In the second part of the talk, continuing our discussion concerning continuity of semigroups in the Calkin algebra $Q(H)$, we shall present a sketch of proof of the characterization of SOT-continuity of a dyadic semigroup $(q(t))_{t \in D} \subset Q(H)$ induced by the zero element of the extension group $\text{Ext}(X)$, where $X$ is an admissible compact metric space. Namely, if $X = \lim_{\leftarrow} X_n$, where $X_n = \exp(2^{-n}Z)$, $Z \subset \mathbb{C}$ is a closed set lying in some left half-plane, and $\pi_n: X \to X_n$ stands for the $n^{th}$ projection, for each $n \in \mathbb{N}$, then for the semigroup $(q(t))_{t \geq 0}$ induced by $\Theta \in \text{Ext}(X)$ the following conditions are equivalent:

1. $(q(t))_{t \in D}$ is strongly continuous with respect to a fixed Calkin's representation $\gamma: Q(H) \to B(H)$;
2. $(q(t))_{t \in D}$ is strongly continuous with respect to all Calkin's representations;
3. $\lim_{n \to \infty} \pi_n(\xi) = 1$ for every $\xi \in X$.

Next, we shall discuss the lifting problem for $C_0$-semigroups $(q(t))_{t \geq 0}$ in $Q(H)$, i.e. the question whether one can find a (strongly continuous) operator semigroup $(Q(t))_{t \geq 0}$ in $B(H)$ such that $\pi Q(t) = q(t)$ for $t \geq 0$. Let $A$ be the generator of $(q(t))_{t \geq 0}$. By using Milnor’s exact sequence, we show that if each $q(t)$ has a normal lift, then the question whether the extension $\Gamma$ induced by $(q(t))_{t \geq 0}$ is trivial reduces to the question whether the corresponding first derived functor $\lim_{\leftarrow} \text{Ext}_2(X_n)$ for the suspensions of $X_n = \exp(2^{-n}\sigma(A))$ vanishes. With the aid of the CRISP property and Kasparov’s Technical Theorem, we propose two results which provide some geometric conditions on $\sigma(A)$ guaranteeing splitting of $\Gamma$.

The talk will be mostly based on the preprint: *Compact perturbations of operator semigroups*, arXiv:2203.05635v2

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