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On the determination of extended classes and some other problems in Java programming language

PhD dissertation

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Preface

This thesis is a part of a more general work concerning the problem of identifier binding in Java. Java is concerned here as most general of languages admitting both inheritance and inner classes. The problem of identifier binding consist in assigning the proper meaning to the occurrences of identifiers either at compile time or at run time. At the compile time we distinguish between defining and applied occurrences. The meaning of a defining occurrence of an identifier is the definition which the identifier is connected with. Each applied occurrence of an identifier must be bound to the proper defining occurrence. If we restrict our problem to the variable identifiers then at the compile time we will be interested in types of these variables. But at runtime we will be interested first of all in the location of the variable in the memory, and in most cases in the value of the variable. Since a type of the variable can be read from the declaration of this variable we can say that at compile time each variable is bound to its declaration while at runtime each variable is bound to the location in memory, which was reserved specially for this variable.

Now consider class identifiers. During compilation we are interested in binding each applied occurrence of class identifier to the declaration of the corresponding class. Then during runtime, while creating an object of a class K, we are looking for an environment which will be used for binding the nonlocal variables occurrences within methods of the class K.

Unexpectedly it turned out that the problem of binding class identifiers is especially difficult during the compilation of a program. The source of the difficulty is the fact that in Java the extended class may be denoted not only by stand alone class identifier but also by qualified type, which is a sequence of class identifiers separated by dots. We must deal here with the mutually recursive definition of binding. Binding of qualified types uses the inheritance function, while the definition of this function uses the binding of qualified type. The problem reduces to inventing the proper order of computing of the inheritance function as it will be shown in sections 3.1 and 4.1. The additional problems are checking of well-formedness of a program and the detection and correction of eventual errors.

The structure of the thesis is as follows. The first chapter “Introduction” exposes the problem through a series of examples. The next chapter formulates the problem of identification of direct superclass in the language of a graph of classes of a given program. Chapter 3 presents a non-deterministic algorithm and the proof of correctness. In the subsequent chapter a deterministic algorithm is presented and analyzed. Chapter 5 analyzes earlier work on the problem and in particular the work of A.Igarashi and B.Pierce. The structure of the chapter is as follows: The section 5.2 presents the calculus of Igarashi and Pierce. Examples are given showing that the calculus is inconsistent. We are giving a simple remedy to it. In the section 5.3 we translate the inference rules of the IPET calculus in such a way that the phrase "the meaning of the type X in the environment P is the class T" is now expressed by the formula \( \text{bind}(X \text{ in } P) = T \) and we show that the function \( \text{bind} \) calculated by the algorithm of the section 3.1 is a model of the IPET calculus. Next section 5.4 shows that there is another concept \( \text{Bind} \) of binding function and that \( \text{Bind} \) is modeling the IPET calculus as well. In the section 5.5 we show that the intersection of two
models needs not be a model. In this way one should abandon the hope that by adding the, metatheoretic, phrase, "take the least model" of the models of IPET calculus the task of identifying the direct superclass of classes of a Java program may be completed.

The last chapter exposes the application of the results obtained above during compilation and latter during the execution of programs.
Chapter 1

Introduction

1.1 Motivation

In this thesis we address the problem of determining direct superclasses. This problem becomes hard to answer when an object-oriented programming language admits inner classes. Inner classes of Simula67 [9], Loglan’82 [1] [3], BETA [2] and Java (Java 1.2 and subsequent versions) [6], [7], [8] introduce block structure for class declarations. Consider an arbitrary Java-program. The set of classes of the program together with the relation \textit{class A is an inner class of class B} is a forest structure. (The roots of trees are top level classes.) Therefore several classes of a program may be given the same name (See the examples 1 and 2). Classes use the clause \textbf{extends} C, (to be read as \textit{this class inherits from a class named C}). However, the meaning of the name C is not unique. Which of possibly many classes named C is the direct superclass of our class? The consequences of a possible error in inheriting may be dramatic if the author(s) and readers of a program interpret the meaning of inherited classes differently. Obviously, compilers are readers of programs. Therefore we postulate as a matter of course: for every program \(P\), its author and the compilers should identify the direct superclasses in the same way. This implies the necessity of a clear and compact criterion which would guarantee the existence of a solution of the problem of determining direct superclasses, whether the solution was guessed by a programmer in an intuitive way or whether it was computed mechanically by a compiler. To find a solution is so challenging because the Java Language Specification JLS [7] is defining inheritance or superclassing rather implicitly: 1) Inheritance is defined by the help of the binding function which binds applied identifier occurrences in a program to their declaring occurrences. 2) The binding function on the other hand is defined by use of the inheritance. Therefore we need a more formalized specification of the problem and a constructive way of solving it, i.e. an algorithm. The algorithm should be applied as the first step in the static semantic analysis performed by the compiler.

1.2 Four examples

\textbf{Example 1.2.1.} \textit{In the program below there are three classes named B. We can identify them as follows: class B, class \texttt{A$B}, class \texttt{A$D$B}.}

```java
class B { }
class A {
    class B { }
    class C extends B { }
```

1
Which class $\text{inh}(C)$ is the direct superclass of the class $C$? Which class $\text{inh}(E)$ is inherited by the class $E$?
The answer is easy, $\text{inh}(C) = A\$B, \text{inh}(E) = A\$D\$B$. We guessed the first answer following the usual method of static binding that for any applicative occurrence of an identifier finds a declaration of this identifier that is appropriate. The second answer was found in two steps: first we searched a declarative occurrence of $D$ which is $A\$D$, next, we searched a class named $B$ declared inside the class $D$. □

The second example shows that the complexity of the problem is non-trivial.

\begin{example*}
Consider the following classes
\begin{verbatim}
  class D {
    class B { }
  }
  class E extends D.B { }
  //end A

  class E { 
    class G {} // this can be class B or A$B
  }
  //end E

  class A extends B { // this can be class B or A$B
    class C extends D { // this can be class B$D or E$D
      class F extends G {} // this can be class A$E$G or E$G
    } //end C
  }
  //end A

  class B { 
    class D extends E {} // this can be class E or A$E
  } // end B

  class E { 
    class G extends B {} // this can be class B or A$B
    class D {} 
  } // end E
\end{verbatim}
\end{example*}

In this example the function $\text{inh}$ that for every class $K$ associates with it its direct superclass $\text{inh}(K)$ may be defined on $2^6$ possible ways. For we have six clauses $\text{extends}$ and for each clause there are two classes of the name mentioned in it. □

This example shows that searching all possible candidates for $\text{inh}$ function is not a good approach as in general the number of candidates for the mapping $\text{inh}$ may be exponential function of the length of a given Java program. The compiler would be stuck for indefinite amount of time.
The next example shows that the solutions may be counter intuitive.

\begin{example*}
This example shows that the requirement (taken from JLS[7]) “a class may not depend on itself” is essential. The natural, however complicated, requirement that all types mentioned in the extends phrases have some meaning, is not sufficient. Adding a natural, additional requirement that there is no cycle in the inheritance relation does not
help. We show that there exist two different and astonishing functions, candidates for \( inh \).

Two candidate functions: \( inh_1 \) and \( inh_2 \) may be guessed

<table>
<thead>
<tr>
<th></th>
<th>( A )</th>
<th>( B )</th>
<th>( G$\S$D$\C )</th>
<th>( I$\S$C$\D )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( inh_1 )</td>
<td>( G$\S$D$\C )</td>
<td>( G$\S$D )</td>
<td>( G )</td>
<td>( I )</td>
</tr>
<tr>
<td>( inh_2 )</td>
<td>( I$\S$C )</td>
<td>( I$\S$C$\D )</td>
<td>( G )</td>
<td>( I )</td>
</tr>
</tbody>
</table>

Both functions seem to be correct since \( inh_i (K) \) is reestablished by binding the extends type of every class declaration occurrence \( K \). Both functions \( inh_1 \) and \( inh_2 \) satisfy conditions mentioned in JLS [7] in 6.5.5 (later in Problem 2.1.6 formalized as condition \( I_1 \)). Still we feel uneasy because the program would have two different dynamic semantics what is not appropriate for a well-formed program. Which of two is the right inheritance function? Or are both of them wrong with respect to Java’s static semantics? The answer, namely the rejection of well-formedness of program Example 3 due to JLS [7], will be given later by Theorems 3.2.10 and 3.2.20. Example 1.2.3 uncovers the decisive rôle of the dependency relation which JLS [7] is introducing. This example violates the requirement “no class depends on itself” of JLS [7] 8.1.4 (later in Problem 2.1.6 formalized as condition \( I_2 \)).
This implicit condition can not be directly included into a compiler. However, we need a precise criterion, in form of an algorithm, to be used by the constructors of compilers for some code of any length.

The fourth example demonstrates that compilers may compile the same program in different ways. It has little to do with the problem of determining direct superclasses, yet, it shows the scale of disagreement on the meaning of program.

**Example 1.2.4.** We asked several Java programmers to tell what the program would print. The answers came in two classes: "prints 1", "prints 2". Next, we tested the program by five Java compilers {javac, gcj, jikes, kopi, ecj }. The results were in three (!) classes. For one compiler said: The program has an error. In this way an open question arises: Is Java an unambiguous programming language?

```
abstract class AF {
    abstract int f();
    class A { int x = f(); }
}
class B1 extends AF {
    int f() { return 1; }
    class B2 extends AF {
        int f(){return 2;}
        class B extends B1.A {}
    }
}
class Test {
    public static void main( String[] argv) {
        System.out.println(new B1().new B2().new B().x);
    }
}
```

1.3 Discussion

These examples show that the problem of determining direct superclasses is of importance for compiler writers and for programmers as well. Even short programs may create problems in proper determination of their meaning. It seems that the majority of programmers is unaware of subtle problems that can appear during their work with programs. The help provided by the reference book Java Language Specification [7] is clumsy. The book requires that a programmer reads several sections (e.g. 8.1.4 Superclasses and Subclasses, 8.5 Member Type Declarations, 6.5.5 Meaning of Type Name.) before he/she is able to understand what is happening in a given program.

Someone may say: "your examples are unrealistic. The programmers do not write such weird programs". Let us remark that the programs become longer and more complicated than our examples. Compilers must be prepared to detect any possible error in any source code.

Another doubt may appear: "are inner classes needed at all? Some descendants of Simula67 such as SMALLTALK, C++, C# forbid inner classes."
It turns out that the combination of inheritance and inner classes offers many interesting possibilities:

- it allows to obtain most of the effects of multiple inheritance c.f. [12] Chapter 10,
- instead of passing classes as parameters one can extend abstract inner classes which serve as counterparts of formal parameters [13] p.176,
- provides a convenient way to express call back objects [12],
- allows to inherit certain patterns of architecture, e.g. a class pattern of the model-view-controller system can be defined and extended by inheriting classes [1], [12],
- allows to inherit protocols [1] p.112-113,
- enables inheritance of a class put earlier into a tree-like library of classes,
- and many others.
Chapter 2

Formulation of the problem

2.1 Preliminaries

We assume that the reader is accustomed to the programming language Java [7] and the notions of class, top-level class, direct superclass.

In this section we generalize the intuitions which arise from the examples. We begin with the observation that instead of Java-programs it suffices to consider their structures of classes. Let \( P \) be a given Java-program. We add two predefined classes \{\( \text{Root}, \text{Object} \}\). We may assume that \( P \)’s top level classes are contained in the additional class \( \text{Root} \). We assume also that the class \( \text{Root} \) contains the additional class \( \text{Object} \). Now, we strip the

\[ \text{class} \ A \ \text{O}r \ \text{class} \ A \ \text{extends} \ B \ \text{O}r \ \text{* class} \ A \ \text{extends} \ B_1, B_2, ... B_n \]

and the corresponding closing braces \}

In this way one obtains a Java-program which exhibits the structure of all classes. It contains all classes declared in the program and two predefined classes \( \text{Root} \) and \( \text{Object} \). Each user declared class has its name - an identifier introduced by the declaration. Additionally we assume that the name of class \( \text{Object} \) is the identifier Object. Let \( \text{Classes} \) be the set of user defined class declarations occurrences in a program \( P \). The structure of classes is equipped with a partial function \( \text{decl} \) which for every class except \( \text{Root} \) and \( \text{Object} \) points to that class in which class \( K \) is declared, i.e. the textually directly enclosing class. It is easy to observe that the structure \( \langle \text{Classes} \cup \{\text{Root, Object}, \text{decl} \} \rangle \) of classes is a tree. Class \( \text{Root} \) is the root of this tree.\(^2\)

**Definition 2.1.1.** Qualified type, is a finite sequence of class identifiers separated by dots.

The clauses \textit{extends} bring another function defined on the set of user defined classes, namely, for each class except \( \text{Root} \) and \( \text{Object} \) we have a extension type associated to it. For some classes the type is empty (in these cases the keyword \textit{extends} is omitted – it means that that for these classes the direct superclass is the predefined class \( \text{Object} \)), for some other classes the type consists of one class identifier, for other classes the type is a qualified type. If a type consists of just one identifier then it is the name of the direct

\(^1\)i.e. we throw away all instructions and all declarations other than class declarations

\(^2\)There is a bijection between the set of \( \text{Classes} \) and the set \( T \) of finite sequences of names of classes such that a sequence \( s \in T \) iff it is a code of a path leading from a top-level class to a given class. This concept is present already in [11]. For example, \( \text{A.B.F} \) denotes the class \( \text{F} \) declared inside the class \( \text{B} \) which is declared in a top level class \( \text{A} \).
superclass (the directly inherited class). One program may contain many classes of the same name which makes the problem of determining which class is direct superclass of a given class a non-trivial task. Every well-formed program satisfies the local distinctness property which says that every two different directly inner classes directly declared in a class have different names. The property will be useful in our considerations. Now, Java allows the types of length > 1, c.f.\(^\star\). The identifiers occurring in a type are names of classes. The declaring occurrence of class named \(B_1\) should be visible from the place where the class \(A\) is declared and for every \(1 \leq i < n\) the class \(B_{i+1}\) is a member (an attribute) \(^3\) of the class named \(B_i\) (i.e. an inner class of \(B_i\) or an inner class of a direct or indirect superclass of class \(B_i\)).

Let \(P\) be a (stripped) Java program. Sometimes we shall use a formal description of the structure of classes of the program \(P\):

\[
S_P = \langle \text{Classes}, \text{Id}, \text{Types}, \text{decl}, \text{name}, \text{ext}, \text{Root}, \text{Object} \rangle
\]

where

- \(\text{Classes}\) is the set of classes declared (more distinctly: class declaration occurrences) in the stripped program \(P\),
- \(\text{Id}\) is the set of identifiers found in the stripped program \(P\) plus the identifier Object,
- \(\text{Types}\) is the set of types found in the stripped program \(P\) after the keyword \text{extends} extended by a special empty type \(\varepsilon\) see function \text{ext} below,
- \(\text{decl} : \text{Classes} \cup \{\text{Object}\} \rightarrow \text{Classes} \cup \{\text{Root}\}\)
  is the function which for each class \(K \in \text{Classes} \cup \{\text{Object}\}\) returns the textually directly enclosing class of the class \(K\),
- \(\text{name} : \text{Classes} \cup \{\text{Object}\} \rightarrow \text{Id}\)
  is the function that returns the identifier of a given class. The additional class \(\text{Root}\) has no name. For the class \(\text{Object} \ \ \ \text{name}(\text{Object}) = \text{Object}\),
- \(\text{ext} : \text{Classes} \rightarrow \text{Types}\) is the function which for each class \(K \in \text{Classes}\) returns the (extension) type found in its extension clause. If the extension clause is omitted in the declaration of class \(K\) then \(\text{ext}(K) = \varepsilon\). This empty type will denote a special class \(\text{Object}\) as a direct superclass of class \(K\).

