# Complexity of Multiagent BDI Logics with Restricted Modal Context

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**Abstract**

We present and discuss a novel language restriction for modal logics for multiagent systems that can reduce the complexity of the satisfiability problem from EXPTIME-hard to NPTIME-complete. In the discussion we focus on a particular BDI logic, called TeamLog, which is a logic for modelling cooperating groups of agents and which possesses some of the characteristics typical to other BDI logics. All the technical results can be found in the dissertation [7].

**Keywords:** Multiagent Theories, BDI, Teamwork, Modal Logic, Satisfiability

## 1 Introduction

Modal logics of agency based on BDI model [2] are important formalisms used to specify individual agents in terms of their beliefs, goals/desires and intentions. One of the characteristics of such logics is adopting, along with standard modal systems $K_n$, $KD_n$ or $KD45_n$, mixed axioms that interrelate modalities representing different aspects of agent description. Well known examples of such axioms are *realism axioms* [3, 19, 20] and *introspection axioms* [18, 4, 6]. In the case of BDI logics for single agent, addition of these axioms does not change the complexity of the satisfiability problem, and they all remain PSPACE-complete [19, 9].

In the multiagent case such logics are extended by lifting individual modalities representing beliefs, goals or intentions to the group level by introducing fixpoint modalities representing common beliefs, mutual goals or mutual intentions [15, 20, 1, 6]. Adding such modalities leads to EXPTIME-hard satisfiability problem [10], and existence of mixed axioms does not affect this result [9].

One of the ways of dealing with high complexity of logical formalisms is restricting their language, so that the complexity of the satisfiability problem is 'reduced'.[1] As was shown in [11], restricting modal depth of formulas by a constant may lead to NPTIME-complete satisfiability problem, while combining this with restricting the number of propositional symbols leads to linear time solvability of

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[1] In many cases the term 'reduction' is used under the assumption that NPTIME, PSPACE and EXPTIME are different and increasing in this order.

the problem. This, however, is with a constant that depends exponentially on the number of those symbols. [16] shows that Horn fragment of some of modal systems has NPTIME-complete satisfiability problem, which can become PTIME-complete when combined with modal depth restriction.

In the case of modal logics with fixed point modalities the restrictions mentioned above are not that promising, as the satisfiability problem remains EXPTIME-hard even when modal depth of formulas is bounded by 2 [9]. To deal with this problem we propose a new kind of language restriction called *modal context restriction*. In [8] we applied this restriction to standard systems of multimodal logics enriched with fixpoint modalities and showed that it leads to PSPACE-completeness and, when combined with modal depth restriction, to NPTIME-completeness of the satisfiability problem. In this paper we present modal context restrictions for BDI logics, choosing, as a 'working' formalism, TeamLog [6], a well known and important formalism that focuses on teamwork.

We show that the restrictions lead to PSPACE-completeness of the satisfiability problem. However, existence of the introspection axioms results in PSPACE-hardness of the problem, even if modal depth of formulas is bounded by 2. An interesting feature of the considered logic is the fact that models satisfying the formulas may be exponentially deep (with respect to their size). For this reason a 'standard' tableau based algorithm has to be extended so that it uses polynomial space to decide the satisfiability.

# 2 The Formalism

TeamLog is a logical framework proposed to formalize individual and group aspects of multiagent systems [6]. It is a multimodal formalism with the set of modal operators based on a non-empty and finite set of agents, $\mathcal{A}$:[2]

$$\Omega^{\mathrm{T}} = \Omega^{\mathrm{B}^+} \cup \Omega^{\mathrm{G}} \cup \Omega^{\mathrm{I}^+},$$

where

$$\begin{aligned}
\Omega^{\mathrm{B}^+} &= \Omega^{\mathrm{B}} \cup \{[\mathrm{B}]^+_G : G \in \mathrm{P}(\mathcal{A}) \setminus \{\varnothing\}\},\\
\Omega^{\mathrm{I}^+} &= \Omega^{\mathrm{I}} \cup \{[\mathrm{I}]^+_G : G \in \mathrm{P}(\mathcal{A}) \setminus \{\varnothing\}\},\\
\Omega^{\mathrm{B}} &= \{[\mathrm{B}]_j : j \in \mathcal{A}\},\\
\Omega^{\mathrm{G}} &= \{[\mathrm{G}]_j : j \in \mathcal{A}\},\\
\Omega^{\mathrm{I}} &= \{[\mathrm{I}]_j : j \in \mathcal{A}\}.
\end{aligned}$$

Operators $[\mathrm{B}]_j$, $[\mathrm{G}]_j$ and $[\mathrm{I}]_j$ stand for beliefs, goals and intentions of agent $j$, respectively, while $[\mathrm{B}]^+_G$ and $[\mathrm{I}]^+_G$ are fixpoint modalities standing for common beliefs and mutual intentions of group $G$, respectively.[3] The propositional multimodal language $\mathcal{L}^{\mathrm{T}}$ of TeamLog and its semantics are defined in the usual way (see [6] for details).

[2] For the sake of conciseness we use a more compact notation for operators of TeamLog, replacing that standard ones from [6].

[3] Thus $\Omega^{\mathrm{B}}$, for example, is the set of all modal operators related to beliefs of individual agents and $\Omega^{\mathrm{B}^+}$ is the set of all modal operators related to beliefs of individual agents and to common beliefs of groups of agents.

Deduction system of TeamLog combines systems $\mathrm{K}_n$ (for goals), $\mathrm{KD}_n$ (for intentions) and $\mathrm{KD45}_n$ (for beliefs). An important aspect of the formalism are mixed axioms, interrelating different attitudes of individual agents. The fact that for each agent $j$ intentions are a subset of goals, is reflected in the **goals-intentions compatibility** axiom[4]

$$[\mathrm{I}]_j\varphi \to [\mathrm{G}]_j\varphi.$$

The fact that each agent $j$ is fully aware of his goals and intentions is reflected in **positive** and **negative introspection** axioms:

$$[O]_j\varphi \to [\mathrm{B}]_j[O]_j\varphi,$$
$$\neg[O]_j\varphi \to [\mathrm{B}]_j\neg[O]_j\varphi,$$

where $O \in \{\mathrm{G}, \mathrm{I}\}$.

For fixpoint modalities $[\mathrm{B}]^+_G$ and $[\mathrm{I}]^+_G$ axioms

$$[O]^+_G\varphi \leftrightarrow [O]_G\left(\varphi \wedge [O]^+_G\varphi\right)$$

and rules

$$\text{from } \varphi \to [O]_G(\psi \wedge \varphi) \text{ infer } \varphi \to [O]^+_G\psi$$

where

$$[O]_G\varphi \leftrightarrow \bigwedge_{i\in G}[O]_G\varphi$$

and $O \in \{\mathrm{B}, \mathrm{I}\}$, are adopted.

