# MAGIT Exercises, Series 4

Exercise 1.

Let $X$ be a topological space. Prove that the sequence of maps

$$... \to S_2(X) \to S_1(X) \to S_0(X) \to 0$$

defined in lecture IV is a complex.

Exercise 2.

Compute the singular homology groups of a point.

Exercise 3.

Prove that if $f : X \to Y$ is a continuous map of topological spaces then we have an induced map of singular chain complexes $S_\bullet(X) \to S_\bullet(Y)$. Prove that this induces a homomorphism of the corresponding singular homology modules. Check that in this way we get a functor $H_n(\cdot, R) : Top \to Mod{-}R$.

Exercise 4.

Show that the inverse limit functor $\varprojlim$ on diagrams $\mathbb{N} \to \mathrm{Ab}$ is not right exact.

Exercise 5.

Let $\mathcal{A}$ be an abelian category. Show that the category of short exact sequences in $\mathcal{A}$ need not be abelian. Is the category of chain complexes in $\mathcal{A}$ that are zero in degrees $< 0$ abelian? Is the category of chain complexes in $\mathcal{A}$ that are zero in large negative degrees abelian?