Below we list properties of the structure \(S_P\).

- \(\text{decl}(\text{Object}) = \text{Root}\).
- The pair \(\langle \text{Classes} \cup \{\text{Root}, \text{Object}\}, \text{decl} \rangle\) is a tree. The class \(\text{Root}\) is its root.
- If \(\text{decl}(K) = \text{decl}(M)\) then \(\text{name}(K) \neq \text{name}(M)\) or \(K = M\).

Let \(C\) be an identifier. In the remainder of this paper we shall use partial function

\[
.C : \text{Classes} \cup \{\text{Root}\} \rightarrow \text{Classes} \cup \{\text{Object}\}.
\]

Let \(K\) be a class. The expression \(K.C\) denotes a class \(Y\) which is declared within class \(K\) and its name is \(C\), \(K.C\) is defined \(\iff (\exists Y)(\text{decl}(Y) = K \land C = \text{name}(Y))\). The

\(^3\)Attribute\(^7\) is the jargon of Simula67, Loglan’82 and BETA, ”member” is the jargon of SMALLTALK, C++ and Java.
well-definedness of \( C \) follows from the third property listed above.

One can conceive this structure as a graph. The set \( \text{Classes} \) is the set of nodes of the graph. Each node has two attributes associated with it: \textit{name} - the name of the class, \textit{ext} - the type designating its direct superclass. The function \( \text{decl} \) defines the edges of the graph. An example of the graph is shown in Fig. 1.1.

The same graph may be obtained from the SymbolTable - the data structure built by the compiler for the program \( P \). One takes the SymbolTable and throws away (ignores) all irrelevant information about declarations of variables, methods, constructors, etc., only information about classes is retained.

Speaking informally, for a given structure \( S \) the problem is to obtain a total function \( \text{inh} \), (or equivalently, a set of edges of colour \( \text{inh} \)), which for every given class \( K \in \text{Classes} \) returns the direct superclass of class \( K \) or to assure that such a function does not exist, signalling that the class structure \( S \) is not a (static semantically) correct one. Fig. 1 has continuous edges - edges showing the \( \text{decl} \) function and dashed edges - edges showing \( \text{inh} \) function.

In the sequel we shall use the following notations. Let \( f \) be a partially defined mapping \( f : X \rightarrow X \). An \( i \)-th iteration of the function \( f \) is defined by induction

\[
  f^0(x) = x \quad f^{i+1}(x) = f(f^i(x)).
\]

The notation \( f^* \) denotes zero or more iterations of the function \( f \), while \( f^+ \) denotes one or more iterations of the function \( f \). Let \( Y \) be a subset of the set \( X \). Notation \( f\mid_Y \) denotes the restriction of the function \( f \) to the set \( Y \). Before we specify the problem, we need definitions of a partial function \( \text{bind} \), which is based on a given function \( \text{inh} \), partially defined on a subset of \( \text{Classes} \).

Below we shall give an inductive definition of the partial function

\[
  \text{bind} : \text{Types} \times \text{Classes} \cup \{\text{Root}\} \rightarrow \text{Classes} \cup \{\text{Object}\}
\]

which to a given pair \( \langle \text{type } T, \text{class } C \rangle \) associates a class \( D \). An equation \( \text{bind}(T \text{ in } C) = D \) reads informally as: the meaning of type \( T \) inside the class \( C \) is the class \( D \), or in other words inside the class \( C \) the type \( T \) is bound to the class \( D \). Note that the same type \( T \) may have a different meaning inside another class \( C' \).

\begin{definition}

A1) (base of induction 1) For any class \( K \) the meaning of the empty type \( \varepsilon \) is bound to Object. We define

\[
  \text{bind}(\varepsilon \text{ in } K) \overset{df}{=} \text{Object}.
\]

A2) (base of induction 2) Let \( K \) be a class. An applied occurrence of a (class) identifier \( C \) in the class \( K \) is bound to a class named \( C \) such that

\[
  \text{bind}(C \text{ in } K) \overset{df}{=} (\text{inh} \text{'} \text{decl} \text{'}(K)).C
\]

where the pair \( (j, i) \), \( j \geq 0, i \geq 0 \), is the least pair in the lexicographic order such that the class \( (\text{inh} \text{'} \text{decl} \text{'}(K)).C \) is defined. The pairs are compared according to the lexicographical order, i.e. the pair \( (j, i) \) is less than the pair \( (q, p) \) if \( j < q \) or \( j = q \) and \( i < p \). The value of \( \text{bind}(C \text{ in } K) \) is undefined in the remaining cases.

B) (inductive step) Let \( X \neq \varepsilon \). For any class \( K \) the meaning of a type of the form \( X.C \) in the class \( K \) is determined in two steps.

\[
  \text{bind}(X.C \text{ in } K) \overset{df}{=} (\text{inh} \text{'}(\text{bind}(X \text{ in } K))).C
\]

\end{definition}
where $i \geq 0$, is the least natural number such that the class
\((\text{inh}^i(\text{bind}(X \text{ in } K))).C\) is defined,
the value of \(\text{bind}(X.C \text{ in } K)\) is undefined in the remaining cases.

Compare our definition with the text of JLS [7](Sections 6.3, 6.5.5, 8.1.4). We believe that
our definition of the function \(\text{bind}\) corresponds most closely to the lengthy and scattered
description of meaning of Java’s type name. Notice that the partial function \(\text{bind}\) can be
given an algorithmic definition (in the form of a procedure) as well, c.f. 2.2.

Example 2.1.3. This example illustrates our definition of the function \(\text{bind}\).

\[
\begin{align*}
\text{bind}(\varepsilon \text{ in } \text{Root}) &= \text{Object} = \text{inh}(C) \\
\text{bind}(\varepsilon \text{ in } C) &= \text{Object} = \text{inh}(C$G) \\
\text{bind}(\varepsilon \text{ in } C$B) &= \text{Object} = \text{inh}(C$B$A) \\
\text{bind}(C \text{ in } \text{Root}) &= C = \text{inh}(F) \\
\text{bind}(C \text{ in } E) &= \text{decl}(E).C = \text{Root}.C = C \\
\text{bind}(G \text{ in } A) &= \text{inh} \text{decl}(A).G = E$G \\
\text{bind}(G.G \text{ in } A) &= \text{inh}^i(\text{bind}(G \text{ in } A)).G \\
&= \text{inh}(E$G).G = C.G = C$G
\end{align*}
\]

The following relation \(\text{dep}\) plays an important rôle in the further considerations. In the
description of the Java [7] it is called the dependency relation (induced by a superclasing
function \(\text{inh}\)).

Let \(\text{ext}(K)\) be the following type \(C_1.C_2.\.C_i.\.C_n.\) Then \(\text{ext}(K)^i\) denotes the initial
segment \(C_1.C_2.\.C_i.\) of length \(i\), of the type \(\text{ext}(K)\).

Definition 2.1.4. The dependency relation \(\text{dep}\) is
\[
\text{dep} \overset{df}{=} \{ (K, \text{bind}(\text{ext}(K)^i \text{ in } \text{decl}(K))) : \\
K \in \text{Classes}, \\
0 < i \leq \text{length}(\text{ext}(K)) \text{ for } \text{ext}(K) \neq \varepsilon, \\
i = 0 \text{ for } \text{ext}(K) = \varepsilon\}.
\]

The above definition can be read as follows: let a class \(K\) be of the form: \textbf{class } C \\
\textbf{extends } C_1.C_2.\.C_i.\.C_n \{ \text{... } \} \) then the class \(K\) depends on every class designated
by the type \(\text{ext}(K)^i\). In JLS [7] (Section 8.1.4) one finds the following sentence: \textit{It is a compile-time error if a class depends on itself.} The word depends in this sentence is to be
meant as the transitive closure of the relation \(\text{dep}\).

Figure 2.1 illustrates the way of computing the value of the function \(\text{inh}\) and the relation
\(\text{dep}\).

Definition 2.1.5. A stripped Java program \(P\) is well-formed iff its class’ structure \(S_P\) can
be extended by a function
\[
\text{inh} : \text{Classes} \rightarrow \text{Classes} \cup \{ \text{Object} \}
\]
such that it satifies the following two conditions \(I_1\) for every class \(K \in \text{Classes}\) the value
\(\text{inh}(K)\) is defined and the following equality holds
\[
\text{inh}(K) = \text{bind}(\text{ext}(K) \text{ in } \text{decl}(K)).
\]
\(I_2\) The relation \(\text{dep}\) contains no cycle .
2.1 Preliminaries

Let $K$ denote the class named $C$.

The diagram can be viewed from several angles.
First, if we delete the arrows $dep$ and $decl$ and keep the arrow $decl^+$, then the diagram obtained in this way commutes. This is so for every class in a well-formed structure of classes. Note however, that the diagrams themselves may differ accordingly to the length of type mentioned in the `extends` clause.

Second, the commutativity of the modified diagram illustrates the condition $I_1$, i.e. $inh(K) = bind(ext(K) \text{ in } decl(K))$.

Third, the diagram may help to understand how to calculate the $inh$-arrow for class $K$ (Note, this is not an algorithm!). In this case we assume that all other $inh$-arrows appearing in the diagram were calculated earlier. We are to identify class $M_1$ of name $C_1$, $M_1 = (inh^{i}(decl^{i}(K))).C_1$ where the pair $(j, i)$ is the least pair such that the value of the expression $inh^{i}(decl^{i}(K)).C_1$ defined, next the class $M_2$ of name $C_2$, $M_2 = inh^{i}(M_1).C_2$ where $i$ is the least integer, such that the value of $inh^{i}(M_1).C_2$ is defined, ... class $M_n$ of name $C_n$, $M_n = inh^{i}(M_{n-1}).C_n$ where $i$ is the least integer such that the value of $inh^{i}(M_{n-1}).C_n$ is defined, in this order.

Now we can put an arrow $inh$ leading from $K$ to $M_n$.

During the above process, we put arrows $dep$ leading from the class $K$ to the classes $M_i$, $1 \leq i \leq n$. The diagram of the structure of classes enriched with $inh$-arrows and $dep$-arrows may not contain a cycle of $dep$-arrows.
Chapter 2.

Now we are ready to specify the problem of determining the direct superclasses.

**Problem 2.1.6.** For a given structure of classes $\mathcal{S}$, find whether the structure is well formed, and if it is so compute the function $\text{inh}$ which satisfies conditions $I_1$ and $I_2$, in the case of the negative answer signal an error.

### 2.2 The algorithm computing function bind

In the following two chapters we shall use two auxiliary functions.

**Bind** (Id id, Classes $\cup$ \{Root\} K): Classes $\cup$ \{Object, null\}

```plaintext
var Class $\cup$ \{null\} L
while K $\neq$ null do
    L := BindInh(id, K)
    if L $\neq$ null then return L endif
    K := decl(K)
endwhile
return null
end Bind
```

**BindInh** (Id id, Classes $\cup$ \{Root\} K): Classes $\cup$ \{Object, null\}

```plaintext
var Class $\cup$ \{null\} L
while K $\neq$ null do
    L := K.id
    if L $\neq$ null then return L endif
    K := inh(K)
endwhile
return null
end BindInh
```

For algorithmic purposes we assume that these functions are working on a data structure $\mathcal{S}^{\text{prog}}$ being a slightly modified structure $\mathcal{S}$ from the previous chapter.

$$\mathcal{S}^{\text{prog}} = \langle \text{Classes, Id, Types, \{null\}, decl^{prog}, inh, name, ext, Root, Object} \rangle$$

where Classes, Id, Types, name, ext, Root, Object are as previously defined, null is a special constant representing an undefined class.

$\text{decl}^{prog}$ extends the previous function $\text{decl}$ by adding $\text{decl}^{prog}(\text{Root}) = \text{null}$

$\text{inh} : \text{Classes} \cup \{\text{Object, Root}\} \rightarrow \text{Classes} \cup \{\text{Object, null}\}$

is a partial acyclic function defined step by step by the algorithm in the next section.

Next we modify the function $\cdot C$ defined in the previous chapter. We extend it from partial to the total function $\cdot C$

$$\cdot C : \text{Classes} \cup \{\text{Root, Object}\} \rightarrow \text{Classes} \cup \{\text{Object, null}\}$$

by putting $K.C = \text{null}$ for all pairs for which the former value of this function was undefined.

For the readers convenience the upper subscript $\text{prog}$ was omitted in the text of programs as it is clear which function should be taken into account.

Now we are ready to give a program which defines the function $\text{bind}^{\text{prog}}$ for the purpose of the next section.
2.2 The algorithm computing function bind

\[\text{bind}_\text{prog}(\text{Types Path in Classes } \cup \{\text{Root}\} M) : \text{Classes } \cup \{\text{Object, null}\}\]

\vspace{1em}

\begin{verbatim}
var Class \cup \{\text{null}\} L
if length(Path) = 0 then return Object;
L := Bind(Path[1], M);
if L = null then return null;
for i := 2 to length(Path) do
    L := BindInh(Path[i], L);
    if L = null then return null;
endfor;
return L;
end bind\_prog
\end{verbatim}

Lemma 2.2.1. For every class K if K = bind(T in M) for some type T and class M then also bind\_prog(T in K) = M.
Chapter 3

Non-deterministic algorithm

3.1 The algorithm

Below, we present a non-deterministic, abstract algorithm named LSWA, which computes the function $inh$. The algorithm uses function $bind$ defined in the previous section as $bind_{prog}$ which satisfies the corresponding lemma 2.2.1

Data Structure: The class structure $S_{prog}$ of a program.

Argument: The graph $G$ representing the structure $S_{prog}$.

Result: The function $inh$ which for each class $A \in Classes$ shows a class $B$, the direct superclass of class $A$, or executes a command $Error$ signalling that the structure of classes is erroneous.

Specification: See the Problem 2.1.6.

Algorithm LSWA:

\[
\begin{align*}
\text{Visited} & := \{ \text{Root, Object} \}; \\
\text{inh} & := \{ (\text{Root, null}), (\text{Object, null}) \}; \\
\text{while} \ \text{Visited} \neq (Classes \cup \{ \text{Root, Object} \}) & \text{ do} \\
& \quad \text{Candidates} := \{ K : decl(K) \in \text{Visited} \land K \notin \text{Visited} \} \\
& \quad \text{if} \ (\exists K \in \text{Candidates}) \ bind(\text{ext}(K) \ in \ decl(K)) \in \text{Visited} \\
& \qquad \text{then} \\
& \qquad \quad \text{let } K \text{ be a } \text{Candidate} \text{ taken arbitrary from the above test} ; \\
& \qquad \quad \{ \text{this choice is made in the nondeterministic way} \} \\
& \qquad \quad M := bind(\text{ext}(K) \ in \ decl(K)); \\
& \qquad \quad \text{inh} := \text{inh} \cup \{ (K, M) \}; \\
& \qquad \quad \text{Visited} := \text{Visited} \cup \{ K \} \\
& \quad \text{else} \\
& \quad \quad Error \\
& \quad \text{endif} \\
\text{endwhile}
\end{align*}
\]

Observe that the command “let ...” is instruction of non-deterministic choice. The values of function $bind$ are computed using the current diagram of function $inh$ computed so far.