As was shown in [9], the TeamLog satisfiability problem is EXPTIME-complete.

## 3 Modal context restriction

We start by defining the notion of modal context restriction for general language of multimodal logic. First we need a notion of modal context of a formula within a formula. Let $\mathcal{L}$ be a multimodal language defined over some set of unary modal operators $\Omega$.[5]

**Definition 1** *Let* $\{\varphi, \xi\} \subseteq \mathcal{L}$*. The* modal context of formula $\xi$ within formula $\varphi$ *is a set of finite sequences over* $\Omega$*,* $\mathrm{cont}(\xi,\varphi) \subseteq \Omega^*$*, defined inductively as follows:*

- $\mathrm{cont}(\xi,\varphi) = \varnothing$, *if* $\xi \notin \mathrm{Sub}(\varphi)$,
- $\mathrm{cont}(\varphi,\varphi) = \{\varepsilon\}$,
- $\mathrm{cont}(\xi,\neg\psi) = \mathrm{cont}(\xi,\psi)$, *if* $\xi \in \mathrm{Sub}(\varphi)$ *and* $\xi \neq \neg\psi$,
- $\mathrm{cont}(\xi,\psi_1 \wedge \psi_2) = \mathrm{cont}(\xi,\psi_1) \cup \mathrm{cont}(\xi,\psi_2)$, *if* $\xi \in \mathrm{Sub}(\varphi)$ *and* $\xi \neq \psi_1 \wedge \psi_2$,
- $\mathrm{cont}(\xi,\Box\psi) = \Box \cdot \mathrm{cont}(\xi,\psi_j)$, *if* $\xi \in \mathrm{Sub}(\varphi)$, $\xi \neq \Box\psi$ *and* $\Box \in \Omega$,

*where* $\mathrm{Sub}(\varphi)$ *denotes the set of all subformulas of* $\varphi$ *and* $\Box \cdot S = \{\Box \cdot s : s \in S\}$*, for* $\Box \in \Omega$ *and* $S \subseteq \Omega^*$*.*

[4] This is also called *strong realism* axiom in [19].

[5] The definition can be adopted to modalities of higher arity in a natural way.

To illustrate the notion of modal context, consider a formula $\varphi \stackrel{\text{def}}{=} [\mathrm{B}]_1[\mathrm{I}]^+_{\{1,2\}}(p \vee q) \wedge [\mathrm{I}]_2 \neg p$. Then $\mathrm{cont}(p, \varphi) = \{[\mathrm{B}]_1[\mathrm{I}]^+_{\{1,2\}}, [\mathrm{I}]_2\}$, $\mathrm{cont}(q, \varphi) = \{[\mathrm{B}]_1[\mathrm{I}]^+_{\{1,2\}}\}$, $\mathrm{cont}(r, \varphi) = \varnothing$ and $\mathrm{cont}(\neg p, \varphi) = \{[\mathrm{I}]_2\}$.

**Definition 2** *A* modal context restriction *is a set of finite sequences over $\Omega$, $R \subseteq \Omega^*$, constraining possible modal contexts of subformulas within formulas. We say that a formula $\varphi \in \mathcal{L}$* satisfies a modal context restriction *$R \subseteq \Omega^*$ iff for all $\xi \in \mathrm{Sub}(\varphi)$ it holds that $\mathrm{cont}(\xi, \varphi) \subseteq R$.*

In this paper we propose two modal context restrictions of the language of TeamLog that lead to PSPACE completeness of the satisfiability problem. The restrictions are presented below.

**Definition 3** *Let*

$$\mathbf{R_1} = \Omega^* \setminus \left( \Omega^* \cdot \left[ \bigcup_{G \in \mathrm{P}(\mathcal{A}) \setminus \{\varnothing\}} (S_{\mathrm{I}}(G) \cup S_{\mathrm{IB}}(G)) \cup \bigcup_{G \in \mathrm{P}(\mathcal{A}), |G| \geq 2} S_{\mathrm{B}}(G) \right] \cdot \Omega^* \right),$$

*where*

$$\begin{aligned} S_{\mathrm{IB}}(G) &= \bigcup_{j \in G} [\mathrm{I}]^+_G \cdot ([\mathrm{B}]_j)^* \cdot T_{\mathrm{B}}(\{j\}) \cdot T_{\mathrm{I}}(\{j\}), \text{ and} \\ S_O(G) &= [O]^+_G \cdot T_O(G), \\ T_O(G) &= \{[O]_j : j \in G\} \cup \{[O]^+_H : H \in \mathrm{P}(\mathcal{A}), H \cap G \neq \varnothing\}, \end{aligned}$$

*for $O \in \{\mathrm{B}, \mathrm{I}\}$. The set of formulas in $\mathcal{L}^{\mathrm{T}}$ satisfying restriction $\boldsymbol{R}_1$ will be denoted by $\mathcal{L}^{\mathrm{T}}_{\mathbf{R_1}}$.*

**Definition 4** *Let*

$$\mathbf{R_2} = \Omega^* \setminus \left( \Omega^* \cdot \left[ \bigcup_{G \in \mathrm{P}(\mathcal{A}) \setminus \{\varnothing\}} (S_{\mathrm{I}}(G) \cup S_{\mathrm{IB}}(G)) \cup \bigcup_{G \in \mathrm{P}(\mathcal{A}), |G| \geq 2} \tilde{S}_{\mathrm{B}}(G) \right] \cdot \Omega^* \right),$$

*where*

$$\tilde{S}_{\mathrm{B}}(G) = [\mathrm{B}]^+_G \cdot \left( \{[\mathrm{G}]_j : j \in G\} \cup \bigcup_{O \in \{\mathrm{B}, \mathrm{I}\}} T_O(G) \right)$$

*and $S_{\mathrm{IB}}$, $S_{\mathrm{I}}$ and $T_O$, for $O \in \{\mathrm{B}, \mathrm{I}\}$, are defined like in the case of restriction $\boldsymbol{R}_1$. The set of formulas in $\mathcal{L}^{\mathrm{T}}$ satisfying restriction $\boldsymbol{R}_2$ will be denoted by $\mathcal{L}^{\mathrm{T}}_{\mathbf{R_2}}$.*

Restriction $\mathbf{R}_1$ forbids any operator $[O]_j$ or $[O]^+_H$, with $O \in \{\mathrm{B}, \mathrm{I}\}$ in the context of $[O]^+_G$, if $j \in G$ or $H \cap G \neq \varnothing$. Additionally the restriction forbids subsequences contained in $S_{\mathrm{IB}}$. Forbidding subsequences from $S_{\mathrm{IB}}$ is related to axioms of positive and negative introspection of intentions. Restriction $\mathbf{R}_2$ is a refinement of restriction $\mathbf{R}_1$, forbidding any operator $[O]_j$ or $[O]^+_H$, with $O \in \{\mathrm{B}, \mathrm{G}, \mathrm{I}\}$ in the context of $[\mathrm{B}]^+_G$, if $j \in G$ or $H \cap G \neq \varnothing$. Thus any formula $\varphi \in \mathcal{L}^{\mathrm{T}}$ satisfying restriction $\mathbf{R}_2$, satisfies restriction $\mathbf{R}_1$ as well, that is $\mathcal{L}^{\mathrm{T}}_{\mathbf{R_2}} \subseteq \mathcal{L}^{\mathrm{T}}_{\mathbf{R_1}}$. Notice that if $|\mathcal{A}| = 1$, then $\mathcal{L}^{\mathrm{T}}_{\mathbf{R_2}} = \mathcal{L}^{\mathrm{T}}_{\mathbf{R_1}}$.

