**Weighted Inequalities, Homework II. Deadline: 16.06.2019**

*Throghout, we assume that all martingales have continuous paths.*

**1.** Prove that for any probabilistic weight $W$ (i.e., any nonnegative, uniformly integrable martingale) we have

$$\mathbb{E}\exp W^* \leq e\mathbb{E}\exp W.$$

**2.** Let $X$ be an arbitrary uniformly integrable martingale and let $W$ be an $A_1$ weight. Prove that

$$\mathbb{E}[X,X]W \leq C[W]_{A_1}\mathbb{E}X^2W$$

for some universal constant $C$.

**3.** Let $X$ be an arbitrary uniformly integrable martingale and let $W$ be an $A_2$ weight. Prove that

$$\mathbb{E}X^2W \leq C[W]_{A_2}^{\alpha}\mathbb{E}[X,X]W$$

for some universal constants $C$ and $\alpha$.

**4.** Let $T^{\mathcal{S}}$ be a dyadic shift associated with some sparse family $\mathcal{S}$ of dyadic cubes in $\mathbb{R}^d$. Prove that for any $1 < p < \infty$ we have

$$\|T^{\mathcal{S}}\|_{L^p(\mathbb{R}^d)\to L^p(\mathbb{R}^d)} \leq \frac{2p^2}{p-1}.$$