For the algorithm of $bind$ consult section 2.2

The word $Error$ is an abbreviation of the following instruction
if $\exists K \in \text{Candidates} \exists i_1 \leq i \leq \text{length}(\text{ext}(K)) \forall j_1 \leq j < i (\text{bind}(\text{ext}(K)|^j \text{ in } \text{decl}(K)) \in \text{Visited}$

$\land \text{bind}(\text{ext}(K)|^i \text{ in } \text{decl}(K)) \text{ is undefined}$

then

\begin{verbatim}
    throw new Signal_ConditionI1_violated() // a direct superclass of K cannot
    // be detected. There is no declaration
    // of a class named ext(K)[i]
\end{verbatim}

else
    if $\forall K \in \text{Candidates} \exists i_1 \leq i \leq \text{length}(\text{ext}(K)) \forall j_1 \leq j < i (\text{bind}(\text{ext}(K)|^j \text{ in } \text{decl}(K))$

    $\in \text{Visited} \land \text{bind}(\text{ext}(K)|^i \text{ in } \text{decl}(K)) \in \text{Candidates}$

    then
        throw new Signal_ConditionI2_violated() // there is a cycle in the dep-relation
    endif

We shall prove:

- This non-deterministic algorithm is determinate, it means that for every data, i.e. a structure of classes, any two acceptable computations of the algorithm give the same result.

- The algorithm either computes the correct solution, i.e. the function $inh$ satisfying conditions $I_1$ and $I_2$ or

  it adequately signals that no solution exists for the given structure of classes.

3.2 Analysis of the algorithm

3.2.1 Correctness

We are going to prove that the algorithm terminates and is correct i.e. that if it halts in a successful way (i.e. without signalling an error) then the computed function $inh$ is satisfying the conditions $I_1$ and $I_2$. Moreover the algorithm is complete meaning that if it signals an error that this diagnosis is correct, i.e. there is no function $inh$ satisfying both $I_1$ and $I_2$. Finally, we are going to prove the uniqueness. We show that any solution satisfying the conditions $I_1$ and $I_2$ is identical to the solution computed by the algorithm.

Lemma 3.2.1. (on termination) The algorithm terminates.

Proof. In each step of iteration of the algorithm either the set $\text{Visited}$ is increased or the algorithm signals an error and stops. It is easy to observe that the number of iterations is not bigger than the number of declared classes.

Definition 3.2.2. A state $S$ is the pair $\langle \text{Visited}_S, inh_S \rangle$ of values of corresponding variables, computed by the algorithm at the moment of testing the condition of the while instruction.

Obviously, $inh_S$ is a set of pairs of classes, $\text{Visited}_S$ denotes a subset of the set $\text{Classes} \cup \{\text{Root, Object}\}$.

Remark 3.2.3. For every state $S$, the graph

$$G_{1S} = \langle \text{Visited}_S, \text{decl}|_{\text{Visited}_S} \rangle$$

is a tree with the root $\text{Root}$. Graph

$$G_{2S} = \langle \text{Visited}_S \setminus \{\text{Root}\}, inh_S \rangle$$

is a tree with the root $\text{Object}$. 
3.2 Analysis of the algorithm

Proof. Proof goes by induction w.r.t. \( n \), number of iterations of the algorithm. For \( n = 0 \) the tree \( G_{2S} \) contains only its root. Suppose that the thesis is true for a number \( k \) of iterations. In the next iteration one adds an edge going from outside the set \( Visited \) to a certain node in this set. Therefore the new graph is a tree again.

Lemma 3.2.4. For every state \( S = \langle Visited_S, inh_S \rangle \), for every class \( K \in Visited_S \) and for every type \( P \) :

If \( bind_{inh_S}(P \text{ in } K) \in Visited_S \) then \( bind_{inh_S}(P|^i \text{ in } K) \in Visited_S \) for \( 1 \leq i < \text{length}(P) \).

Proof. Assume the thesis of the lemma is wrong. Then there is a greatest \( i_0 \) such that \( 1 \leq i_0 < \text{length}(P) \) and class \( C_{i_0} = bind_{inh_S}(P|_0 \text{ in } K) \notin Visited_S \). The subsequent class \( C_{i_0+1} = bind_{inh_S}(P|^i \text{ in } K) \in Visited_S \), and, from the definition of bind, we have: \( C_{i_0+1} = inh_S(C_{i_0}).name(C_{i_0+1}) \) where \( k \) is the smallest integer such that the right side is defined. From Remark 3.2.3 we obtain that since \( C_{i_0} \notin Visited_S \) then \( k = 0 \). Then \( C_{i_0+1} = C_{i_0}.name(C_{i_0+1}) \) and \( decl(C_{i_0+1}) = C_{i_0} \). Again from Remark 3.2.3 since \( C_{i_0+1} \in Visited_S \) then also \( C_{i_0} \in Visited_S \). Contradiction.

Lemma 3.2.5. Let \( S = \langle Visited_S, inh_S \rangle \) be a state.

Let \( inh \) be an arbitrary extension of function \( inh_S \) on the set Classes.

A) For every class \( K \in Visited_S \), and \( i \geq 0 : inh^i_S(K) = inh^i(K) \) or both sides are undefined.

B) For every class \( K \in Visited_S \) and for every type \( P \):

if for every \( 1 \leq i < \text{length}(P) \), \( bind_{inh_S}(P|^i \text{ in } K) \in Visited_S \) then

\[
\forall M \in \text{Classes} (bind_{inh_S}(P \text{ in } K) = M \iff bind_{inh}(P \text{ in } K) = M).
\]

Proof. Proof of A)

First we are going to prove that for every \( K \in Visited_S \), \( inh_S(K) = inh(K) \).

Case 1) \( K \notin \{\text{Root, Object}\} \). Then \( inh_S(K) \) is defined due to Remark 3.2.3. Hence \( \langle K, inh_S(K) \rangle \in inh \) since \( inh_S \subseteq inh \).

Case 2) \( K \in \{\text{Root, Object}\} \). Then \( inh_S(K) \) and \( inh(K) \) are both undefined.

Using the remark on graph \( G_{2S} \) we conclude that \( inh^i_S(K) = inh^i(K) \) or both sides are undefined.

Proof of B)

0) (base of induction) For types of length 0 the lemma is obvious.

1) (base of induction) Consider types of length 1. Let \( P = C \), \( C \) is a name of a class. We are going to prove

\[ \langle bind_{inh_S}(P \text{ in } K) \rangle = M \iff bind_{inh}(P \text{ in } K) = M. \]

By definition \( bind_{inh_S}(P \text{ in } K) = M \) iff \( M = (inh^i_S(decl^i(K))).C \) where the pair \( \langle j, i \rangle \) is the least pair in the lexicographic order such that the expression \( (inh^i_S(decl^i(K))).C \) has a value. We are going to show that for any pair \( \langle m, l \rangle \) less or equal the pair \( \langle j, i \rangle \) and for any class \( N = (inh^l_S(decl^l(N))).C \) we have \( N = (inh^i(decl^i(K))).C \).

If \( decl^i(K) \) has a value then it denotes a class in \( Visited_S \). Put \( K_0 = decl^m(K) \). Using A) we see that for any \( p : inh^p_S(K_0) = inh^p(K_0) \) or both sides are undefined.

From here one obtains

\[ \langle bind_{inh_S}(P \text{ in } K) \rangle = M \iff bind_{inh}(P \text{ in } K) = M. \]

I) (induction step) Let us assume that the lemma is true for types \( P \) of length not greater than \( n \), \( n \geq 1 \). Let us consider type \( P.C \). From the assumptions of this lemma we have

\[ bind_{inh_S}(P.C|^i \text{ in } K) \in Visited_S \] for \( 1 \leq i \leq \text{length}(P) \).

Therefore \( bind_{inh_S}(P|^i \text{ in } K) \in Visited_S \) for \( 1 \leq i < \text{length}(P) \). By inductive assumption \( bind_{inh_S}(P \text{ in } K) = bind_{inh}(P \text{ in } K) \). Now we use the definition to calculate \( bind_{inh}(P.C \text{ in } K) \). Arguments similar to those of point 1) lead to the result.
bind_{inh_S}(P.C \text{ in } K) = bind_{inh}(P.C \text{ in } K)

or to the conclusion that both sides of the equality are undefined, which ends the proof of the lemma.

Lemma 3.2.6. Let $S = \langle Visited_S, \text{ inh}_S \rangle$ be an arbitrary state. Let inh be a function satisfying condition $I_1$. Then $\text{ inh}_S \subseteq \text{ inh}$.

Proof. Consider the sequence of states (a computation) leading to the state $S$. Let us consider the longest initial segment $\{S_i\}_{i=0, \ldots, q}$ of the computation such that for every state $S_i$ the inclusion $\text{ inh}_{S_i} \subseteq \text{ inh}$ holds. Such segment exists for $\emptyset = \text{ inh}_{S_0} \subseteq \text{ inh}$. If $S_q = S$ then the thesis of the lemma is true. Suppose $S_q \neq S$. Then for a certain candidate $K$ in the state $S_q$ we added the pair $(K, \text{ bind}_{inh_{S_q}}(\text{ ext}(K) \text{ in } \text{ decl}(K)))$. Now, the function $\text{ inh}_{S_q}$, the state $S_q$ and the function inh satisfy the premises of the preceding lemma. We put $\text{ decl}(K)$ instead of $K$ and we put $\text{ ext}(K)$ as $P$. From the algorithm we know that $\text{ bind}_{inh_{S_q}}(\text{ ext}(K) \text{ in } \text{ decl}(K)) \in Visited_{S_q}$. Using Lemma 3.2.4 we have $\text{ bind}_{inh_{S_q}}(\text{ ext}(K))^{\uparrow}$ in $\text{ decl}(K) \in Visited_{S_q}$ for $i = 1, \ldots, \text{ length}(\text{ ext}(K)) - 1$. Let $M \overset{def}{=} \text{ bind}_{inh_{S_q}}(\text{ ext}(K) \text{ in } \text{ decl}(K))$. Now we can apply the preceding lemma to conclude that $M = \text{ bind}_{inh}(\text{ ext}(K) \text{ in } \text{ decl}(K))$. Since inh satisfies condition $I_1$ we get $(K, M) \in \text{ inh}$. Therefore the state $S_{q+1}$ satisfies $\text{ inh}_{S_{q+1}} \subseteq \text{ inh}$ which contradicts our assumption.

Remark 3.2.7. The set $S$ of states is partially ordered by the relation $\prec$ being the transitive closure of the relation of immediate successorship of states.

Given two states $S_1$ and $S_2$, if $S_1 \prec S_2$ then then $\text{ inh}_{S_1} \subseteq \text{ inh}_{S_2}$ and $\text{ Visited}_{S_1} \subseteq \text{ Visited}_{S_2}$

Lemma 3.2.8. Let $S$ be a state $S = \langle Visited_S, \text{ inh}_S \rangle$. $S$ satisfies condition $I_1$ restricted in this way that in this condition the set $\text{ Visited}_S$ takes place of set $\text{ Classes}\cup\{\text{ Root, Object}\}$, and function $\text{ inh}_S$ the place of inh.

Proof. Let $K$ be a class from $\text{ Visited}_S \setminus \{\text{ Root, Object}\}$. Let $S_1$ be a state earlier than $S$, $S_1 \prec S$, such that instruction $\text{ inh} := \text{ inh} \cup \{(K, M)\}$ is going to be executed, i.e. the state $S_2$ next to $S_1$ is the first state such that $(K, \text{ inh}_{S_2}(K)) \in \text{ inh}_{S_2}$. Then $\text{ inh}_{S_2}(K) = \text{ bind}_{inh_{S_1}}(\text{ ext}(K) \text{ in } \text{ decl}(K))$. By Remark 3.2.7 $\text{ inh}_{S_1} \subseteq \text{ inh}_{S_2} \subseteq \text{ inh}_S$. From the algorithm the class $\text{ bind}_{inh_{S_1}}(\text{ ext}(K) \text{ in } \text{ decl}(K)) \in \text{ Visited}_{S_1}$. Then, by Lemma 3.2.4 for every $1 \leq i < \text{ length}(\text{ ext}(K))$ the class $\text{ bind}_{inh_{S_1}}(\text{ ext}(K))^{\uparrow}$ in $\text{ decl}(K) \in \text{ Visited}_{S_1}$. Now by the Lemma 3.2.5

\begin{align*}
\text{ bind}_{inh_S}(\text{ ext}(K) \text{ in } \text{ decl}(K)) &= \text{ bind}_{inh_{S_1}}(\text{ ext}(K) \text{ in } \text{ decl}(K)) \\
\text{ bind}_{inh_{S_1}}(\text{ ext}(K) \text{ in } \text{ decl}(K)) &= \text{ inh}_{S_2}(K) \text{ (see above)} \\
in_{inh_{S_1}}(K) &= \text{ inh}_{S}(K) \text{ (by the above inclusion)}
\end{align*}

This ends the proof of property $I_1$.

Lemma 3.2.9. With the assumptions of the previous lemma we observe that if there exists a cycle in the relation $\text{ dep}_S$ then no one of the classes of this cycle will ever be included to the set $\text{ Visited}$.

Proof. Suppose that there exists a cycle in the relation $\text{ dep}_S$.

Let $V = \{K_1 \rightarrow K_2 \rightarrow \ldots \rightarrow K_p\}$ be this cycle. I.e. for $j = 1, \ldots, p - 1$ pairs $(K_j, K_{j+1}) \in \text{ dep}$ and pair $(K_p, K_1) \in \text{ dep}$. W.l.g. assume that $K_1$ is the class which was added to the set $\text{ Visited}$ as the first one. Then $K_2 = \text{ bind}_{inh_S}(\text{ ext}(K_1))^{\uparrow}$ in $\text{ decl}(K_1)$ for some $1 \leq i \leq \text{ length}(\text{ ext}(K_1))$. Let $S_0, S_1$ be two consecutive states such that $\text{ inh}_{S_1}(K_1)$ is computed in
3.2 Analysis of the algorithm

the state $S_0$. By the algorithm $inh_{S_0}(K_1) = bind_{inh_{S_0}}(ext(K_1) \in decl(K_1)) \in Visited_{S_0}$.

By Lemma 3.2.4, for every $1 \leq j < length(ext(K_1))$ the class $bind_{inh_{S_0}}(ext(K_1)|j in decl(K_1)) \in Visited_{S_0}$.

According to Lemma 3.2.5 $bind_{inh_{S_0}}(ext(K_1)|i in decl(K_1)) = bind_{inh_{S_0}}(ext(K_1)|i in decl(K_1)) = K_2$. In this way we proved that $K_2 \in Visited_{S_0}$ and $K_1 \not\in Visited_{S_0}$. Contradiction!

The above lemma leads immediately to the following

**Theorem 3.2.10.** (on correctness) Suppose that the algorithm stops without signalling an error. Then the resulting function $inh$ satisfies the conditions $I_1$, $I_2$ and the structure of classes is well-formed.