We have the following results regarding the complexity of the TeamLog satisfiability problems for formulas from $\mathcal{L}^{\mathrm{T}}_{\mathbf{R_1}}$ and $\mathcal{L}^{\mathrm{T}}_{\mathbf{R_2}}$.

**Theorem 1** *The* TeamLog *satisfiability problem for formulas from $\mathcal{L}^{\mathrm{T}}_{\mathbf{R_2}}$ is PSPACE-complete. Moreover, it is NPTIME-complete if model depth of formulas from $\mathcal{L}^{\mathrm{T}}_{\mathbf{R_2}}$ is bounded by a constant.*

**Theorem 2** *The* TeamLog *satisfiability problem for formulas from $\mathcal{L}^{\mathrm{T}}_{\mathbf{R_1}}$ is PSPACE-complete, even if modal depth of formula is bounded by a constant $\geq 2$.*

We omit the proof of the theorems in this paper (all the details can be found in dissertation [7]). In Section 4 we present the algorithm used to obtain the result in the case of $\mathcal{L}^{\mathrm{T}}_{\mathbf{R_1}}$. The algorithm for $\mathcal{L}^{\mathrm{T}}_{\mathbf{R_2}}$ is similar to the one presented in [8] for standard modal logics with fix point modalities but without mixed axioms.

## 3.1 Discussion

We would like to note first the effect of mixed axioms on the restrictions required and associated complexity results. The axioms that have the effect here are the introspection axioms. Firstly, because of the introspection axioms for intentions, it is required to forbid subsequences contained in $S_{\mathrm{IB}}$ (for notice that with these axioms the formulas of the form $[\mathrm{I}]_j\varphi \leftrightarrow [\mathrm{B}]_j[\mathrm{I}]_j\varphi$ are valid). Secondly, it is because of the introspection axioms for intentions and goals, that some of the formulas from $\mathcal{L}^{\mathrm{T}}_{\mathbf{R_1}}$ require a tree-like model with depth depending exponentially on their modal depth to be satisfied.[6] For this reason, to obtain reduction to NPTIME-solvability by restricting modal depth of formulas, we need the restriction $\mathbf{R}_2$.

Let us start the discussion of the two restrictions with formulas specifying beliefs of groups of agents. When interpreted in the context of BDI agents, $\mathbf{R}_1$ can be seen as forbidding common introspection of beliefs within a group of agents. In other words, it forbids any formula of the form $[\mathrm{B}]^+_G\varphi$ where $\varphi$ contains, within the scope of propositional operators, any formulas referring to beliefs of agents from $G$. For example the following formula specifies that group $G$ commonly beliefs that group $H$ generally beliefs that $\varphi$ holds:

$$[\mathrm{B}]^+_G[\mathrm{B}]_H\varphi.$$

If $G \cap H = \varnothing$, then this formula satisfies restriction $\mathbf{R}_1$. However, if $G \cap H \neq \varnothing$, then this formula does not satisfy the restriction. Restriction $\mathbf{R}_2$ forbids, additionally,

[6] We omit the argument here, but see [7] for a formula enforcing exponentially long path in a model, that uses introspection axioms.

common introspection of goals and intentions within a group of agents. Hence the following formula

$$[\mathrm{B}]^+_G[\mathrm{I}]_H\varphi$$

with $H \cap G \neq \varnothing$ is forbidden by $\mathbf{R}_2$, while it is allowed by $\mathbf{R}_1$.

Now let us consider formulas specifying group intentions. In this case there is no difference between the two restrictions and both of them forbid mutual intentions towards intentions and beliefs about intentions within a group of agents. For example the formula

$$[\mathrm{I}]^+_G[\mathrm{I}]_H\varphi$$

does not satisfy neither $\mathbf{R}_1$ nor $\mathbf{R}_2$, if $G \cap H \neq \varnothing$. In this case specifying that $G$ mutually intends that some of the agents from this group have certain intentions is forbidden. The formula

$$[\mathrm{I}]^+_G[\mathrm{B}]_H[\mathrm{I}]_H\varphi$$

does not satisfy the restrictions as well, if $G \cap H \neq \varnothing$. In this case specifying that $G$ mutually intends that some group $H$ containing agents from $G$ generally beliefs that it generally intends something is forbidden. The main difference between the two restrictions is seen when group beliefs about motivational attitudes are considered. For example the definition of collective intention, the notion characterising the situations when a group of agents achieves something in a fully cooperative and coordinated manner [4, 6],

$$[\mathrm{I}]^+_G\varphi \wedge [\mathrm{B}]_G[\mathrm{I}]^+_G\varphi$$

does not satisfy $\mathbf{R}_2$, while it satisfies $\mathbf{R}_1$ (as long as it is satisfied by formulas $\varphi$ and $[\mathrm{I}]^+_G\varphi$). Similar problem appears when definitions of other notions involving beliefs about intentions are considered, like for example various versions of collective commitments, describing a concrete manner of how the common goal is achieved [5, 6].

Methodologies of agent oriented modelling and design, like KGR [14, 13, 12], divide the process of agents modelling by separating construction of belief model, goal model and plan model. Similarly, in specification of a multiagent systems using formalisms like TeamLog, separate parts could be distinguished, where purely informational and purely motivational aspects of individual agents and groups of agents are specified and parts where interrelations between these parts are specified. In such cases restriction $\mathbf{R}_2$ can be applied to the purely informational or purely motivational parts, while restriction $\mathbf{R}_1$ can be applied to the mixed parts.

# 4 The Complexity of The Satisfiability Problem

To check the satisfiability of formulas of the logic in question we will use and extend the tableau method, as presented in [10]. The method is based on the notion of *modal tableau*, a *model graph* satisfying additional properties related to the considered logic.

**Definition 5 (Model graph)** *A* model graph *is a pair* $(\mathcal{F}, L)$ *where* $\mathcal{F}$ *is a Kripke frame and* $L$ *is a* labelling function *associating with each world* $w \in W$ *a set of formulas* $L(w)$*.*

Labels of modal tableau for the logic considered in this paper must be closed propositional tableaux. We define this notion first.

