From analytical foundations to numerical results.**

Karolina Kropielnicka

IMPAN and University of Gdańsk

Abstract

Fredrickson–Hoppensteadt model describes the evolution of the age-structured, two-sex populations. This system consists of three structured population equations of McKendrick type, which are built of transport terms and are coupled by (nonlinear and nonlocal) influx terms.

To track the dynamics numerically we need to resort to the Escalato Boxcar Train (EBT) method. Due to the fact that all the problem and its analysis are embedded in a space of nonnegative Radon measures equipped with flat metric (also known as bounded Lipschitz or Fournet–Mourier distance), large part of the talk will be devoted to the justification of the choice of the proper metric space. We will also describe difficulties occurring in the measurement of error in the space of measures, and present developed recently remedies.

Based on joint work with José A. Carrillo (Imperial College London), Piotr Gwiazda (IMPAN), Anna Marciniak-Czochra (University of Heidelberg).

Discrete and continuous transitions in models of cell differentiation

Anna Marciniak-Czochra

Heidelberg University

Abstract

Cell differentiation is a multi-step process, in which a relatively small population of stem cells undergo asymmetric cell divisions leading to formation of more mature cells (differentiation process) and subsequent replenishment of cells at different maturation stages. Understanding of the mechanisms governing cell differentiation is of central interest for stem cell biology, especially because of its clinical impact. One established method of modeling of such hierarchical cell systems is to use a discrete collection of ordinary differential equations, each of which describes a well-defined differentiation stage. However, there are indications that the differentiation process is less rigid and that it involves transitions which are continuous, along with discrete ones. Population dynamics based on transitions within a continuum of intermediate stages can be described using so called structured population models. Comparing both the discrete and continuous framework to describe dynamics of cell differentiation show that the models may exhibit different dynamics and, in particular, the structure and stability of stationary states may be different. We will discuss different models, biological evidence concerning the underlying processes and address the question of the choice of the right class of models.

Cucker-Smale equation

Piotr B. Mucha

University of Warsaw

Abstract

The Cucker-Smale flocking model belongs to a wide class of kinetic models that describe a collective motion of interacting particles that exhibit some specific tendency e.g. to aggregate, flock or disperse. I will talk about the kinetic Cucker-Smale equation with a singular communication weight. Given a compactly supported measure as an initial datum I will show a construct a global in time weak measure-valued solution. The solution is defined as a mean-field limit of the empirical distributions of particles, which dynamics is governed by the Cucker-Smale particle system. What is interesting the choice of the range of singularity admits sticking of characteristics/trajectories. I will mention also about the weak-atomic uniqueness property stating that a weak solution initiated by a finite sum of atoms preserves its atomic structure. Hence they coincide with unique solutions to the system of ODEs associated with the Cucker-Smale particle system.

The talk will be based on the results from: *The Cucker-Smale equation: singular communication weight, measure-valued solutions and weak-atomic uniqueness*, Piotr B. Mucha, Jan Peszek, ARMA 2017

Bacterial collective movement: models, patterns, pathways, scales

Benoît Perthame

Laboratoire J.-L. Lions, Paris

Abstract

Cell communities exhibit remarkable patterns which result from complex and poorly understood mechanisms. The simplest of them are traveling pulses. Many partial differential equations have been proposed in this field and two classes are remarkable

- semilinear parabolic systems for the cell population density and the nutrients,
- Fokker-Planck equations as the famous Keller-Segel model for chemotaxis.

To go further, address finer experimental observations, fit precise measurements and understand the various scales, new classes of models have appeared. At the mesoscopic scale, they have been based on Boltzmann-kinetic models after the experimental observation of the run and tumble movement of bacteria. They are useful to include informations at the individual cell level through the tumbling kernel. They also allow to derive macroscopic models (at the population scale), as the Keller-Segel system, in the diffusion limit..

We will finally explain how the microscopic behavior of *E. coli*, and its modulation of the tumbles, can be explained using molecular pathways. This gives rise to the Flux Limited Keller-Segel equation in the diffusion limit. In opposition to the traditional Keller-Segel system, this new model can sustain robust traveling bands as observed in the famous experiment of Adler.

The graph structure impact on a singular limit of the generalized network transport

Aleksandra Puchalska

University of Warsaw

Abstract

In the talk we present the family of perturbed transport problems in which a domain consists of countable intervals coupled by transition conditions at the ends. Using the theory of convergence of sequences of semigroups, we present different convergence results in the case of velocities of transport that accelerates to infinity being balanced by certain conditions at the boundary. We compare the structure of graph in primal problems with the properties of a network of the limit solution. The motivation of this model is giving the description of proliferation of the genetic mutation. The results can be applied to give the qualitative comparison of models based on transport equation and appropriate ODE model, that are used for the analysis of this problem.

On ill-posedness of Euler system with non-local terms

Agnieszka Świerczewska-Gwiazda

University of Warsaw

Abstract

We discuss several modifications of the Euler system of fluid dynamics including its pressureless variant driven by non-local interaction repulsive-attractive and alignment forces. These models arise in the study of self-organisation in collective behavior modeling of animals and crowds. We adapt the method of convex integration to show the existence of infinitely many global-in-time weak solutions for any bounded initial data. Then we consider the class of dissipative solutions satisfying, in addition, the associated global energy balance (inequality). We identify a large set of initial data for which the problem admits infinitely many dissipative weak solutions. Finally, we establish a weak-strong uniqueness principle for the pressure driven Euler system with non-local interaction terms as well as for the pressureless system with Newtonian interaction.

Hydrodynamic models of collective behavior with damping and nonlocal interactions

Aneta Wróblewska-Kamińska

Imperial College London

Abstract

Hydrodynamic systems for interacting particles where attraction is taken into account by nonlocal forces derived from a potential and repulsion is introduced by local pressure arise in swarming modelling. We focus on the case where there is a balance between nonlocal attraction and local pressure in presence of confinement in the whole space. Under suitable assumptions on the potentials and the pressure functions, we show the global existence of solutions for these hydrodynamic models with viscosity and linear damping. By introducing linear damping into the system, we ensure the existence and uniqueness of compactly supported stationary densities with fixed mass and center of mass whose associated velocity field is zero in their support. Moreover, we show that global weak solutions converge for large times to the set of these stationary solutions in a suitable sense. In particular cases, we can identify the limiting density uniquely as the global minimizer of the free energy with the right mass and center of mass. This is a joint result with Jose A. Carrillo.

Incompressible limit of the Navier-Stokes model with a growth term

Ewelina Zatorska

University College London

Abstract

In this talk I will analyze the Navier-Stokes equations that generalize the fluid-based models of tumors. Starting from isentropic compressible Navier-Stokes equations with growth term in the continuity equation, I will rigorously justify that performing an incompressible limit one arrives to the two-phase free boundary fluid system.

This talk is based on the joint paper with Nicolas Vauchelet.