3.2.2 Completeness

Now we are going to prove the completeness property of the algorithm. Namely, if the algorithm stops and signals error then no total function $inh$ exists of desired properties. We begin with the remark that the instruction $Error$ may be considered as an abbreviation of a conditional statement, look at Section 3.1. This splitting in two cases is motivated by the following observation: Should the algorithm come up with $Error$ then the set $Candidates$ is not empty and $ext(K) \neq \epsilon$ for all $K \in Candidates$. For every such $K$ there is a uniquely associated $i$ such that $1 \leq i \leq length(ext(K))$ with $bind(ext(K)|i in decl(K)) \in Visited$ for all $1 \leq j < i$ and $bind(ext(K)|i in decl(K)) \not\in Visited$. There are two possible reasons for the latter situation: Either $bind(ext(K)|i in decl(K))$ is undefined or a class $M \in Candidates$.

The following program examples illustrate these two cases.

**Example 3.2.11.** In this example no class named $C$ is visible in the place where class $A$ is declared which is to inherit a class named $C$.

```java
class A extends C {}
class B {
    class C {}
}
```

The algorithm terminates erroneously in final state

$$S_{fin} = \langle Visited, inh \rangle = \{(Root, Object, B, C), (B, Object), (C, Object)\}.$$ 

Class $A$ is the only one candidate. The value of $bind(C$ in $Root)$ is undefined what is showing up the first case.

A compiler should report “there is no appropriate class $C$ declared”.

**Example 3.2.12.** We consider the instructive Example 1.2.3 again. It is showing up the second case. Our algorithm terminates erroneously in final state shown on Fig.3.1. $A$ and $B$ are the two candidates. The value $bind(A$ in $Root)$ is $A$, the value $bind(B$ in $Root)$ is $B$.

**Example 3.2.13.** This example has exactly one solution $inh$ which fulfills $I_1$. $inh$ has no cycles, but its dependency relation $dep$ has a cycle.
Figure 3.1: Final state for Example 3.2.12 (and Example 1.2.3)

\[
\text{class } A \text{ extends } B \cdot D \{
\text{class } C \{
\}
\}
\text{class } B \text{ extends } A \cdot C \{
\text{class } D \{
\}
\}
\]

Our algorithm terminates erroneously in final state
\[S_{\text{fin}} = \langle \{\text{Root, Object}\}, \emptyset \rangle.\]

A and B are the candidates. \(\text{bind}(B \text{ in Root}) = B\) and \(\text{bind}(A \text{ in Root}) = A\), so there is a cycle \(A \xrightarrow{\text{dep}} B \xrightarrow{\text{dep}} A\).

This example is so instructive also because it is showing that Igarashi’s and Pierce’s so called sanity conditions 6) and 7) in [11] are more liberal than the condition “no class depends on itself” taken from JLS [7]. Condition 6) is saying that \(\text{inh}\) has no cycles. Condition 7) is saying that no class \(A\) is inheriting its own inner class (no \(A \xrightarrow{\text{inh}} \rightarrow B \xrightarrow{\text{decl}} A\) is allowed).□

The example 3.2.11 motivates the following

**Definition 3.2.14.** We say that in a given state \(S = \langle \text{Visited}_S, \text{inh}_S \rangle\) the permanent lack of a class to be inherited occurs iff there exists a class \(K\) such that for a certain \(1 \leq i \leq \text{length}(\text{ext}(K))\), \(\text{decl}(K) \in \text{Visited}_S\) and \(\text{bind}_{\text{inh}_S}(\text{ext}(K)[i] \text{ in } \text{decl}(K))\) is undefined and for all \(1 \leq j < i\) \(\text{bind}_{\text{inh}_S}(\text{ext}(K)[j] \text{ in } \text{decl}(K)) \in \text{Visited}_S\).

**Lemma 3.2.15.** If in a certain state \(S = \langle \text{Visited}_S, \text{inh}_S \rangle\) occurs the permanent lack of a class to be inherited then no function \(\text{inh}\) exists which satisfies condition \(I_1\).

**Proof.** Suppose that a function \(\text{inh}\) satisfying \(I_1\) exists. Let \(K\) and \(i\) be a class and an integer that have properties mentioned in Definition 3.2.14. By Lemma 3.2.6, \(\text{inh}_S \subseteq \text{inh}\). Moreover, for all \(1 \leq j < i\) \(\text{bind}_{\text{inh}_S}(\text{ext}(K)[j] \text{ in } \text{decl}(K)) \in \text{Visited}_S\). The values \(\text{inh}(\text{Root})\) and \(\text{inh}(\text{Object})\) are undefined. Observe that \(\text{decl}(K) \in \text{Visited}_S\). By Lemma 3.2.5 we have \(\forall M \in \text{Classes}(\text{bind}_{\text{inh}_S}(\text{ext}(K)[i] \text{ in } \text{decl}(K)) = M \Leftrightarrow \text{bind}_{\text{inh}_S}(\text{ext}(K)[i] \text{ in } \text{decl}(K)) = M)\).

Since \(\text{inh}\) satisfies \(I_1\), \(\text{bind}_{\text{inh}_S}(\text{ext}(K) \text{ in } \text{decl}(K)) = \text{inh}(K)\) is defined. Hence the right-hand side of the equivalence above holds for some class \(M\). So \(\text{bind}_{\text{inh}_S}(\text{ext}(K)[i] \text{ in } \text{decl}(K))\) is defined contrary to our assumption! □

Now we shall analyse the remaining case and prove that if for a certain state \(S = \langle \text{Visited}_S, \text{inh}_S \rangle\) the following condition holds
c1) the algorithm signals an error in this state, and
c2) the property permanent lack of a class to be inherited does
not hold for S
then there exists a cycle in the dep relation for every dep relation induced by any function
inh satisfying \( I_1 \).

**Definition 3.2.16.** Each state \( S \) determines the set \( \text{Candidates}_S \) which is evaluated just after the test \( \text{Visited} \neq (\text{Classes} \cup \{ \text{Root}, \text{Object} \}) \) is performed.

We begin with an auxiliary lemma

**Lemma 3.2.17.** Let \( S \) be a state such that the conditions c1) and c2) are satisfied then
for every class \( K \in \text{Candidates}_S \) there exists a class \( M \in \text{Candidates}_S \) such that for a
certain \( 1 \leq i \leq \text{length}(\text{ext}(K)) \), \( M = \text{bind}_{\text{inh}_S}(\text{ext}(K))^i \) in \( \text{decl}(K) \) and for all \( 1 \leq j < i \),
\( \text{bind}_{\text{inh}_S}(\text{ext}(K))^j \) in \( \text{decl}(K) \) \( \in \text{Visited}_S \).

**Proof.** Let \( I_K = \{ l : 1 \leq l \leq \text{length}(\text{ext}(K)) \wedge (\forall 1 \leq j < l) \text{bind}(\text{ext}(K))^j \) in \( \text{decl}(K) \) \} \in \text{Visited}_S \}. \)
First, we show that the set \( I_K \) is non-empty. We demonstrate that \( 1 \in I_K \). We
can assume that \( \text{length}(\text{ext}(K)) \geq 1 \) for in the opposite case of \( \text{length}(\text{ext}(K)) = 0 \) the
algorithm would not signal error and add the pair \( \langle K, \text{Object} \rangle \) to \( \text{inh} \). Consider \( l = 1 \), the
formula \( (\forall 1 \leq j < l) \text{bind}(\text{ext}(K))^j \) in \( \text{decl}(K) \) \( \in \text{Visited}_S \) is valid. Hence the set \( I_K \)
contains 1 and is non-empty.
An upper bound of the set \( I_K \) is \( \text{length}(\text{ext}(K)) \), hence \( \text{max}(I_K) \) is defined, denote it by \( i \).
Suppose now, that the value of \( \text{bind}_{\text{inh}_S}(\text{ext}(K))^i \) in \( \text{decl}(K) \) is undefined. It would mean
that the condition of permanent lack of a class to be inherited occurs which was excluded
by the assumption. Therefore there exists a class \( M \) defined by this expression
\( M = \text{bind}_{\text{inh}_S}(\text{ext}(K))^i \) in \( \text{decl}(K) \). We are going to show that \( M \notin \text{Visited}_S \). Let us assume
that \( M \in \text{Visited}_S \). Suppose moreover that \( i = \text{length}(\text{ext}(K)) \). In this case the algorithm
would add the pair \( \langle K, M \rangle \) to \( \text{inh}_S \) instead of signalling error. Hence \( i < \text{length}(\text{ext}(K)) \).
In this case \( i + 1 \in I_K \) which contradicts the assumption \( i = \text{max}(I_K) \). In this way we
proved that \( M \notin \text{Visited}_S \).
It remains to be proved that \( M \in \text{Candidates}_S \). In order to do so it suffices to show that
\( \text{decl}(M) \in \text{Visited}_S \). Put \( C = \text{name}(M) \).
Consider the case \( i = 1 \). We have \( M = \text{bind}_{\text{inh}_S}(C \in \text{decl}(K)) \). By the definition of \( \text{bind}_{\text{inh}_S} \)
\( M = (\text{inh}_S(\text{decl}^k \text{decl}(K))).C \) for some \( l \) and \( k \). Hence \( \text{decl}(M) = \text{inh}_S^l(\text{decl}^k \text{decl}(K)) \).
Since \( K \in \text{Candidates}_S \) we know \( \text{decl}(K) \in \text{Visited}_S \). From Remark 3.2.3 we obtain
\( \text{decl}(M) \in \text{Visited}_S \).
Consider the case \( i > 1 \). From the definition of \( i \) we have
\( \text{bind}_{\text{inh}_S}(\text{ext}(K))^{i-1} \) in \( \text{decl}(K) \) \( \in \text{Visited}_S \). Let \( P = \text{ext}(K)^i \) then \( \text{ext}(K)^i = P.C \).
Since \( M = \text{bind}_{\text{inh}_S}(P.C \text{ in } \text{decl}(K)) \) then by definition of \( \text{bind}_{\text{inh}_S} \) we obtain
\[ \text{decl}(M) = \text{inh}_S^l(\text{bind}(P \text{ in } \text{decl}(K)) \] for some \( l \geq 0 \).
Now we apply Remark 3.2.3 and obtain \( \text{decl}(M) \in \text{Visited}_S \).

**Corollary 3.2.18.** Suppose that the assumptions of lemma 3.2.17 are satisfied. Let \( M \) be
a class such that
\[
M = \text{bind}_{\text{inh}_S}(\text{ext}(K))^i \text{ in } \text{decl}(K)
\]
for a certain \( 1 \leq i \leq \text{length}(\text{ext}(K)) \) and
for all \( 1 \leq j < i \) \( \text{bind}_{\text{inh}_S}(\text{ext}(K))^j \text{ in } \text{decl}(K) \) \( \in \text{Visited}_S \). Suppose that a function \( \text{inh} \)
satisfying \( I_1 \) exists.
Then \( M = \text{bind}_{\text{inh}}(\text{ext}(K))^i \text{ in } \text{decl}(K) \).
Proof. By the lemmas 3.2.5 and 3.2.6.

**Lemma 3.2.19.** If in a state $S$ the algorithm signalled an error and there exists a function $inh$ satisfying the condition $I_1$ (it implies no permanent lack of a class to be inherited due to Lemma 3.2.15) then for any natural number $n$ one can find a sequence $K_1 \xrightarrow{dep} K_2 \xrightarrow{dep} \ldots \xrightarrow{dep} K_n$, such that all classes $K_i \in \text{Candidates}_S$, the relation $\xrightarrow{dep}$ is determined by the solution $inh$, according to the definition.

Proof. (by induction) Let $n = 1$. The set $(\text{Classes} \setminus \text{Visited}_S)$ is non-empty. Consider a class $M \in \text{Classes} \setminus \text{Visited}_S$. Let $i$ be the least natural number such that $\text{decl}^i(M) \in \text{Visited}_S$. Since $\text{Root} \in \{\text{decl}^j(M) : j > 0 \text{ and } \text{decl}^j(M) \text{ is defined}\}$ and $\text{Root} \in \text{Visited}_S$, we know that $i$ exists. Since $M \notin \text{Visited}_S$, $i \geq 1$. Let $K_1 = \text{decl}^{i-1}(M)$. $K_1 \in \text{Candidates}_S$.

(*inductive step*) Suppose that there exists a sequence $K_1 \xrightarrow{dep} K_2 \xrightarrow{dep} \ldots \xrightarrow{dep} K_n$ which satisfies the thesis of the lemma, i.e. $K_n \in \text{Candidates}_S$. By Lemma 3.2.17 and Corollary 3.2.18 there exists a class $M = \text{bind}_{inh}(\text{ext}(K_n) \mid i \text{ in } \text{decl}(K_n))$, $M \in \text{Candidates}_S$ for a certain $1 \leq i \leq \text{length} (\text{ext}(K))$. We define $K_{n+1} = M$ and have $K_n \xrightarrow{dep} M$ which ends the proof.

From the above considerations one obtains:

**Theorem 3.2.20.** (on completeness) The algorithm signals Error iff the structure of classes is erroneous and no function $inh$ satisfying conditions $I_1$ and $I_2$ exists.

Proof. Suppose that a solution $inh$ exists. Since an error is signalled then either there is permanent lack of a class to be inherited (c.f. Lemma 3.2.15) and consequently $inh$ does not enjoy the property $I_1$ or (by Lemma 3.2.19) there exists a sequence $K_1 \xrightarrow{dep} K_2 \xrightarrow{dep} \ldots \xrightarrow{dep} K_n$ of length greater than the cardinality of set $\text{Classes}$. It means that there is a cycle of $\text{dep}$ arrows, it is impossible to extend the function $inh$ computed so far in a way satisfying conditions $I_1, I_2$ and $A_1, A_2, B$ of Definition 2.1.2.
Chapter 4

A deterministic algorithm

4.1 The algorithm

This section presents a deterministic algorithm which elaborates direct superclasses. This algorithm has several advantages over the non-deterministic one of c.f. section 3.1

- The algorithm is deterministic. All details necessary to implement it in practice are given.

- The algorithm is equipped with diagnostics and error recovery mechanism. It consists in coloring the visited classes i.e. nodes of classes structure $S$. The diagnostics recognizes two types of errors: a) undeclared class or b) cycle in dependence relation. The case of cycle in dependence relation is easy identifiable for the algorithm lists the pairs $(\text{header of a class declaration, path (name) of class on which the declared class depends on})$

- In both cases the error recovery uses the class $\text{Object}$ as a target of inheritance. The error recovery makes possible to continue the static semantic analysis of Java programs.

In order to define proper data structure for the algorithm we extend the structure $S_{\text{prog}}$ defined in section 2.2 by the function $\text{sons} : \text{Classes} \cup \{\text{Root}\} \rightarrow \text{OrderedCollection of Classes}$ Members of the ordered collection $\text{sons}(K)$ are defined by the following equivalence:

$$M \in \text{sons}(K) \equiv \text{decl}(M) = K$$

The function $\text{sons}$ together with its the order of its result collection is produced (like other elements of the structure $S_{\text{prog}}$) as result of syntax analysis. The simplest ordering is that induced by the textual order of classes declarations. The ordering of a collection $\text{sons}(K)$ in relevant in the execution of a loop $\text{for (Classes L : sons(K))}$

The algorithm uses functions $\text{Bind}$ and $\text{BindInh}$ defined in section 2.2, and consists of a few procedures.