**Algorithm 1:** DecideSatisfiability

```
Input: a formula φ
Output: a decision whether φ is satisfiable or not
/* Pre-tableau construction                                                   */
Construct a pre-tableau consisting of single node root, with L(root) = {φ} and all
successor relations being empty;
repeat
    Let Z be the set of all leaves of the pre-tableau with labelling sets that are not
    blatantly inconsistent;
    if there is n ∈ Z such that n is not a state and ψ ∈ L(n) is a witness to that then
        FormState(n,ψ);
    else if there is s ∈ Z then
        foreach ψ ∈ L(s) do
            CreateSuccessorsB(s,ψ);
            CreateSuccessorsG(s,ψ);
            CreateSuccessorsI(s,ψ);
until no change occurred;
/* Marking nodes and deciding satisfiability                                  */
repeat
    MarkNodes;
until no new node marked;
if root is marked sat then
    return sat;
else
    return unsat;
```

**Definition 6 (Closed set of formulas)** *A set of formulas $\Phi$ is* closed *if it satisfies the following condition, for all $G \subseteq \mathcal{A}$:*

**Cl** *If $[\mathrm{B}]^+_G\varphi \in \Phi$, then $\{[\mathrm{B}]_j[\mathrm{B}]^+_G\varphi, [\mathrm{B}]_j\varphi : j \in G\} \subseteq \Phi$,*

Given a set of formulas $\Phi$ we will use $\mathrm{Cl}(\Phi)$ to denote the minimal closed set having $\Phi$ as a subset and $\neg\Phi$ to denote the negation closure of $\Phi$, i.e. $\neg\Phi = \Phi \cup \{\sim\varphi : \varphi \in \Phi\}$, where $\sim$ denotes a single negation, i.e. $\sim\varphi = \neg\varphi$, if $\varphi$ does not start with $\neg$, and $\psi$, if $\varphi$ is of the form $\neg\psi$. A set of formulas $\Phi$ is called *blatantly inconsistent* if there is a formula $\varphi \in \Phi$ such that $\neg\varphi \in \Phi$.

**Definition 7 (Propositional tableau)** *A* propositional tableau *is a set $\mathcal{T}$ of formulas such that $\mathcal{T}$ is not blatantly inconsistent and:*

1. *If $\neg\neg\psi \in \mathcal{T}$ then $\psi \in \mathcal{T}$.*
2. *If $\varphi \wedge \psi \in \mathcal{T}$ then $\varphi \in \mathcal{T}$ and $\psi \in \mathcal{T}$.*
3. *If $\neg(\varphi \wedge \psi) \in \mathcal{T}$ then either $\{\sim\varphi, \psi\} \subseteq \mathcal{T}$ or $\{\varphi, \sim\psi\} \subseteq \mathcal{T}$ or $\{\sim\varphi, \sim\psi\} \subseteq \mathcal{T}$.*

A *propositional tableau for a formula $\varphi$* is a minimal propositional tableau $\mathcal{T}$ such that $\varphi \in \mathcal{T}$. It is easy to see that $\mathcal{T}$ is a propositional tableau for $\varphi$ iff it is a maximal consistent subset of $\neg\mathrm{PT}(\varphi)$, where $\mathrm{PT}(\varphi)$ contains subformulas of $\varphi$ taken with respect to propositional operators, i.e. it is defined inductively as follows:

1. $\mathrm{PT}(p) = \{p\}$, where $p \in \mathcal{P}$,
2. $\mathrm{PT}(\neg\psi) = \{\neg\psi\} \cup \mathrm{PT}(\psi)$,
3. $\mathrm{PT}(\psi_1 \wedge \psi_2) = \mathrm{PT}(\psi_1) \cup \mathrm{PT}(\psi_2)$,
4. $\mathrm{PT}(\Box\psi) = \{\Box\psi\}$, where $\Box \in \Omega^{\mathrm{T}}$.

Now we are ready to define the modal tableau for the logic in question.

**Definition 8 (Modal tableau)** *A* modal tableau *is a model graph $\mathcal{T} = (\mathcal{F}, L)$ such that for all $w \in W$, $L(w)$ is a closed propositional tableau, and the following conditions are satisfied for all $w \in W$, $j \in \mathcal{A}$ and $G \in \mathrm{P}(\mathcal{A}) \setminus \{\varnothing\}$:*

**T1** *If $[O]_j\varphi \in L(w)$ and $v \in R_j^{[O]}(w)$, then $\varphi \in L(v)$, for all $O \in \{\mathrm{B}, \mathrm{G}, \mathrm{I}\}$.*

**T2** *If $\neg[O]_j\varphi \in L(w)$, then there exists $v \in R_j^{[O]}(w)$ such that $\sim\varphi \in L(v)$, for all $O \in \{\mathrm{B}, \mathrm{G}, \mathrm{I}\}$.*

**TD** *If $[O]_j\varphi \in L(w)$, then either $\varphi \in L(w)$ or $R_j^{[O]}(w) \neq \varnothing$, for all $O \in \{\mathrm{B}, \mathrm{I}\}$.*

**T4** *If $v \in R_j^{[\mathrm{B}]}(w)$ and $[\mathrm{B}]_j\varphi \in L(w)$, then $[\mathrm{B}]_j\varphi \in L(v)$.*

**T5** *If $v \in R_j^{[\mathrm{B}]}(w)$ and $[\mathrm{B}]_j\varphi \in L(v)$, then $[\mathrm{B}]_j\varphi \in L(w)$.*

**TB*O*4** *If $v \in R_j^{[\mathrm{B}]}(w)$ and $[O]_j\varphi \in L(w)$, then $[O]_j\varphi \in L(v)$, for all $O \in \{\mathrm{G}, \mathrm{I}\}$.*

**TB*O*5** *If $v \in R_j^{[\mathrm{B}]}(w)$ and $[O]_j\varphi \in L(v)$, then $[O]_j\varphi \in L(w)$, $O \in \{\mathrm{G}, \mathrm{I}\}$.*

**TIG** *If $v \in R_j^{[\mathrm{G}]}(w)$ and $[\mathrm{I}]_j\varphi \in L(w)$, then $\varphi \in L(v)$.*

**TC** *If $\neg[O]_G\varphi \in L(w)$, then there exists $v \in {R_G^{[O]}}^+(w)$ such that $\sim\varphi \in L(v)$, for all $O \in \{\mathrm{B}, \mathrm{I}\}$.*

*Given a formula $\varphi$, we say that a tableau $\mathcal{T}$ is a* tableau for $\varphi$, *if there exists a state $w \in W$ such that $\varphi \in L(w)$.*

The following theorem links the satisfiability problem and the notion of tableau.

