2nd homework of the Functional Analysis, 7.11.2014
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The deadline: November, 14st 2014, h14.30.
The place of submition: envelope on the door of the room no 5020.

Ex 1. Show that the set $L^2(0,1)$ is of the first Baire category in Banach space $L^1(0,1)$.

Ex 2. Find the norms of the following functionals $F \in X^*$

a) $X = \ell^1, \quad F(x_1, x_2, x_3, ...) = \sum_{n=1}^{\infty} (x_{2n} - 5x_{2n-1}),$

b) $X = C([0,1]), \quad F(f) = \sum_{k=1}^{\infty} \frac{(-1)^k}{k^2} f\left(\frac{1}{k^2}\right),$

c) $X = L^p(0,1), \quad F(f) = \int_0^1 t f(t) \mathrm{dt}.$

Ex 3. Let us assume that X is a Banach space and $\Lambda : X \to \mathbf{R}$ is a linear functional. Show that operator Λ is continuous, if and only if the kernel (i.e. the set $\{x \in X : \ \Lambda x = 0\}$) is closed.

Ex 4. Let us assume that X is a Banach space and $A : X \to X$ is a linear bounded operator. We assume further that there exist complex numbers $c_1, \ldots, c_n$, $n \geq 1$ such that

$$Id + c_1 A + \ldots + c_n A^n = 0.$$

Show existence of continuous operator which is an inverse of A.