**Data Structure:** The class structure $S_{\text{prog}}$ of a program and the function $\text{sons}$

**Argument:** The graph $G$ representing the structure $S_{\text{prog}}$ and the function $\text{sons}$.

**Result:** The function $\text{inh}$ which for each class $A \in \text{Classes}$ shows a class $B$, the direct superclass of class $A$, such that $\text{inh}$ satisfies problem 2.1.6 or signaling an error if such function $\text{inh}$ does not exist. In the case when error is signalled error recovery is performed in order to define $\text{inh}$ to the rest of remaining classes.
Algorithm

DCStack := EmptyStack
{this stack is a global structure and will be used for diagnostics of a cycle in dep relation}
Mark Root and Object White and all other classes mark Black
{Root and Object have inh undefined and all other classes were not processed yet}
Let inh(K)=null for every K ∈ Classes
call Preorder(Root)
end Algorithm

Preorder(Classes K)
if K is Black then
{we proceed only with such K which was not processed yet by ComputeInhFor called form itself}
call ComputeInhFor(K)
endif
for (Classes L : sons(K)) do
call Preorder(L)
endfor
end Preorder

CycleMessage(Classes K)
{dump DCStack in order to write the whole dependency cycle}
writeln("Dependence cycle:"
writeln(header(K))
writeln("depends on class named ", Pop(DCStack), "which is: ")
do
writeln(header(Pop(DCStack)))
if empty(DCStack) then exit endif
writeln(" which in turn depends on class named:"Pop(DCStack), "which is: ")
endo
end CycleMessage

ComputeInhFor(Classes K)
var Classes ∪ {null} L i.e. value of variable L is either a Class or null
{variable L is used to compute consecutive types found in ext(K)}
{ decl(K) is White }
{ decl(K) was yet processed and has its inh computed }
Mark K Grey
{denotes the presence of K in the stack of ComputeInhFor activation records and is used for detection of cycle in dep relation}
if length(ext(K)) =0 then
  L := Object;
  {empty type denotes, by default, inheritance form Object}
else
  i:=1
  L:=Bind(ext(K)[i], decl(K))
  {L is the meaning of the first identifier in ext(K)}
  {Invariant of the loop: L=bind(ext(K)[i] in decl(K)) }
  while ¬ (i= length(ext(K)) and L is White) do
if L=null then
    writeln("undeclared class", ext(K)[i], “in the header of class”, header(K))
    inh(K):=Object; Mark K White -- recovery
    return
endif

if L is Grey then
    {L in stack of ComputeInhFor – cycle in dep detected}
    Push(DCStack, L); Push(DCStack, ext(K)[i])
    Mark K Black; Mark L Red
    mark red the bottom element of the cycle in stack of ComputeInhFor
    if K=L then -- one element cycle detected
        call CycleMessage(K) -- diagnostics
        L := Object -- recovery
        exit
    endif
endif

if L is Black then
    {L was not processed yet}
    call ComputeInhFor(L)
    {compute inh for L by demand}
    if not empty(DCStack) then
        {L was involved in dep cycle, so push it to DCStack}
        Push(DCStack, L) ; Push(DCStack, ext(K)[i])
        if K is Red then
            {bottom of the cycle find}
            call CycleMessage(K) -- produce diagnostics
            L:=Object -- recovery
            exit
        endif
        Mark K Black
        {bottom of the cycle lies below current ComputeInhFor activation record
        K will be treated as unprocessed for further analysis
        and will be pushed onto DCStack immediately after return}
        return
    endif -- ComputeInhFor has returned with the proper value of inh(L)
endif -- the value of inh is properly set for l

if i=length(ext(K)) then exit endif
{Assertion1: L=bind(ext(K)[i] in decl(K)) and L is White }

if i=length(ext(K)) then exit endif
{if the value of L is the whole type ext(K) exit while
otherwise compute the next element of ext(k)}
i:=i+1 ; L:=BindInh(ext(K)[i], L)
endwhile
{Assertion2: L=bind(ext(K) in decl(K)) and L is White or K is Red and L=Object }
{L is either the proper value of ext(K) for nonempty type
or is set to Object by error recovery}

def
{L is either the proper value of ext(K) or is set to Object by error recovery}

\[ \text{inh}(K) := L ; \text{Mark K White} \]

end ComputeInhFor

\section{Analysis of algorithm}

We are going to prove that if the algorithm computes the function \( \text{inh} \) without signalling any error, then it is a correct solution of the problem.

\subsection{Experiments}

We did some experimenting before proceeding to wording of lemmas and proving them. Two animations of experiments are offered at the URL: http://duch.mimuw.edu.pl/~salwicki/CSP2007/presentationLagow/prezentacja21Listop2007.pdf. Here we quote a slide of detection of cycle in dep relation.

\subsection{Proof of correctness}

\textbf{Lemma 4.2.1.} The following observations are valid:

\begin{enumerate}
  \item Marking a node \( K \) White and assigning a value \( \neq \text{null} \) to \( \text{inh}(K) \) takes place together.
  \item If a node is marked White then it is never marked again.
  \item If value \( \neq \text{null} \) was assigned to \( \text{inh}(K) \) then it is never changed.
  \item In procedure ComputeInhFor node \( K \) may get color Black. It is done only if instruction Push is executed.
  \item If a node is marked Grey then the stack of activation records of ComputeInhFor contains a record such that its parameter \( K \) points to the node.
  \item The procedure ComputeInhFor will never be called with the parameter \( K \) equal Root or Object.
\end{enumerate}

\textit{Proof.} (i) Look at the code of ComputeInhFor and check that there are only two places in the algorithm where a node \( K \) is marked White. In both cases this instruction is accompanied by the instruction of the form \( \text{inh}(K) := \text{expression} \).

(ii) Check the code of ComputeInhFor.

(iii) It follows from the previous observations.

(iv) Check the code of ComputeInhFor.

(v) Obvious.

(vi) This is obvious consequence of the fact that Root and Object are White and all procedure calls of ComputeInhFor have the argument \( K \) Black. As the consequence we observe that the expressions of the form \( \text{ext}(K) \) have always the well defined value.
4.2 Analysis of algorithm

Figure 4.1: The cycle of dep relation of dependency is: A \(\text{dep} \ B \text{dep} \ F \text{dep} \ A\)

Explanation of the diagram:
- in left upper corner you see the structure of classes,
- in right lower corner the content of DCStack,
- to the left the stack of activation records is shown,
- in left lower corner we exhibit a part of code of currently executed method.

Relation dep is visible in activation records of ComputeInhFor
the node K \textit{depends} on node L.

Attention! This picture uses colour \textit{Yellow} instead of colour \textit{Black} – it was done in order to avoid blackening the rectangles and texts in them, simply read Yellow as Black.
Chapter 4.

The next lemma analyzes actions taken when a node is marked Red. The condition necessary to mark a node Red is as follow: The algorithm detects that the newest activation record of ComputeInhFor has computed the value \( L := \text{Bind}(\text{ext}(K)[i], \text{decl}(K)) \) and found that \( L \) is marked Grey. It means that the stack of activation records contains an activation record of ComputeInhFor where the parameter \( K \) is equal to the recently computed value of the variable \( L \). The algorithm pushes two elements on DCStack. Having this in mind we formulate the following

**Lemma 4.2.2.** If during an execution of the algorithm a node \( B \) is marked Red then

- activation records of ComputeInhFor are closed one after another until the current activation record has the object \( K \) marked Red,
- when an activation record of ComputeInhFor is going to be closed and node \( K \) is Grey then the algorithm puts two elements onto DCStack and marks \( K \) Black, then the activation record is closed, instruction return is executed,
- observe that these actions do not allow to reach Assertion1,
- when the current activation record of ComputeInhFor has the object \( K \) marked Red then the instructions \{call CycleMessage(K); L := Object: exit\} are executed causing that the algorithm skips Assertion1 and reaches Assertion2.

**Lemma 4.2.3.** Whenever the algorithm reaches Assertion1 the node \( L \) is White

*Proof.* It is clear that when the algorithm reaches Assertion1 then \( L \) cannot be Grey nor Black nor null. From the above lemma we know that \( L \) is not marked Red. □

**Lemma 4.2.4.** The set \( W = \{ n \in \text{Classes} : n \text{ is White} \wedge n \neq \text{Root} \} \) of nodes marked White excluding Root, together with the function \( \text{inh} \) is a tree. The class \( \text{Object} \) is the root of the tree.

*Proof.* Proof goes by induction w.r.t. \( n \) – the number of executed instructions “Mark K White”. For \( n = 0 \) the tree contains only its root \( \text{Object} \). Suppose that the thesis is true for a number \( k \). At the next execution of instruction "Mark K White" we add an edge going from a non-White node \( K \) to a certain White node and we mark \( K \) White. It follows from (ii) and (iii) of Lemma 4.2.1 that the new graph is a tree again. □

**Corollary 4.2.5.** In every step of computation, if a node \( K \) is White then all nodes reachable by a path \( \text{inh}^* \) from \( K \) are White too.

**Lemma 4.2.6.** The following statements are invariants of every computation of the algorithm

A) If \( K \) is White then any element of the form \((\text{decl} | \text{inh})^*(K)\) is White.

B) If the instruction call ComputeInhFor(K) is going to be executed then \( K \) is Black and \( \text{decl}(K) \) is White.

*Proof.* Proof of B) We entered ComputeInhFor either from Preorder, and then the thesis is obvious or from ComputeInhFor. In this case we execute the instruction call ComputeInhFor(L) and either a) \( L = \text{Bind}(\text{ext}(K)[1], \text{decl}(K)) \)

or

b) \( L = \text{BindInh}(\text{ext}(K)[i], L') \) and \( L' \) is White.

If a) then \( \text{decl}(L) \in (\text{decl} | \text{inh})^*(\text{decl}(K)) \) therefore by inductive assumption A \( \text{decl}(L) \) is
4.2 Analysis of algorithm

White.

If b) then \( \text{decl}(L) \in \text{inh}^*(L') \) therefore \( \text{decl}(L) \) is White. It ends the proof of B).

Proof of A) From B) and the inductive assumption it follows that when \( K \) is marked White then \( (\text{decl} | \text{inh})^*(\text{decl}(K)) \) is White. Using corollary 4.2.5 we have \( \text{inh}^*(K) \) is White hence \( (\text{decl} | \text{inh})^*(K) \) is White which ends the proof of A). \( \Box \)

**Lemma 4.2.7.** When the algorithm computes \( \text{Bind} \) or \( \text{BindInh} \) then its second argument is a White node.

**Proof.** Check the places where the instructions call \( \text{Bind} \), respectively call \( \text{BindInh} \), occur. Instruction call \( \text{Bind} \) occurs once before the while statement. Its second argument, \( \text{decl}(K) \) is White, c.f. lemma 4.2.6. Instruction call \( \text{BindInh} \) occurs once, inside the while statement. Its second argument \( L \) is White c.f. lemma 4.2.3. \( \Box \)

**Lemma 4.2.8.** If each element of the form \( (\text{decl}|\text{inh})^*(K) \) is White then for every identifier \( \text{id} \) and for any later moment in the execution of the algorithm the evaluation of expression \( \text{Bind}(\text{id}, K) \) (respectively, \( \text{BindInh}(\text{id}, K) \)) give the same result.

**Lemma 4.2.9.** In any moment of execution of the algorithm, i.e. for any function \( \text{inh} \), and for every type name \( \text{Path} \), such that \( \text{length}(\text{Path}) > 0 \) and for every class \( M \)

\[
R = \text{bind}(\text{Path} \text{ in } M) \equiv L := \text{Bind}(\text{Path}[1], M); \\
\text{for } i := 2 \text{ to } \text{length}(\text{Path}) \text{ do} \\
L := \text{BindInh}(\text{Path}[i], L) \\
\text{done } \{ L = R \}
\]

here \( \text{Path} \) is conceived as an array of identifiers, \( \text{Path}[i] \) denotes the \( i \)-th identifier of qualified type name.

**Lemma 4.2.10.** Assume that a computation of the algorithm terminates without signalling an error. The following formula is the invariant of the loop while in the ComputeInhFor(\( K \))

\[
L = \text{bind}(	ext{ext}(K)[i] \text{ in } \text{decl}(K))
\]

**Lemma 4.2.11.** Algorithm terminates.

**Proof.** It suffices to show that the stack of activation records of ComputeInhFor is of depth limited by the number of classes. Now, recall that when an instruction call ComputeInhFor(\( K \)) is executed then node \( K \) is marked Black. Upon entrance to the procedure, node \( K \) is marked Grey. Its colour may change to White or to Black, and then the computation leaves the activation record. The colour of node \( K \) may change to Red and ComputeInhFor(\( K \)) cannot be called again. Hence it is impossible to have stack of depth bigger then \( n \), where \( n \) is number of classes.

Observe that the number of iterations of instruction while is limited by \( \text{length}(	ext{ext}(K)) \).

□ Putting all the lemmas together we obtain the following

**Theorem 4.2.12.** If the algorithm terminates without signalling any error, then it computes function \( \text{inh} \) such that the conditions \( I_1 \) and \( I_2 \) mentioned in problem 2.1.6 are satisfied.

The qualitative analysis of the algorithm is completed by the following theorem

**Theorem 4.2.13.** If the algorithm terminates and signals an error then no solution exists, i.e. the structure of classes is erroneous.
Proof. If during an execution of the algorithm a node is marked Red then a cycle in dep relation has been detected and printed out. C.f. lemma 4.2.2. If during an execution of the algorithm a message was printed "undeclared class X in the header of the class header(Y) then no solution exists. Here X stands for a type name, and in the structure of classes, no class named X is accessible from the place of declaration of the currently analyzed class Y. For the proof see [17]. □

4.2.3 Complexity of the algorithm

As concerns the complexity of the algorithm it can be estimated as follow for the well-formed programs: Each node is visited twice: once by the procedure ComputeInhFor and second by the procedure Preorder. During these visits the while instruction of the procedure ComputeInhFor is executed. The number of iterations is equal to the length of path appearing after extends. To this cost one must include $O(n^2)$ – the cost of operations Bind executed. In a pessimistic case the cost may be as high as $O(n^3)$. In real programs the paths occurring after the key word extends are not too long. The cost of Bind can be also less then pessimistic $O(n^2)$. In practical cases the cost of the algorithm is linear.

4.3 Remarks on efficiency and error recovery

The algorithm solves the system of two recursively defined functions. Hence, it was not obvious how to prove its correctness and completeness. The proof of correctness of non-deterministic algorithm took 10 pages.

The method of elaborating types proposed by Igarashi & Pierce [11] is highly ineffective, for it requires elaboration of each segment of a qualified type anew. The nondeterministic algorithm (see section 3.1) stores the elaborated types and makes possible the further usage of earlier stored results. The deterministic algorithm uses pebbling by pebbles of different colours thus making the algorithm more efficient.

The algorithm continues its work after discovering an error in the structure of classes. The first error found is accompanied by the unquestionable diagnostic. Hence one can confide in it. The diagnostic of subsequent errors has a lower level of credibility. Can we trust the subsequent error signals? Is it possible to prepare a better scheme of error recovery?

The compilers of Java do error recovery only in selected phases (most of them interrupts the parsing after a syntactic error found). On the other hand, the errors found during static semantic analysis are always recovered. The errors discovered by our algorithm belong to this class. We felt obliged to include such a mechanism in our deterministic algorithm. During the work on deterministic algorithm we concentrated on its correctness and completeness (i.e. diagnostic of errors) and the error recovery came as an easy extension. In the case when the structure of classes of a program is not well-formed the estimation of the cost is an open problem.