**Theorem 3** *A formula $\varphi \in \mathcal{L}$ is satisfiable iff there is a tableau for $\varphi$.*

Proof of the theorem is by standard arguments. For details see [7]

## 4.1 The Algorithm

To show that the satisfiability problem for the logic in question is PSPACE complete we will use an algorithm extending the method presented in [10]. The general idea of algorithms proposed there is as follows. Given an input formula $\varphi$, the algorithm tries to construct a *pre-tableau*, a tree-like structure that forms a basis for a modal tableau for $\varphi$. The construction proceeds in the manner of depth first search, developing branches of the pre-tableau separately. Execution of the algorithm requires storing each constructed branch in the memory. Therefore the space required depends polynomially on the length of the longest branch. In [8] we extended this method for standard multimodal logics with fixpoint modalities such as common knowledge or common beliefs and with restricted modal context. This extension relied on the fact that with such restrictions the depth of the model required to satisfy the formulas was bounded polynomially with respect to their lengths. This is still true in the case of TeamLog with restriction $\mathbf{R}_2$, but ceases to hold in the case of restriction $\mathbf{R}_1$, as stated by the proposition below.

**Procedure 2:** CreateSuccessorsB

**Input**: a state $s$ and a formula $\psi \in L(s)$

**if** *$\psi$ is of the form* $\neg[\mathrm{B}]_j\xi$ **then**
  **if** *there is no* $R_j^{[\mathrm{B}]}$*-Predecessor $t$ of $s$ such that* $\neg[\mathrm{B}]_j\xi \in L(t)$ *and* $L^{\neg[\mathrm{B}]_j}(t,\xi) = L^{\neg[\mathrm{B}]_j}(s,\xi)$ **then**
    **if** $\xi = [\mathrm{B}]^+_G\zeta$ *with* $j \in G$ *and there is a* $\neg[\mathrm{B}]_k[\mathrm{B}]^+_G\zeta$*-Predecessor $t$ of $s$ such that* $L(s)/[\mathrm{B}]_j \cup \{\zeta\} = L(t)/[\mathrm{B}]_k \cup \{\zeta\}$ **then**
      **if** $[\mathrm{B}]_j\zeta \in L(s)$ **then**
        **if** $\zeta \notin L(s)$ **then**
          Create a $[\mathrm{B}]_j$-Successor $v$ of $s$ with $L(v) = L^{\neg[\mathrm{B}]_j}(s,\xi)$;
        **else if** `DecideSatisfiabilityAux`$(L(s),\, G,\, \zeta,\, j)$ $=$ `unsat` **then**
          Mark $s$ `unsat`;
    **else** Create a $[\mathrm{B}]_j$-Successor $v$ of $s$ with $L(v) = L^{\neg[\mathrm{B}]_j}(s,\xi)$;

**else if** *$\psi$ is of the form* $[\mathrm{B}]_j\xi$ *and there are no formulas of the form* $\neg[\mathrm{B}]_j\zeta \in L(s)$ **then**
  If there is no $R_j^{[\mathrm{B}]}$-Predecessor $t$ of $s$ such that $[\mathrm{B}]_j\xi \in L(t)$ and $L^{[\mathrm{B}]_j}(t) = L^{[\mathrm{B}]_j}(s)$ and $L^{[\mathrm{B}]_j}(s) \nsubseteq L(s)$, then create an $R_j^{[\mathrm{B}]}$-successor $u$ of $s$ with $L(u) = L^{[\mathrm{B}]_j}(s)$;

**Proposition 1** *Let* $|\mathcal{A}| \geq 2$. *Then there exists a satisfiable formula* $\varphi \in \mathcal{L}^{\mathrm{T}}_{\mathbf{R_1}}$ *such that any model for it contains a sequence of pairwise different worlds of length exponential with respect to* $|\varphi|$.

Proposition 1 relies on introspection axioms and uses formulas of the form $[\mathrm{B}]^+_G\varphi$ with subformulas of the form $[\mathrm{I}]_jp$ (or, alternatively, $[\mathrm{G}]_jp$), with $j \in G$, in $\neg\mathrm{PT}(\varphi)$. The proposition suggests that we cannot use algorithms analogous to those from [10, 8] in the case of the logic considered in this paper, as they try to explicitly construct a tree-like model for an input formula, storing whole each of the subsequently constructed branches in the memory. The algorithm proposed in this section combines the method from [10, 8] with the idea of Savitch algorithm for checking reachability in graphs, that uses quadratic logarithmic space with respect to the number of vertices (c.f. [17]). Before we describe the algorithm we need to introduce some new notions.

Given an input formula, the algorithm will try to construct a pre-tableau for it. We omit formal definition of this notion here, for the lack of space, but see [10]. Roughly speaking a pre-tableau for a formula $\varphi$ is a tree-like structure with nodes connected with a successor relations and labelled with subsets of $\neg\mathrm{Cl}(\neg\mathrm{Sub}(\varphi))$. These subsets consist of properly selected formulas and to define them we need to introduce the following notions.

**Definition 9 ($[O]^+$-expanded set of formulas)** *A set of formulas* $\Phi \subseteq \mathcal{L}$ *is* $[O]^+$-expanded, *with* $O \in \{\mathrm{B}, \mathrm{I}\}$ *and* $G \subseteq \mathcal{A}$, *if the following condition is satisfied:*

**CE** *If* $\neg[O]^+_G\psi \in \Phi$, *then for all* $j \in G$, $\{[O]_j\psi, [O]_j[O]^+_G\psi\} \subseteq \neg\Phi$ *and there exists* $j \in G$ *such that either* $\neg[O]_j\psi \in \Phi$ *or* $\neg[O]_j[O]^+_G\psi \in \Phi$.

**Definition 10 ([B]-expanded tableau)** *A* [B]-expanded tableau *is a* $[\mathrm{B}]^+$*-expanded and* $[\mathrm{I}]^+$*-expanded closed propositional tableau* $\mathcal{T}$ *such that for all* $j \in \mathcal{A}$*:*

*4. If* $[\mathrm{B}]_j\varphi \in \neg\mathcal{T}$ *and* $[O]_j\psi \in \neg\mathrm{PT}(\varphi)$, *then* $[O]_j\psi \in \neg\mathcal{T}$, *where* $O \in \{\mathrm{B}, \mathrm{G}, \mathrm{I}\}$.

5. *If* $[\mathrm{B}]_j\varphi \in \neg\mathcal{T}$ *and* $[O]^+_G\psi \in \neg\mathrm{PT}(\varphi)$ *with* $j \in G$*, then* $[O]_j\psi \in \neg\mathcal{T}$ *and* $[O]_j[O]^+_G\psi \in \neg\mathcal{T}$*.*

A [B]-*expanded tableau for a formula* $\varphi$ is a minimal [B]-expanded tableau $\mathcal{T}$ such that $\varphi \in \mathcal{T}$. Nodes of pre-tableau which are [B]-expanded tableaux are called *states* and all the remaining nodes are called *internal nodes*.

