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A Report on the PhD Dissertation by Mr Tomasz Was

The PhD dissertation of Mr Was is titled “Axiomatization of the Walk-Based Centrality Measures”. It builds on three papers that were published in leading AI conferences (AAAI/IJCAI): specifically, the dissertation consists of an introduction (which includes a detailed survey of the related work), a preliminaries section, and three chapters, each of which is an extended version of a conference paper.

Each of the three technical chapters provides an axiomatization of a graph centrality measure or a family of such measures: Chapter 3 focuses on PageRank (and provides the first axiomatization of this very important measure), Chapter 4 considers Random Walk Decay Centrality and Personalized Decay Centrality, and Chapter 5 offers an axiom system for feedback centralities, which is able to distinguish among several well-known centralities in this class.

More specifically, Chapter 3 formulates six axioms – Node Deletion, Edge Deletion, Edge Multiplication, Edge Swap, Node Redirect and Baseline. These axioms are quite attractive in that they speak of the behaviour of centrality measures under simple graph transformations, without directly referring to walks on graphs. It first shows that PageRank satisfies all of these axioms, and then establishes that it is the only centrality measure to do so. The argument proceeds by first characterizing the behaviour of any centrality measure that satisfies these six axioms on a very simple graph, which consists of one directed edge; the argument is then extended to the so-called k -arrow graphs (graphs with a single source and k sinks), then to cycle-free graphs and finally to arbitrary graphs. Moreover, the author shows that all six axioms are independent; to this end, for each of these axioms he constructs a centrality measure that fails this axiom, but satisfies the other five. The chapter concludes by exploring the differences between PageRank and other closely related centrality measures.

Chapter 4 identifies a feature of PageRank that may be undesirable from the perspective of strategic behaviour: namely, a node can increase its own centrality by changing its outgoing links. The author then puts forward a variant of this centrality measure, which he calls Random Walk Decay Centrality, which does not have this weakness. He then provides an axiomatic characterization for this new centrality measure, as well as an alternative axiomatization of PageRank and another centrality measure, namely, Personalized Decay Centrality; these axiomatizations are obtained from the one for Random Walk Decay Centrality by modifying one of the axioms. In contrast with the axioms in Chapter 3, the axiom systems in this chapter contain an axiom that specifically requires the centrality measure to be based on random walks. While in some ways this makes the characterization result less surprising, it still turns out to be very demanding from a technical perspective: the proof proceeds by considering non-trivial graph-theoretic concepts, such as cactus graphs. The thesis then offers a useful comparison between PageRank and Random Walk Decay, which takes into account strategyproofness and diversity of incoming edges.



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Chapter 5 is the most ambitious of the three, in that it introduces a system of axioms that can capture four centrality measures in a systematic manner: there are three axioms shared by all four measures, and two pairs of axioms such that each centrality measure is obtained by choosing one element from each pair. The axioms are simple and intuitively appealing; however, just as in the previous chapters, the proofs are far from trivial.

Overall, the thesis forms a coherent body of outstanding work: the quality of writing is excellent, the research topic is well-motivated, and the proofs are highly non-trivial, yet clearly explained. The thesis offers useful examples that are very helpful to the reader, and provides intuitive explanations for the technical results. Perhaps the only shortcoming is that it does not offer a conclusion or directions for future work; it would have been useful to include what the author sees as the most promising research directions in this field.

I deem the thesis as sufficient to grant a PhD. Indeed, given the quality of the work, I believe that the PhD may be granted with an honorary distinction.

A handwritten signature in black ink, appearing to be 'JA' or similar initials, written over a horizontal line.

May 11, 2022