4.4 Signalling the errors

In this appendix we present a source of an erroneous Java programs and the diagnostic produced by our algorithm.

```java
class A extends B.C {
    class D {}
}
```
class B extends E.F.G {
    class C {}
}

class E {
    class F extends A.D {
        class G {}  
    }
}

Below is a message of our algorithm

Dependence cycle:
line1: class A extends B.C {}
depends on class named B which is:
line5: class B extends E.F.G {}
which in turn depends on class named E.F which is:
line10: class F extends A.D {}
which in turn depends on class named A which is:
line1: class A extends B.C {}

Observe that no existing Java compiler gives so many details on cycle in dependence relation.
Chapter 5

Remarks on earlier work

5.1 Introduction

We begin this Section with the comments on SIMULA67 and LOGLAN’82 programming languages.

SIMULA67
In this language the type of direct superclass is designated by a single identifier. The direct superclass (or the prefixing class, in the jargon of SIMULA) has to fulfill much simpler condition

\[ I_{SIMULA} \] for every class X, \( decl(\text{inh}(X)) = inh^i(decl(X)) \)

where \( i \) is the least non-negative integer such that above equality is holding.

It can be proved that if \( \text{inh}(X) \) is defined then \( \exists k \; decl^k(\text{inh}(X)) = decl^k(X) \), it means that the direct superclass of a given class is either a sibling of the extended class or more generally, must be found on the same level of \( decl \)-tree as the class itself. We say that SIMULA67 admits the horizontal inheritance.

Due to this simpler requirement, the task of determining the direct superclass is easy. For example, the algorithm bind may be much shorter and simpler. The same applies as well to the algorithm of determining the direct superclass.

On the other hand the requirement that the inherited class and the inheriting class are on the same level of the structure of inner classes (also called the tree of nesting modules) makes extensions of library of classes impossible. Simula67 has only two classes in its library: SIMSET and SIMULATION. On the other hand, due to the above mentioned restriction, Simula67 can use the Display Vector mechanism [4] of Algol60 without problems.

LOGLAN’82
As in Simula the type denoting the direct superclass is designated by a single identifier. No restriction on the level of direct superclass is imposed. It means that the class named B must be visible from the place where the class A is declared

\[ \text{inh}(A) = \text{bind}(B \text{ in } decl(A)). \]

This kind of inheritance can be described as upward skew inheritance, for the direct superclass is on not lower level of \( decl \)-tree than the class itself. The algorithm determining the direct superclasses for LOGLAN’82 is much simpler than the LSWA algorithm presented above. It can be compared to topological sort.

The library of classes may be extended at will. It is worthwhile to mention that LOGLAN’82
admits inheritance in all modules: procedures, functions, blocks, classes, coroutines and processes. In the papers [5,6,7] the problem of maintaining the Display Vector was addressed and solved.

**BETA**
The situation in BETA[3] is different, but no less complex than the one in Java. Inheritance in BETA is dynamic, it involves objects, not only names of classes. Note, BETA as LOGLAN'82 admits inheritance of patterns in procedures, functions, classes.

**JAVA**
The problems of Java inheritance have been studied among others by Igarashi and Pierce [16]. The scope of their paper is much broader, they present a formal semantics for (essentially) a subset FJI of new Java [11], Featherweight Java with Inner classes. Usual Java-programs are assigned their semantics via semantics of corresponding FJI-programs. It is characteristics of FJI that every extension clause is the complete names path of the direct superclass plus of all enclosing classes where there are not allowed identifier repetitions in a path. Due to the local distinctness property and the required visibility of top level class names there is exactly one inheritance function $inh$ per program which satisfies condition $I_1$ of Section 2.

But not every syntactically correct FJI-program is a well-formed FJI-program, i.e. one which can be assigned an appropriate dynamic semantics. Igarashi and Pierce require so called sanity conditions to be fulfilled. Condition 6) says: $inh$ has no cycles. Condition 7) says: There is no class which has any direct or indirect inner class as its direct or indirect superclass. These sanity conditions should correspond to condition $I_2$ in Section 2 which expresses that the dependency relation is free of cycles, see Java Language Specification [11], (Section 8.1.4, Superclasses and Subclasses).

However, the sanity conditions and the conditions $I_1$, $I_2$, restricted to FJI, are not equivalent. FJI is more liberal. Example program 3.2.13 in Section 4.1 is a drastic counter example of equivalence. Example 3.2.13 is a well-formed FJI-program, but algorithm LSWA reports an error as we have seen in Section 4.1: Condition $I_2$ is violated, the dependency relation has a cycle.

Igarashi and Pierce propose an Elaboration of Types calculus – let us call it IPET – which allows to infer binding. Inheriting is a special case of binding. $P \vdash X \Rightarrow T$, read “type $X$ is elaborated to class $T$ in class $P$”, is what we would express $bind_{inh}(X \text{ in } P) = T$ or $bind_{inh}(X \text{ in } P) = T$. Inferring in a general top-down manner does not work because there is one inference rule, namely ET-SimpEncl, with a metatheoretic premise $P \vdash X. D \uparrow$ which means: “There is no derivation of $P \vdash X. D \Rightarrow T$ for any class $T$”.

In our opinion it is a serious methodological error to mix a theory and its metatheory. Such mixing leads to paradoxes frequently. The authors of [16] give no evidence that such a paradox will not appear.

They recommend to read the rules in a bottom-up manner and so to interpret them as a generalized program procedure, implemented and executed by help of consecutive run-time stacks of procedure activation records [4]. Generalized means: We have not only pushing-down and popping-up of activation records, but we have also backing-up in case there is some evidence that all actions whatsoever after an activation $P.C \vdash D$ are never resulting in any class $T$ (see rule ET-SimpEncl). Even if such evidence is showing up, e.g. by cycling or other infinitely expanding run-time stacks, we are to know which are correct back-up states in order to guarantee determinate, non-multivalued results.

Program examples demonstrate that we need clearer correctness, completeness and termination criteria for IPET. We would like to consider calculus IPET rather a method
than an algorithm. It is possible to repair calculus IPET and to transform algorithm LSWA and its binding function towards a calculus without metatheoretic premises where all inferences can be done in a top-down manner, see a forthcoming article.

The subsection 5.2.1 of the section 5 of [11] is devoted to the elaboration of types, which makes the identification of direct superclasses possible. The Table 1 cites six inference rules of the paper [11][p.35]. They define a calculus. We shall name it the IPET calculus. We are analyzing the calculus. The aim of the IPET calculus is to help in identifying the direct superclasses in any Java program. We present some observations.

1. The calculus is not determinate. It means that it is possible to derive two or more different classes as a direct superclass of certain class. One may say the calculus is inconsistent.

2. Moreover, there exist two different models of the calculus.

3. Moreover, the models do not enjoy the property: the intersection of two models is a model. Therefore it is difficult to say what is the meaning of the calculus.

The authors are aware of the non-determinacy. They say the calculus plus a metatheoretic hint: apply the rules from bottom up may be called an algorithm. They have chosen an inadequate word. It is not an algorithm however. For it does not enjoy the termination property c.f. [11][p. 34] . Therefore we propose to call it a method. Again the method may lead towards different answers. We shall show that the method can be specified in two different manners. The IPET calculus may be used to define the inheritance function inh from classes to classes. We can take another approach and ask: has the IPET calculus one or more models? It turns out that it has several, non-isomorphic, models. Let us remark that each model can be constructed by the corresponding algorithm. Hence it is necessary to add some hint of metatheoretical nature. Usually, a calculus (or a theory) is accompanied by the metatheoretical hint: choose the least model. We are going to show that this will not work. For the intersection of two models needs not be a model.

It seems that the source of the problems is in admitting an inference rule (ET-SimpEncl). One of the premises of this rule is a metatheorem: \( P \vdash X.D \uparrow \). This formula expresses the following property: for every class \( T \) there is no proof of the formula \( P \vdash X.D \Rightarrow T \). One remedy would be to eliminate the rule and to replace it with some rules that do not introduce metatheoretic premises and such that the premises are positive formulas. Another approach would consists in extending the language of the theory, such that the expression \( P \vdash X.D \uparrow \) were a well formed expression of the language, and adding some inference rules to deduce formulas of this kind. Nothing of this kind happens in [11].

We are stressing the fact that the discussed paper is one of many papers of various authors, where the reader discovers a metatheorem as a premise of an inference rule. Therefore our remarks are of general character.

5.2 Igarashi’s and Pierce’s calculus IPET for elaboration of types

Igarashi and Pierce [11][5.2.1 pp.34-36] are presenting a calculus IPET of derivation rules for a so called elaboration relation of types. The formulae of the calculus have the form (are written as) \( P \vdash X \Rightarrow T \) to be read: The simple or qualified class type \( X \) (i.e. a non-empty sequence of class identifiers separated by periods) occurring inside the directly enclosing body of class declaration occurrence \( P \) is elaborated to (resp. is bound to) the class declaration occurrence \( T \). In other words: the meaning of the type \( X \) in the class \( P \) is the
class T. We have to differentiate between a syntactical entity and its occurrences, see the
ALGOL68-report [14], because a class declaration (class for short) may occur several times
in a given program.

Observe that there is a bijection between class occurrences like P (or T) and their so
called absolute types (paths) C₁,···,Cₙ where Cₙ is the name of class P and Cₙ₋₁,···,C₁
are the names of the successive class occurrences which enclose class occurrence P. To
understand this one should remark that the classes of a program form a tree. The root of
the tree is a fictitious class that encloses all the top level classes of the program. Let n be
an internal node of the tree. It can be identified with the path leading from the root to
it. Such a path consists of the names of classes. All direct inner classes declared in the
class which is the node n are the sons of the node n. Therefore we are entitled to identify
an occurrence of a class declaration and the absolute path of it. Beside the user declared
class occurrences in a Java-program there are two implicit, fictitious class occurrences:

1. Root = {···} which is enclosing all top level classes (i.e. other class occurrences) of
the Java-program and which has no name nor extends clause. The authors of [11]
represent Root by its fictitious name ⋆ which users are not allowed to write.
⋆.C₁,···,Cₙ is identified with C₁,···,Cₙ.

2. Object = class Object {···} the name of which is Object, which is directly enclosed
by Root and which has also no extends clause. Without loss of generality we can
assume that there are no classes declared inside the body of Object.

Let us explain the meaning of some premises in the inference rules. In three rules one finds
the premise of the form CT(P,C) = class C extends X {···}. In this way the authors
Igarashi and Pierce express the fact that the class P.C extends the type X i.e. the class
which is the meaning of the type X in this place where the declaration the class P.C occurs.
The formulas of the form P.C ∈ Dom(CT) mean the program contains the class named
C in its directly enclosing class to be identified with the path P. Obviously, the formula
of the form P.C.D /∈ dom(CT) expresses the fact that the class to be identified with the
path P.C does not contain any class named D. In the Table 1 we present Igarashi’s and
Pierce’s calculus IPET for elaboration of types.
Below we collect some observations and comments.

1. The system IPET is inconsistent! Consider a Java program with a user declared class
Object. From the axiom (I ET-OBJECT) one obtains that the meaning of the name
Object is Object, or ⋆.Object. From the rule (II ET-IN-CT) one obtains P.Object
where P is the class containing the user declared class Object.
Remedy: Consider only programs without user declared class named Object.

2. The rule (III ET-SimpEnc) has four premises. The fourth premise of the form
\[ P \vdash X \uparrow \] is in fact a metatheorem “there is no class T such that the triplet P \vdash
X ⇒ T has a formal proof”\]. This rule is a source of severe problems as we shall see
below.

3. There is no definition of the notion of proof in the system IPET of inference rules.
Should one accept the classical definition of the notion of formal proof then the lack
of possibilities to derive premises of the form P \vdash X \uparrow becomes evident. We know,
the standard answer to this remark is: but everything is finite and therefore one can
control the situation. Is this one person added to the definition of the proof? What
instructions are given to her/him making the task of recognition of the impossibility
of the proof possible?
### Table 5.1: Igarashi’s & Pierce’s rules of elaboration

<table>
<thead>
<tr>
<th>Rule Description</th>
<th>Rule Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. (ET-Object)</td>
<td>( P \vdash \text{Object} \Rightarrow \text{Object} )</td>
</tr>
<tr>
<td>II. (ET - In CT)</td>
<td>( \frac{P.C \in \text{dom}(CT)}{P \vdash \text{Object} \Rightarrow \text{Object}} )</td>
</tr>
<tr>
<td>III. (ET-SimpEncl)</td>
<td>( \frac{CT(P.C) = \text{class} C \ \text{extends} \ { \cdot \cdot \cdot }}{P \vdash X.D \uparrow} )</td>
</tr>
<tr>
<td>IV. (ET-SimpSup)</td>
<td>( \frac{CT(P.C) = \text{class} C \ \text{extends} \ { \cdot \cdot \cdot }}{P \vdash X.D \Rightarrow T} )</td>
</tr>
<tr>
<td>V. (ET-Long)</td>
<td>( \frac{P \vdash X \Rightarrow T \quad T.C \in \text{dom}(CT)}{P \vdash X.C \Rightarrow T.C} )</td>
</tr>
<tr>
<td>VI. (ET-LongSup)</td>
<td>( \frac{CT(P'.D) = \text{class} D \ \text{extends} \ { \cdot \cdot \cdot }}{P' \vdash Y.C \Rightarrow U} )</td>
</tr>
</tbody>
</table>

4. A reader may hesitate what does the following sentence mean "A straightforward elaboration algorithm obtained by reading the rules in a bottom-up manner might diverge." [11][p.34] Two questions appear immediately. Is there an implicitly defined elaboration algorithm? What does it mean "reading the rules in a bottom-up manner"?

Our first guess is that the authors think of Gentzen-style proofs. The textbook on mathematical logic [15] describes an algorithm constructing a formal proof of a logical formula. The system of inference rules must enjoy some properties and the algorithm must precisely describe which rule is to be applied in every step of the algorithm.

Our second guess is as follow: Subcase 1. Consider an open question \( [P \vdash X \Rightarrow?] \) and apply the rules trying to construct the formal proof of some triplet \( P \vdash X \Rightarrow T \). Depending on the rule applied we create some new open questions. In this way a tree is constructed with the nodes decorated by open questions or axioms. Once an inner node has all its sons decorated by closed triplets, one can close an open question by application of the rule that constructed the sons of the current node. Should we come back to the root with an answer the formal proof is constructed and the searched class \( T \) is found. Subcase 2. As previously consider an open triplet and build a formal proof of some triplet \( P \vdash X \Rightarrow T \) applying the rules from the sixth to the first one.

Which guess is a proper one?

5. The authors are aware that construction of the proof is not always possible. They give an evidence of the fact that the algorithm we guessed may loop without exit [11][p.34].

6. In fact the task of type elaboration is divided in two subtasks: a) to find if the program is a well formed one, b) to define a function \( \text{inh} \) which for every user declared class \( C \) returns the direct superclass of \( C \). It is evident that the IPET does not help in detecting the possible errors in typing.
7. Seeing the incompleteness of the IPET calculus (c.f. the rule III) one may ask a slightly different questions: is it true that the IPET has exactly one model? We shall see that there are several models.

8. The next question appears: Is it possible to equip the calculus with an extra hint of the kind: consider the least one of all models as THE model of the IPET calculus.