Algorithm 1 consists of three main stages. Firstly, a pre-tableau for an input formula is constructed, starting from a single node `root`. Next, the nodes of the pre-tableau are marked. Lastly, the satisfiability of the input formula is decided on the basis of how the `root` node is marked.

During the stage of pre-tableau construction, successors of internal nodes are created, with labels extending those of their predecessors, until states or nodes with blatantly inconsistent labels are obtained. During this stage, formulas violating properties of [B]-expanded tableau, called *witnesses*, are sought and appropriate extensions to labels of the nodes are made to remove the violation. Given an internal node $n$ we will use $SS(n)$ to refer to the set of states that are descendants of $n$ and all ancestor nodes on the path from $n$ to them are internal nodes. Given a state $n$ we define $SS(n) = \{n\}$. The procedure of forming states is standard and we omit it here (see [7] for details).

Successors of state $s$ are created for formulas of the form $\neg[O]_j\psi$ (with $O \in \{\mathrm{B},\mathrm{G},\mathrm{I}\}$) and $[O]_j\psi$ (with $O \in \{\mathrm{B},\mathrm{I}\}$) in $L(s)$. A node $n$ created as a successor of state $s$ for a formula $\psi$ is called a $\psi$*-successor* of $s$. A $[O]_j$*-successor* of state $s$, where $i \in \mathcal{A}$, is any $\psi$-successor of $s$ with $\psi$ being either of the form $\neg[O]_j\xi$ or $[O]_j\xi$. The procedure of forming $[\mathrm{G}]_j$-successors is standard (see for example [10]). The procedure of forming $[\mathrm{I}]_j$-successors is the same as in the case of restriction $\mathbf{R}_2$, and we omit it here (see [7] for details).

The procedure of creating $[\mathrm{B}]_j$-successors extends the standard method for the modalities associated with the system $\mathrm{KD45}_n$ (c.f. [10, 9]). In the standard method creation of $\neg[\mathrm{B}]_j\psi$-successor of a states is blocked if there is a $R^{[\mathrm{B}]}_j$-Predecessor of that state that already has a $\neg[\mathrm{B}]_j\psi$-successor with the same label. Additionally, in certain situations creation of $\neg[\mathrm{B}]_j[\mathrm{B}]^+_G\psi$-successors can be stopped and Function 3 may be used for checking if the label of the $\neg[\mathrm{B}]_j[\mathrm{B}]^+_G\psi$-successor is satisfiable. If it is decided to be unsatisfiable, then the state is marked `unsat`. Otherwise, the decision on how the state should be marked depends on the other successors and it is done by the Procedure 5 of marking nodes, described below.

The successor relations between nodes introduced above extend to successor relations between subsequent states in a pre-tableau (we will distinguish them from the successor relations between nodes by using capital 'S' in the name). So for example $t$ is a $[O]_j$-Successor of $s$ if $s$ and $t$ are states and there is a $[O]_j$-successor $n$ of $s$ such that $t \in SS(n)$. States and successor relations between them provide the basis for a tableau that can be constructed from the pre-tableau.

**Function 3:** DecideSatisfiabilityAux

**Input**: A formula $\psi$, $G \subseteq \mathcal{A}$, $j \in G$ and a set of formulas $\Phi$ with $\{[\mathrm{B}]_j\psi, \psi\} \subseteq \Phi$

**Output**: A decision whether $\Phi^{\neg[\mathrm{B}]_j}([\mathrm{B}]^+_G\psi)$ is satisfiable or not

Let $H = \mathrm{ag}\Big(\big(\Phi \sqcap [\mathrm{B}]^+_{\{j\}}\big) \cup \{\neg[\mathrm{B}]^+_G\psi\}, [\mathrm{B}]^+\Big)$;

Construct a pre-tableau consisting of single node `root`, with $L(\texttt{root}) = \big(\Phi/[\mathrm{B}]^+_H\big) \cup \{\psi\}$ and all successor relations being empty;

Let $Z$ be the set of all leaves of the pre-tableau with labelling sets that are not blatantly inconsistent;

**repeat**

  **if** *there is* $n \in Z$ *such that* $n$ *is not a state and* $\xi \in L(n)$ *is a witness to that* **then**

    `FormState2`($n$,$\xi$);

**until** *no change occurred*;

**foreach** $n \in Z$ *such* $n$ *is a state* **do**

  **if** `DecideSatisfiability3`($\bigwedge L(n)$) = `sat` **then**

    **foreach** $k \in H$ **do**

      **if** `Reachable`($(\Phi/[\mathrm{B}]^+_H) \cup \{\psi\}$, $(\Phi \sqcap j) \setminus ((\Phi \sqcap [\mathrm{B}]_j) \cup (\Phi \sqcap \neg[\mathrm{B}]_j))$, $L(n)$, $H$, $\{k\}$, $j$, $|L(n)|$) **then**

        **if** `DecideSatisfiability3`($\bigwedge \big(L^{[\mathrm{B}]_k}(n) \cup \Phi/[\mathrm{B}]^+_H \cup \{\sim\psi\}\big)$) = `sat` **then**

          **return** `sat`;

    **if** `Reachable`($(\Phi/[\mathrm{B}]^+_H) \cup \{\psi\}$, $(\Phi \sqcap j) \setminus ((\Phi \sqcap [\mathrm{B}]_j) \cup (\Phi \sqcap \neg[\mathrm{B}]_j))$, $L(n)$, $H$, $\varnothing$, $j$, $|L(n)|$) **then**

      **foreach** $k \in G \setminus H$ **do**

        **if** `DecideSatisfiability3`($\bigwedge \Big(L^{[\mathrm{B}]_k}(n) \cup \Phi^{[\mathrm{B}]^+_{H\cup\{k\}}} \cup \{\sim\psi\}\Big)$) = `sat` **then**

          **return** `sat`;

      **foreach** $k \in G \setminus H$ **do**

        **if** `DecideSatisfiability3`($\bigwedge \big(L^{[\mathrm{B}]_k}(n) \cup \Phi^{[\mathrm{B}]^+_{H\cup\{k\}}} \cup \{\psi, \neg[\mathrm{B}]^+_G\psi, [\mathrm{B}]_k\psi, \neg[\mathrm{B}]_k[\mathrm{B}]^+_G\psi\}\big)$) = `sat` **then**

          **return** `sat`;

**return** `unsat`;

**Function 4:** Reachable

**Input**: Three sets of formulas $\Phi_1$, $\Psi$ and $\Phi_2$, $H \subseteq \mathcal{A}$, $F \subseteq H$, $p \in H$, and $K \geq 0$

