Ground state, bound state, and normalized solutions to semilinear Maxwell and Schrödinger equations

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During this talk, I will present the results contained in my Ph.D. thesis. The first half concerns unconstrained curl-curl problems (arising from Maxwell's equations) and related non-autonomous Schrödinger equations, while the second half is about L^2-constrained problems.

Concerning the first half, its first part is based on [3] and deals with

$$\nabla \times \nabla \times \mathbf{U} = f(x, \mathbf{U}), \quad \mathbf{U}\colon \mathbb{R}^3 \to \mathbb{R}^3 \tag{1}$$

in the Sobolev non-critical case. I will show how a ground state solution and infinitely many bound state solutions are obtained, with the aid of an abstract critical point theory. The second part is based on [1] and considers (1) in a cylindrically symmetric setting together with the related equation

$$-\Delta u + \frac{a}{|y|^2} u = h(x, u), \quad u\colon \mathbb{R}^N \to \mathbb{R}, \tag{2}$$

where y consists of the first K components of $x \in \mathbb{R}^N$, $N > K \geq 2$, and $a > -\frac{(K-2)^2}{4}$. A first result is the rigorous equivalence between (1) and (2) with $a = 1$. The Sobolev non-critical and critical cases are dealt with: I will focus on the latter, showing the existence of an unbounded sequence of solutions.

Regarding the second half, it is mainly concerned with

$$\begin{cases} -\Delta u_j + \lambda_j u_j = \partial_j F(u) \\ \int_{\mathbb{R}^N} u_j^2 \, \mathrm{d}x = \rho_j^2 \\ (\lambda_j, u_j) \in \mathbb{R} \times H^1(\mathbb{R}^N) \end{cases} \qquad \forall j \in \{1, \dots, K\}$$

in the Sobolev subcritical case, where $N, K \geq 1$ may be subjected to further restrictions, $\rho \in]0, \infty[^K$ is given a priori, and λ is part of the unknown. The first part is based on [4] and deals with the L^2 subcritical and critical cases, i.e., when the energy functional is bounded from below for all or some values of ρ; the second one is based on [2] and concerns the L^2 supercritical case, i.e., when the energy functional is unbounded from below. The last part, which I will just sketch, contains new results about L^2-constrained problems of the forms (1) and (2).

References

[1] M. Gaczkowski, J. Mederski, J. Schino, *Multiple solutions to cylindrically symmetric curl-curl problems and related Schrödinger equations with singular potentials*, arXiv:2006.03565v2.

[2] J. Mederski, J. Schino: *Least energy solutions to a cooperative system of Schrödinger equations with prescribed L^2-bounds: at least L^2-critical growth*, arXiv:2101.02611v1.

[3] J. Mederski, J. Schino, A. Szulkin: *Multiple solutions to a semilinear curl-curl problem in $\mathbb{R}^3$*, Arch. Ration. Mech. Anal. **236** (2020), no. 1, 253–288.

[4] J. Schino: *Normalized ground states to a cooperative system of Schrödinger equations with generic L^2-subcritical or L^2-critical nonlinearity*, arXiv:2101.03076.