9. This hope should be abandoned in the light of the section 5.5.

5.3 Langmaack’s, Salwicki’s and Warpechowski’s binding functions $bind_{inh}$ compared with IPET

In [16] Langmaack, Salwicki and Warpechowski developed binding functions $bind_{inh}$ which, based on given inheritance (i.e. direct superclassing) functions $inh$, determine the associated class occurrences $T$ for class types $X$ directly enclosed by the bodies of class occurrences $P$:

$$bind_{inh}(X \text{ in } P) = T.$$ 

Above that the authors developed an algorithm LSWA which determines superclassing function $inh_0$ which is the least fixed point of the continuous functional $Bdfl'(inh)$

$$inh_0 = Bdfl'(inh_0)$$

and is totally defined for all (finitely many) user declared classes $P$ in a given well-formed Java-program. Especially: $inh_0$ satisfies the so called inheritance condition $I_1$, i.e. for all user declared classes $P$ $inh_0(P)$ is defined and the equation

$$inh_0(P) = bind_{inh_0}(X \text{ in } P')$$

is holding where $X$ is the type $ext(P)$ in the extends clause of $P$ and $P'$ is that class occurrence $decl(P)$ which directly encloses $P$. Because both theories of [11] and of [16] ought to agree

the ternary relation $bind_{inh_0}(X \text{ in } P) = T$

should satisfy all six rules of the types elaboration relation

$$P \vdash X \Rightarrow T$$

in calculus IPET. Both theories would agree perfectly iff there were exactly one distinguished satisfying binding function for a well-formed Java-program such that the set of all derivable triplets $(X, P, T)$ is exactly the binding function $bind_{inh_0}$. In order to have an easier way of comparison we translate the rules to the mode of expression in [16] what is yielding the formulation of the definition 5.3.1

**Definition 5.3.1.** The calculus BIPET is defined by the rules of the Table 2.

**Theorem 5.3.2.** The function $bind_{inh_0}$ is satisfying all six rules of BIPET (and hence of IPET) calculus.

**Proof.** Are these six rules (implications) really holding? We shall check them and find that the answer is: Yes.

I. (BET-Object)

As Object is the only class named Object and is directly enclosed by Root the required equation is holding independently of all possible inheritance or direct superclassing functions $inh$ which parametrize $bind_{inh}$.

II. (BET-InCT)

If the class $P$ contains a direct inner class named $C$, i.e. $P.C$ is defined, $P.C \in Dom(CT)$,
then the conclusion the meaning of the name $C$ in the class $P$ is the class $P.C$ is holding independently of all possible inheritance functions.

**III. (BET-SimpEncl)**

Let $P$ be a user declared class. From the first premise $\text{bind}_{\text{inho}}(D \text{ in } P) = T$ we have that there exist natural numbers $i \geq 0$, $l \geq 0$ such that $T = \text{inho}_0(\text{decl}^i(P)).D$ where the pair $(l, i)$ is the least in the lexicographic order such that the right hand side expression is defined.

The third premise "$\text{bind}_{\text{inho}}(\text{ext}(P.C).D \text{ in } P)$ is undefined" says that for every $l \geq 0$ the expression $\text{inho}_0(\text{bind}_{\text{inho}}(\text{ext}(P.C) \text{ in } P)).D$ is undefined, and so $\text{inho}_0(\text{inho}_0(P.C)).D$ is undefined, because the function $\text{inho}_0$ enjoys the property $I_1: \text{inho}_0(P.C) = \text{bind}_{\text{inho}}(\text{ext}(P.C) \text{ in } P)$. We claim that the pair $(j + 1, i)$ is the least pair in the lexicographic order such that $\text{inho}_0(\text{decl}^{j+1}(P.C)).D$ is defined. Suppose that there exists a pair $(k, l)$ such that the expression $\text{inho}_0(\text{decl}^k(P.C)).D$ has a defined value and that the pair $(k, l)$ precedes the pair $(j + 1, i)$. From the second and third premise we know that $k \neq 0$. In other words the path $\text{inho}_0(\text{decl}^k(P.C)).D$ goes from the class $P.C$ through the class $P$ further on. From the previous considerations we know the pair $(j, i)$ is the least in the lexicographic order such that the expression $\text{inho}_0(\text{decl}^j(P.C)).D$ is defined. Hence $k = j + 1$ and $l = i$. The cases where $P = \text{Object}$ or $P = \text{Root}$ can be checked in a separated way.

**IV. (BET-SimpSup)**

Let $P.C$ be a user declared class. The premise $\text{bind}_{\text{inho}}(\text{ext}(P.C).D \text{ in } P) = T$ tells us that there exists a natural number $i \geq 0$ such that the value of $T = \text{inho}_0(\text{bind}_{\text{inho}}(\text{ext}(P.C) \text{ in } P)).D$ is defined. Since the function $\text{inho}_0$ enjoys the property $I_1$ we know that the expression $\text{inho}_0(\text{inho}_0(P.C)).D$ has a value $T$. Hence $T = \text{inho}^{i+1}(\text{decl}^0(P.C)).D = \text{bind}_{\text{inho}}(D \text{ in } P.C)$. For the pair $(0, i + 1)$ is the least pair such that the value of $\text{inho}^{i+1}(\text{decl}^0(P.C)).D$ is defined. The only less candidate $(0, 0)$ is excluded by the second premise. If $P = \text{Object} or

<table>
<thead>
<tr>
<th>Table 5.2: IPET rules translated using $\text{bind}_{\text{inho}}$ function</th>
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<tbody>
<tr>
<td><strong>I. (BET-Object)</strong> $\text{bind}_{\text{inho}}(\text{Object in } P) = \text{Object}.$</td>
</tr>
<tr>
<td><strong>II. (BET - InCT)</strong> $\text{bind}_{\text{inho}}(C \text{ in } P) = P.C.$</td>
</tr>
<tr>
<td><strong>III. (BET-SimpEncl)</strong> $\text{bind}<em>{\text{inho}}(D \text{ in } P) = T,$ no class named D in P.C, (\text{bind}</em>{\text{inho}}(\text{ext}(P.C).D \text{ in } P)) is undefined, $\text{bind}_{\text{inho}}(D \text{ in } P.C) = T.$</td>
</tr>
<tr>
<td><strong>IV. (BET-SimpSup)</strong> $\text{bind}<em>{\text{inho}}(X \text{ in } P) = T,$ class P.C has no direct inner class named D, $\text{bind}</em>{\text{inho}}(\text{ext}(P.C).D \text{ in } P) = T.$</td>
</tr>
<tr>
<td><strong>V. (BET-Long)</strong> $\text{bind}_{\text{inho}}(X.C \text{ in } P) = T.C.$</td>
</tr>
<tr>
<td><strong>VI. (BET-LongSup)</strong> $\text{bind}_{\text{inho}}(X.C \text{ in } P) = U.$</td>
</tr>
</tbody>
</table>
Chapter 5.

P = Root then the conclusion is holding trivially.

V. (BET-Long)
The conclusion is holding due to definition of bind_{inh_0}.

VI. (BET-LongSup)
In case \( P'.D \) is user declared we begin with the third premise \( \text{bind}_{inh_0}(X.C \text{ in } P') = U \).
This means that there exist a natural number \( l \geq 0 \) such that \( inh_0^{l}(\text{bind}_{inh_0}(X \text{ in } P')).C = U \) and \( l \) is the least one that the left hand side is defined. Making use of the first premise and condition \( I_1 \) for \( P'.D \) we obtain \( inh_0^{l+1}(P'.D).C = U \). From the second premise we conclude that the exponent \( l + 1 \) is the least one such that the left hand side is defined. The soundness of this rule follows from the definition 2.1.2 of the function \( \text{bind}_{inh_0} \) and from the first premise.

In case \( P'.D \) is not user declared the conclusion is holding trivially.

Hence the binding function \( \text{bind}_{inh_0}(X \text{ in } P) = T \) satisfies all six rules of IPET where the given Java-program is well-formed. We recall that its least fixed point is the inheritance function \( inh_0 \) and satisfies the conditions \( I_1 \) and \( I_2 \) of the Java Language Specification JLS [7] as formalized in [16]. Our next question is: Is IPET defining \( \text{bind}_{inh_0} \) uniquely? Are there other binding functions which satisfy all rules of IPET? Is there a distinguished binding function of IPET? In what sense does IPET define a distinguished binding function? Or, perhaps, there is no a reasonable way to distinguish a good inheritance function?
We have already seen that IPET does not allow straightforward, constructive top-down application.

5.4 On another binding function \( \text{Bind}_{inh_{B0}} \) which satisfies all rules of IPET

It is astounding that there is a way to define a family of binding functions \( \text{Bind}_{inh}(X \text{ in } P) \) which are different from \( \text{bind}_{inh}(X \text{ in } P) \) and which lead to an analogous satisfaction theorem (of IPET, by an analogous \( \text{Bind}_{inh_{B0}} \) instead of \( \text{bind}_{inh} \)). As previously, we assume that every user declared class has an explicit extends clause with a type of length \( \geq 1 \) as the authors of [11] and their calculus are requiring. \( \text{Bind}_{inh} \) is to become a partial mapping

\[
\text{Bind}_{inh} : \text{Types} \times \mathcal{C}^{\text{RO}} \xrightarrow{\text{part}} \mathcal{C}^{\text{O}}
\]

where \( \mathcal{C} = \text{Classes} \) is the set of user declared class occurrences in a given Java-program with inner classes, \( \mathcal{C}^{\text{O}} = \mathcal{C} \cup \{\text{Object}\} \) and \( \mathcal{C}^{\text{RO}} = \mathcal{C}^{\text{O}} \cup \{\text{Root}\} \). \( \text{Bind}_{inh} \), \( \text{Types} \) is the set of simple or qualified types in the Java-program. The function \( \text{Bind} \) is parametrized by a given partially defined inheritance function (direct superclassing function) \( inh : \mathcal{C}^{\text{part}} \xrightarrow{\text{part}} \mathcal{C}^{\text{O}} \) as \( \text{bind}_{inh} \) is. The values of \( inh(\text{Root}) \) and \( inh(\text{Object}) \) are undefined.

Consider the ordered alphabet \( \mathcal{A} \) of the two operators \( inh \) and \( decl \), where we define \( inh \) to be less than \( decl \), \( inh < decl \). This order is inducing a lexicographical (from the right), total order in the set \( \mathcal{A}^* \) of all words over \( \mathcal{A} \). For example, the words \( inh < decl \cdot inh < decl \cdot inh \cdot decl \) are in this order.

Let \( w = id_1 \cdot id_2 \cdots id_n, \ w \in \mathcal{A}^* \). Let \( P \) be a class. The word \( w \) applied to the class \( P \) is the class \( w(P) = id_1(id_2(\cdots(id_n(P)) \cdots)) \) or the result is undefined. Clearly \( \varepsilon(P) = P \).

Now, we have the following
5.4 On another binding function $Bind_{inh\_bo}$ which satisfies all rules of IPET

**Definition 5.4.1.** Let $X$ be a type of length $\geq 1$, $P \in C^O$ and $inh \in C \xrightarrow{part} C^O$. Then

$$Bind_{inh}(X \text{ in } P) \overset{df}{=} \begin{cases} 
\mu w(P).X & \text{if } \text{length}(X) = 1 \text{ and there exists the least word } \mu w \in A^* \text{ such that } \mu w(P).X \in C^O \text{ is defined and there are no repeated classes on this path } \\
Bind_{inh}(C \text{ in } P) & \text{where } \text{length}(C) = 1 \text{ and } \text{length}(X') \geq 1 \text{ and } \\
\mu w & \text{is defined} \\
\text{undefined} & \text{otherwise}
\end{cases}$$

**Remark 5.4.2.** It is worthwhile to observe that the $Bind_{inh}$ function defined in this way differs from the $bind_{inh}$ function defined earlier. Namely, in the definition of $bind$ we consider only words of the form $inh\_decl$, where $i,j \geq 0$. \qed

**Example 5.4.3.** C.f. the class’ structure of the example 5.4.4. We have $bind(C \text{ in } decl(BD\_F)) = bind(C \text{ in } B) = inh(B).C = Bind(C \text{ in } B) = A\_S.C$.

Now we are going to look for a specific inheritance function $inh\_bo$ such that $Bind_{inh\_bo}$ satisfies all rules of IPET. We go an analogous way as in [17] and look for an appropriate functional $Biddle'$ such that $inh\_bo$ is the least fixed point. The natural functional $BDfl(inh)(P) \overset{df}{=} Bind_{inh}(ext(P) \text{ in } decl(P))$ is, unfortunately, not monotone and continuous in

$$(C \xrightarrow{part} C^O) \xrightarrow{tot} (C \xrightarrow{part} C^O)$$

where $C \xrightarrow{part} C^O$ is a cpo completely partially ordered by the set theoretic inclusion $\subseteq$ of partially defined inheritance functions with bottom function $inh\_bo = \emptyset$.

**Example 5.4.4.** Let us consider the following (structure of a) Java-program:

```java
class A extends Object {
    class E extends Object { } 
    class C extends Object { } 
}
class B extends A {
    class E extends Object { } 
    class D extends C { 
        class F extends E { } 
    }
}
```

We have $\emptyset = inh\_bo \subset BDfl(inh\_bo) \not\subseteq BDfl^2(inh\_bo)$ because $BDfl(inh\_bo)(B) = A$, $BDfl(inh\_bo)(D) = \emptyset$, $BDfl(inh\_bo)(F) = B\_E$, $BDfl^2(inh\_bo)(B) = A$, $BDfl^2(inh\_bo)(D) = A\_S.C$, $BDfl^2(inh\_bo)(F) = A\_E$. \qed
Example 5.4.4 convinces us to modify functional $BDfl$ towards $BDfl'$. But first we introduce the notion of State.

**Definition 5.4.5.** An inheritance function $inh \in (C \xrightarrow{\text{part}} CO)$ is called a State iff for all classes $K \in \text{dom}_{inh}$ the following two relations

- $inh(K) \in \text{dom}_{inh}^O$, where $\text{dom}_{inh}^O \overset{df}{=} \text{dom}_{inh} \cup \{\text{Object}\}$
- $\text{decl}(K) \in \text{dom}_{inh}^R$, where $\text{dom}_{inh}^R \overset{df}{=} \{\text{dom}_{inh} \cup \{\text{Root}\}\}$

and the equation

$$inh(K) = \text{Bind}_{inh}(\text{ext}(K) \text{ in } decl(K))$$

are holding.

Definition 5.4.5 is saying that $\text{dom}_{RO}^{inh} = \text{dom}_{inh} \cup \{\text{Root, Object}\}$ is an initial tree of the whole decl-tree $C_{RO}$, and that for all $K$ inheritance chain $\{inh^i(K) : i = 0, 1, \cdots\}$ is remaining inside $\text{dom}_{RO}^{inh}$ (i.e. either has a cycle or ends up in Object or Root) and condition $I_{B1}$ (see Definition 5.4.7) is satisfied, restricted to $\text{dom}_{inh}$ as a subset of $C$. $inh$ or its dependency relation $Dep_{inh}$ may have cycles:

**Definition 5.4.6.** The dependency relation $Dep_{inh}$ associated to $inh$ is

$$Dep_{inh} \overset{def}{=} \{(K, \text{Bind}_{inh}(\text{ext}(K) |^i \text{ in } decl(K))) : K \in \text{dom}_{inh}, 1 \leq i \leq \text{length}(\text{ext}(K))\}$$

where $\text{ext}(K) |^i$ is the initial segment of length $i$ of type $\text{ext}(K)$.