**Output**: A decision whether there exists a satisfiable set of formulas $\Gamma \in \mathcal{SS}(\Phi_1)$ such that $\Psi \subseteq \Gamma$ and $\Phi_2$ is reachable from $\Gamma$ in $\mathcal{G}_H(\Phi_1)$ in at most $2^K - 1$ steps with a path $\Gamma_0, \ldots, \Gamma_n$ such that if $n = 0$, then $p \notin F$ and if $n \geq 1$, then there exists $j_n \in H \setminus F$ such that $\Gamma_{n-1}$ and $\Gamma_n$ are connected with $j_n$. The algorithm is always used with $F$ being either $\varnothing$ or containing exactly one element.

```
if p ∉ F then
    Construct a pre-tableau consisting of single node root with L(root) := Φ1 ∪ Ψ and all
    successor relations being empty;
    Let Z denote the set of all leaves of the pre-tableau with labelling sets that are not
    blatantly inconsistent;
    repeat
        if there is n ∈ Z such that n is not a state and ξ ∈ L(n) is a witness to that then
            FormState2(n,ξ);
    until all nodes of Z are states;
    foreach n ∈ Z such that n is a state do
        if L(n) = Φ2 then
            return
        true;
if K = 0 then
    return false;
else
    Construct a pre-tableau consisting of single node root with L(root) := Φ1 and all
    successor relations being empty;
    Let Z be the set of all leaves of the pre-tableau with labelling sets that are not
    blatantly inconsistent;
    repeat
        if there is n ∈ Z such that n is not a state and ξ ∈ L(n) is a witness to that then
            FormState2(n,ξ);
    until all nodes of Z are states;
    foreach n ∈ Z do
        if DecideSatisfiability3(⋀ L(n)) = sat then
            if Reachable(Φ1,Ψ,L(n),H,∅,p,K − 1) then
                foreach j ∈ H do
                    if Reachable(Φ1,L(n) ⊓ j,Φ2,H,F,j,K − 1) then
                        return true;
    return false;
```

Given a set of formulas $\Phi$, we define the following sets:

$$\Phi \sqcap [O]_j = \{[O]_j\psi : [O]_j\psi \in \Phi\}, \qquad \Phi \sqcap \neg[O]_j = \{\neg[O]_j\psi : \neg[O]_j\psi \in \Phi\},$$

$$\Phi \sqcap j = \bigcup_{O\in\{\mathrm{B,G,I}\}} ((\Phi \sqcap [O]_j) \cup (\Phi \sqcap \neg[O]_j)\,, \qquad \Phi/[O]_j = \{\psi : [O]_j\psi \in \Phi\},$$

$$\Phi/[\mathrm{B}]^+_G = \{\psi : [\mathrm{B}]^+_H\psi \in \Phi \text{ and } G \subseteq H\},$$

$$\Phi \sqcap [\mathrm{B}]^+_G = \{[\mathrm{B}]^+_H\psi : [\mathrm{B}]^+_H\psi \in \Phi \text{ and } G \subseteq H\},$$

$$\Phi^{[\mathrm{B}]^+_G} = (\Phi/[\mathrm{B}]^+_G) \cup (\Phi \sqcap [\mathrm{B}]^+_G).$$

Labels of successors of states are defined as follows:

$$\Phi^{[\mathrm{B}]_j} = (\Phi/[\mathrm{B}]_j) \cup (\Phi \sqcap j), \qquad \Phi^{[\mathrm{G}]_j} = (\Phi/[\mathrm{G}]_j) \cup \Phi^{[\mathrm{I}]_j},$$

$$\Phi^{[\mathrm{I}]_j} = (\Phi/[\mathrm{I}]_j), \qquad \Phi^{\neg[O]_j}(\psi) = \{\neg\psi\} \cup \Phi^{[O]_j}.$$

Given a state $s$ we will write $L^{[O]_j}(s)$ instead of $(L(s))^{[O]_j}$ and $L^{\neg[O]_j}(s,\psi)$ instead of $(L(s))^{[O]_j}(\psi)$.

Given a formula $\psi$, $G \subseteq \mathcal{A}$, $j \in G$ and a set of formulas $\Phi$ such that $\{[\mathrm{B}]_j\psi, \psi\} \subseteq \Phi$ as an input, Function 3 decides whether the set $\Phi^{\neg[\mathrm{B}]_j}([\mathrm{B}]^+_G\psi)$ is satisfiable or not. To describe the idea of this algorithm, let $\Psi_1$ and $\Psi_2$ be sets of formulas. Given $k \in \mathcal{A}$, we say that $\Psi_1$ and $\Psi_2$ are *connected with* $k$ if $\Psi_1 \sqcap k = \Psi_2 \sqcap k$. Moreover, given a set $H \subseteq \mathcal{A}$, we say that $\Psi_1$ and $\Psi_2$ are $H$*-connected* if they are connected with some $k \in H$. Let $\Gamma$ be a set of formulas and let $\mathcal{SS}(\Gamma)$ be the set of all minimal sets of formulas containing $\Gamma$ as a subset that are [B]-expanded tableaux. Given $H \subseteq \mathcal{A}$, let $\mathcal{G}_H(\Gamma) = (V,E)$ be an undirected graph such that $V$ consists of all elements $\Psi \in \mathcal{SS}(\Gamma)$ such that Algorithm 1 returns `sat` on input $\bigwedge \Psi$ and for all $(\Psi_1, \Psi_2) \in V \times V$, $(\Psi_1, \Psi_2) \in E$ iff they are $H$-connected. A *path* in $\mathcal{G}_H(\Gamma)$ is a sequence $\Gamma_0, \ldots, \Gamma_n$ of elements of $V$ such that for all $1 \leq i \leq n$, $\Gamma_{i-1}$ and $\Gamma_i$ are $H$-connected. The *length of path* $\Gamma_0, \ldots, \Gamma_n$ is $n$. Given a path $\Gamma_0, \ldots, \Gamma_n$ of length $n \geq 1$ in $\mathcal{G}_H(\Gamma)$ we call a sequence $j_1, \ldots, j_n$ of elements from $H$ such that for each $1 \leq i \leq n$, $\Gamma_{i-1}$ and $\Gamma_i$ are connected with $j_i$, a *sequence associated with path* $\Gamma_0, \ldots, \Gamma_n$. If $n = 0$, then the sequence associated with the path is the empty sequence $\varepsilon$. Given two sets of formulas $\Psi_0$ and $\Psi_1$, we say that $\Psi_1$ is *reachable from* $\Psi_0$ in $\mathcal{G}_H(\Gamma)$ (in $n$ steps) iff there exists a path $\Gamma_0, \ldots, \Gamma_n$ in $\mathcal{G}_H(\Gamma)$ such that $\Psi_0 = \Gamma_0$ and $\Psi_1 = \Gamma_n$.

Let $\mathrm{ag}(\psi) = G$ (if $\psi$ is of the form $[\mathrm{B}]^+_G\xi$ or $\neg[\mathrm{B}]^+_G\xi$) and $\mathcal{A} \cup \{\omega\}$ (otherwise), where $\omega \notin \mathcal{A}$. Given a set of formulas $\Phi \neq \varnothing$ we define $\mathrm{ag}(\Phi) = \bigcap_{\psi\in\Phi} \mathrm{ag}(\psi)$ (if $\Phi \neq \varnothing$) and $\mathcal{A} \cup \{\omega\}$ (otherwise). Notice that $\omega \in \mathrm{ag}(\Phi)$ implies that there are no formulas of the form $[\mathrm{B}]^+_G\xi$ nor $\neg[\mathrm{B}]^+_G\xi$ in $\Phi$. Also, when formulas are removed from $\Phi$, then $\mathrm{ag}(\Phi)$ either remains unchanged or increases.

To decide the satisfiability of $\bigwedge \Phi^{\neg[\mathrm{B}]_j}([\mathrm{B}]^+_G\psi)$, Function 3 checks whether there exist two sets of formulas $\{\Psi_0, \Psi_1\} \subseteq \mathcal{SS}((\Phi/[\mathrm{B}]^+_H) \cup \{\psi\})$, with

$$H = \mathrm{ag}\Big(\Big(\Phi \sqcap [\mathrm{B}]^+_{\{j\}}\Big) \cup \{\neg[\mathrm{B}]^+_G\psi\}\Big)\,,$$

such that $\Psi_1$ is reachable from $\Psi_0$ in $\mathcal{G}_H((\Phi/[\mathrm{B}]^+_H) \cup \{\psi\})$, $\Psi_0$ and $\Phi$ are connected with $j$ and to check the satisfiability of $\bigwedge \Psi_1$ a set with either lower modal depth or with larger set $\mathrm{ag}(\cdot)$ has to be considered (see [7] for details).

To check reachability, Function 4 is used. Given sets of formulas $\Phi_1$, $\Psi$ and $\Phi_2$, sets $H \subseteq \mathcal{A}$ and $F \subseteq H$, $p \in H$ and $K \geq 0$, Function 4 checks if there

**Procedure 5:** MarkNodes

```
repeat
    if n is an unmarked state then
        if n has a successor that is marked unsat or undec then
            Mark n unsat;
        else if n contains an unresolved formula then
            Mark n undec;
        else if n does not have an unmarked successor then
            Mark n sat;
    else if n is an unmarked internal node then
        if L(n) is blatantly inconsistent or all successors of n are marked unsat then
            Mark n unsat;
        else if there exists a successor of n marked sat then
            Mark n sat;
        else if n does not have an unmarked successor then
            if there exists a formula ¬[B]^+_G ψ ∈ L(n) which is unresolved in n and
            B(n) = {n} then
                Mark n unsat;
            else
                Mark n undec;
until no new node marked;
```

exists a set of formulas $\Gamma \in \mathcal{SS}(\Phi_1)$ such that Algorithm 1 returns `sat` on input $\bigwedge \Gamma$ and $\Phi_2$ is reachable from $\Gamma$ in $\mathcal{G}_H(\Phi_1)$ in at most $2^K - 1$. The idea of the algorithm is based on Savitch's algorithm for checking reachability in graph that uses quadratic logarithmic space with respect to $|V|$ (c.f. [17]). Notice that all the sets in $\mathcal{SS}((\Psi/[\mathrm{B}]^+_H) \cup \{\psi\})$ have the same number of elements and if $\Gamma \in \mathcal{SS}((\Psi/[\mathrm{B}]^+_H) \cup \{\psi\})$, then $|\mathcal{SS}((\Psi/[\mathrm{B}]^+_H) \cup \{\psi\})| \leq 2^{|\Gamma|}$. Thus to check reachability in $\mathcal{G}_H((\Phi/[\mathrm{B}]^+_H) \cup \{\psi\})$ it is enough to check whether there is reachability in at most $2^{|\Gamma|} - 1$ steps, where $\Gamma \in \mathcal{SS}((\Psi/[\mathrm{B}]^+_H) \cup \{\psi\})$.

During the stage of marking nodes, nodes of the pre-tableau are marked either `sat`, `unsat`, or `undec`. A node $n$ being marked `undec` indicates that satisfiability of $\bigwedge L(n)$ could not be decided due to existence of a formula of the form $\neg[\mathrm{B}]^+_G\psi$ in its label for which an appropriate sequence of states was not constructed yet. We call such formulas *unresolved* in a given node. In the case of $n$ being a state a formula $\neg[\mathrm{B}]^+_G\psi \in L(n)$ is called *unresolved in n* if $n$ is a $\neg[\mathrm{B}]_j[\mathrm{B}]^+_G\psi$-Successor with $j \in G$, $[\mathrm{B}]_k[\mathrm{B}]^+_G\psi \in L(n)$, for all $k \in G \setminus \{j\}$ and $[\mathrm{B}]_k\psi \in L(n)$, for all $k \in G$. In the case of $n$ being an internal node, a formula $\neg[\mathrm{B}]^+_G\psi \in L(n)$ is called unresolved in $n$ if there is $m \in SS(n)$ such that it is unresolved in $m$. Notice that by this definition any node can contain at most one unresolved formula.

As we show in [7], for any formula from $\mathcal{L}^{\mathrm{T}}_{\mathbf{R_1}}$ Algorithm 1 stops and uses polynomial space with respect to the size of the input formula to run. Moreover it returns `sat` if and only if the input formula is satisfiable. Hence it is valid. Thus the satisfiability problem for formulas from $\mathcal{L}^{\mathrm{T}}_{\mathbf{R_1}}$ is in PSPACE. It is also PSPACE-hard, even if input formulas have modal depth bounded by 2. Hardness can be shown by constructing formulas whose models encode the computation of a given polynomial space bounded deterministic Turing machine $T$ on the given input $I$. Modal depth of these formulas is equal to 2 (see [7] for details).

## 5 Conclusions

In this paper we studied the complexity of the satisfiability problem of BDI logic with restricted modal context. We focused on the effect of mixed axioms of introspection on the complexity of the problem. We showed that the restriction leads to PSPACE-completeness of the problem but existence of the introspection axioms leads to PSPACE-hardness of the problem even if modal depth of formulas is bounded by 2. An interesting feature of the studied logic is PSPACE solvability of the satisfiability problem even though the formulas may require models exponentially deep with respect to their size.

Interesting questions arising from the presented research are as follows. Firstly, what modal context restriction, when applied to the logics with introspection axioms, would lead to NPTIME-completeness of the satisfiability problem, when combined with modal depth restriction (like in the case of logics without mixed axioms [8]). Secondly, what we can say about the minimality of the proposed restriction. We know from this paper that the restriction proposed is sufficient to get PSPACE solvability of the satisfiability problem. But is it necessary? Can we allow for more, in terms of modal context, and obtain the same complexity characteristics? We reserve these questions for further research.

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