To remind the reader(c.f. definition 2.1.5):

**Definition 5.4.7.** A Java-program is called Well-Formed iff there exists an inheritance function $inh_{WF}$ which satisfies the following two conditions

1. $inh_{WF}$ is defined for all classes $K \in C$ and the equation
   $$inh_{WF}(K) = \text{Bind}_{inh_{WF}}(\text{ext}(K) \text{ in } decl(K))$$
   is holding for them;
2. the induced dependency relation $Dep_{inh_{WF}}$ has no cycles in $C_{RO}$.

We consider the sub-cpo

$$C \xrightarrow{\text{State}} CO \xrightarrow{\text{part}} CO$$

of inheritance functions which are States. The word "State" is chosen because our later algorithms LSWA$^B_B$ resp. LSWA$B$ which determine the least fixed point $inh_{B0}$ are running through computation states which can be represented by the above mentioned special inheritance functions which are States.

Let’s come to the desired functional $BDfl'$. Let us introduce an abbreviation $\alpha_{inh}(A)$ denoting the following logical formula:

$$\alpha_{inh}(A) : decl(A) \in \text{dom}_{inh}^R \land A \neq \text{Root} \land A \neq \text{Object} \land \text{Bind}_{inh}(\text{ext}(A) \text{ in } decl(A)) \in \text{dom}_{inh}^O.$$  

The desired functional is

$$BDfl'(inh)(A) \overset{df}{=} \begin{cases} \text{Bind}_{inh}(\text{ext}(A) \text{ in } decl(A)) & \text{if } \alpha_{inh}(A) \\ \text{undefined} & \text{otherwise.} \end{cases}$$

**Theorem 5.4.8.** $BDfl'$ is a monotonous functional (and consequently is continuous because $C \xrightarrow{\text{State}} CO$ is finite).

We need four Lemmas 5.4.9 to 5.4.13 for a proof of Theorem 5.4.8.
Lemma 5.4.9. For every State inh, for every class $K \in \text{dom}_{\text{RO}}$ and for every type $X$: If $\text{Bind}_{\text{inh}}(X \mid_C K) \in \text{dom}_{\text{inh}}$ then $\text{Bind}_{\text{inh}}(X \mid_C K) \in \text{dom}_{\text{inh}}$ for $1 \leq i < \text{length}(X)$.  

Proof. Assume the thesis of the Lemma is wrong. Then there is a smallest $i_0$ with $1 \leq i_0 < \text{length}(X)$ and $\text{Bind}_{\text{inh}}(X \mid_{i_0} K) \notin \text{dom}_{\text{inh}}$. Then this class $C_{i_0}$ is such that $\text{decl}(C_{i_0}) \in \text{dom}_{\text{inh}}$ because $\text{inh}$ is a State. Then $\text{inh}(C_{i_0})$ is undefined and $\text{Bind}_{\text{inh}}(X \mid_C K)$ is necessarily a nested class $C_I$ inside or equal $C_{i_0}$ with $\text{decl}^{i_0}(C_I) = C_{i_0}$, $l = \text{length}(X)$, and $C_I$ is necessarily $\notin \text{dom}_{\text{inh}}$. Contradiction!  

Lemma 5.4.10. Let $\text{inh}_0$ be a State. Let inheritance function $\text{inh}$ be an arbitrary extension of function $\text{inh}_0$ on a subset of $\mathcal{C}$.  

A) For every class $K \in \text{dom}_{\text{RO}}$, and every word $w_0 \in \mathcal{A}_0 = \{ \text{decl}, \text{inh}_0 \}^*$ and analogous word $w \in \mathcal{A} = \{ \text{decl}, \text{inh} \}^*$, $w_0(K) = w(K) \in \text{dom}_{\text{inh}}$ or both sides are undefined.  

B) For every class $K \in \text{dom}_{\text{inh}}$ and for every type $X$:

If for every $1 \leq i < \text{length}(X)$, $\text{Bind}_{\text{inh}0}(X \mid_C K) \in \text{dom}_{\text{inh}}$ and for all $1 \leq i < \text{length}(X)$

$\text{Bind}_{\text{inh0}}(X \mid_C K) = \text{Bind}_{\text{inh}}(X \mid_C K)$  

and either $\text{Bind}_{\text{inh0}}(X \mid_C K) = \text{Bind}_{\text{inh}}(X \mid_C K) \in \mathcal{C}_O$ or both sides are undefined.

Proof. Proof of A)

Two cases are to be discussed: A1) $w_0(K) = K_n \in \text{dom}_{\text{inh}0}$ resp. A2) $w_0(K)$ is undefined.  

A1) Because $\text{inh}_0 \subseteq \text{inh}$ we have $w(K) = K_n$ as well.  

A2) Let $K_n \in \text{dom}_{\text{inh}0}$ be the final class in the chain of classes $K = K_0, K_1, \ldots, K_n \in \text{dom}_{\text{inh}0}$ with $n < m$ which $w_0 = \text{inh}_0 \cdots \text{d}_m$ and $K$ are inducing, $\text{d}_i \in \mathcal{A}_0$. Then $\text{d}_{0, n+1}(K_n)$ is undefined. $K_n$ is Object or Root because $\text{inh}_0$ is a State. Therefore $\text{inh}_0 \subseteq \text{inh}$ and $\text{inh}(\text{Root})$ and $\text{inh}(\text{Object})$ are undefined we have for the analogous word $w = \text{d}_1 \cdots \text{d}_{n+1}$, $\text{d}_i \in \mathcal{A}$: Either $K_n = \text{Root}$ and $\text{d}_{0, n+1} = \text{d}_{n+1} = \text{dcl}$ or $K_n \in \{ \text{Root}, \text{Object} \}$ and $\text{d}_{0, n+1} = \text{inh}_0, \text{d}_{n+1} = \text{inh}$. So $w(K)$ is undefined.

Proof of B)

(Base of induction $\text{length}(X) = 1$)

Due to A) the $\mathcal{A}_0$- resp. $\mathcal{A}$-chains of classes starting in $K$ coincide. So Definition 5.4.1 does not differentiate between $\text{inh}_0$ and $\text{inh}$ which the definition is based on.  

(Induction step $\text{length}(X) > 1$)

Let for every $1 \leq i < \text{length}(X)$

$\text{Bind}_{\text{inh}_0}(X \mid_C K) \in \text{dom}_{\text{inh}_0}$  

Due to induction hypothesis and assumption (*) we have for all $1 \leq i < \text{length}(X) - 1$

$\text{Bind}_{\text{inh}_0}(X \mid_C K) = \text{Bind}_{\text{inh}}(X \mid_C K) \in \text{dom}_{\text{inh}_0}$

and

$\text{Bind}_{\text{inh}_0}(X \mid_{\text{length}(X) - 1} C) = \text{Bind}_{\text{inh}}(X \mid_{\text{length}(X) - 1} C) \in \text{dom}_{\text{inh}_0}$.  

So we find an analogous situation as in the induction base and Definition 5.4.1 does not differentiate between $\text{inh}_0$ and $\text{inh}$.  

Lemma 5.4.11. If $\text{inh}$ is a State then the inheritance function $\text{inh}' = \text{BDF} \text{f}'(\text{inh})$ is an extension of $\text{inh}$.

Proof. Let $A \in \text{dom}_{\text{inh}}$. Then $A \neq \text{Root}$, $A \neq \text{Object}$, $\text{decl}(A) \in \text{dom}_{\text{inh}}$. $\text{inh}(A) = \text{Bind}_{\text{inh}}(\text{ext}(A) \in \text{decl}(A))$ because $\text{inh}$ is a State. So $\alpha_{\text{inh}}^n(A)$ is holding and $\text{inh}'(A) = \text{Bind}_{\text{inh}}(\text{ext}(A) \in \text{decl}(A))$ by definition. So $\text{inh}'(A) = \text{inh}(A)$, i.e. $\text{inh}'$ is an extension of $\text{inh}$.  

□
Remark 5.4.12. Let \( \text{inh} \) be a State and \( A' \in C \setminus \text{dom}_{\text{inh}} \) with \( \text{decl}(A') \in \text{dom}_{\text{inh}}^\alpha \) (i.e. \( A' \) is a so called candidate) and \( \text{Bind}_{\text{inh}}(\text{ext}(A')) \in \text{dom}_{\text{inh}}^O \) (i.e. \( A' \) is a so called generating candidate). Then let us denote the extension 
\[
\text{inh} \cup \{ (A', \text{Bind}_{\text{inh}}(\text{ext}(A')) ) \}
\]
of \( \text{inh} \) by \( \text{inh}^{A'} \).

Lemma 5.4.13. If \( \text{inh} \) is a State then \( \text{inh}' = \text{BDfl}'(\text{inh}) \) is also a State.

Proof. Let \( A \in \text{dom}_{\text{inh}}' \). We have to show that \( \text{inh}'(A) \in \text{dom}_{\text{inh}}^O' \) and \( \text{decl}(A) \in \text{dom}_{\text{inh}}^R' \) and that \( \text{inh}'(A) = \text{Bind}_{\text{inh}}'(\text{ext}(A) \in \text{decl}(A)) \) is holding. We have two subcases
A) \( A \in \text{dom}_{\text{inh}} \) and
B) \( A \in \text{dom}_{\text{inh}}' \setminus \text{dom}_{\text{inh}} \).

Subcase A) is straightforward by help of Lemmas 5.4.9, 5.4.10, 5.4.11.

Proof of the subcase B): Because \( \text{inh}'(A) \) is defined, \( \alpha_{\text{inh}}^P(A) \) is holding and \( \text{inh}'(A) = \text{Bind}_{\text{inh}}'(\text{ext}(A) \in \text{decl}(A)) \). Since \( \text{inh}'(A) \in \text{dom}_{\text{inh}}^O' \) and \( \text{inh}' \) is an extension of \( \text{inh} \) we have \( \text{inh}'(A) \in \text{dom}_{\text{inh}}^O \). Since \( \text{decl}(A) \in \text{dom}_{\text{inh}}^R \) we have \( \text{decl}(A) \in \text{dom}_{\text{inh}}^O \). The last fact to prove for subcase B) is: \( \text{inh}'(A) = \text{Bind}_{\text{inh}}'(\text{ext}(A) \in \text{decl}(A)) \). As \( \text{decl}(A) \in \text{dom}_{\text{inh}}^R \), \( \text{inh}' \) is an extension of \( \text{inh} \) and \( \text{Bind}_{\text{inh}}'(\text{ext}(A) \in \text{decl}(A)) \in \text{dom}_{\text{inh}}^O \) then we have due to Lemma 5.4.9 and Lemma 5.4.10 B)
\[
\text{Bind}_{\text{inh}}'(\text{ext}(A) \in \text{decl}(A)) = \text{Bind}_{\text{inh}}'(\text{ext}(A) \in \text{decl}(A)).
\]
The left side is exactly \( \text{inh}'(A) \) by definition of \( \text{BDfl}' \).

Remark 5.4.14. on direct and indirect successors of States:
If in this proof of Lemma 5.4.13 we replace \( \text{inh}' \) by \( \text{inh}^N \) then we have a proof for: \( \text{inh}^A \)

is a State. We call \( \text{inh}^A \)

a direct successor State of \( \text{inh} \) and write \( \text{inh} \prec_{DS} \text{inh}^A \) with

the transitive closure \( \prec^S \) of \( \prec_{DS} \) which is an irreflexive partial order in the set of States

\( C_{\text{State}}^O \).

Proof. of Theorem 5.4.8, on monotonicity of \( \text{BDfl}' \):
Let \( \text{inh}_1 \subseteq \text{inh}_2 \) be two States and \( \text{BDfl}'(\text{inh}_1)(A) = \text{inh}_1'(A) = M \) be defined. We claim
\( \text{BDfl}'(\text{inh}_2)(A) = \text{inh}_2'(A) = M \).

Due to definition of \( \text{BDfl}' \) we have that
\[
\alpha_{\text{inh}_1}(A) \land M = \text{Bind}_{\text{inh}_1}(\text{ext}(A) \in \text{decl}(A))
\]
is holding.

Case 1: \( A \in \text{dom}_{\text{inh}_1} \). Then \( A \in \text{dom}_{\text{inh}_2} \) and \( M = \text{inh}_1'(A) = \text{inh}_1(A) = \text{inh}_2(A) = \text{inh}_2'(A) \).

Case 2: \( A \in \text{dom}_{\text{inh}_1} \setminus \text{dom}_{\text{inh}_1} \). Then \( M = \text{Bind}_{\text{inh}_1}(\text{ext}(A) \in \text{decl}(A)) \in \text{dom}_{\text{inh}_1}^O \).

Lemmas 5.4.9 and Lemma 5.4.10 B) are ensuring
\[
\text{Bind}_{\text{inh}_1}(\text{ext}(A) \in \text{decl}(A)) = \text{Bind}_{\text{inh}_2}(\text{ext}(A) \in \text{decl}(A)).
\]
So \( M \in \text{dom}_{\text{inh}_2}^R \). Furtheron, due to \( \alpha_{\text{inh}_1}(A) : A \neq \text{Root}, A \neq \text{Object}, \text{decl}(A) \in \text{dom}_{\text{inh}_2}^R \subseteq \text{dom}_{\text{inh}_2}^R \). So \( \alpha_{\text{inh}_2}(A) \) is holding and \( M = \text{inh}_2'(A) \).

Remark 5.4.15. on modular confluence:
The relation \( \prec_{DS} \) is modularly confluent, i.e. if \( \text{inh} \prec_{DS} \text{inh}^A \), \( \text{inh} \prec_{DS} \text{inh}^A \) and \( \text{inh} \neq \text{inh}^A \) then there is a common direct successor \( \text{inh} \) with \( \text{inh} \prec_{DS} \text{inh} \) and \( \text{inh} \prec_{DS} \text{inh} \),

especially \( \text{inh} = \text{inh}^{AA} = \text{inh}^{A'A} \) (due to an easy calculation using Lemma 13). If a \( \prec_{DS} \)-

chain
\[
\text{inh}_0 \prec_{DS} \text{inh}_1 \prec_{DS} \text{inh}_2 \prec_{DS} \cdots \prec_{DS} \text{inh}_n
\]
ends up in a maximal \( \text{inh}_n \) then \( \text{inh}_n \) is uniquely determined by \( \text{inh}_0 \). Every State \( \text{inh}_0 \)

has such a uniquely determined maximal successor State \( \text{inh}_0 \).

Obviously
\[
\text{BDfl}'(\text{inh}) = \text{inh} \cup \{ \text{inh} : \text{inh} \prec_{DS} \text{inh} \}
\]
is holding. Therefore a State \( \text{inh} \) is maximal w.r.t. \( \prec^S \) if and only if \( \text{inh} \) is a fixed point of
BDfl'. The maximal successor State $inh_\perp^{\text{max}}$ is the least fixed point of BDfl', obviously. If $inh$ has no cycle then $inh^A$ has none as well since $A \notin \text{dom}_\text{inh}$. If $\text{Dep}_{inh}$ has no cycle then so it is for $\text{Dep}_{inh^A}$, because we may easily deduce by Lemmas 5.4.9 and 5.4.10 B
\[
\text{Dep}_{inh^A} = \text{Dep}_{inh} \cup \{(A, \text{Bind}_{inh}(\text{ext}(A)) \mid i \text{ in } \text{decl}(A))\}:
1 \leq i \leq \text{length}(\text{ext}(A))
\]
where $A \in C \ \setminus \ \text{dom}_\text{inh}$ is the generating candidate for $inh^A$.

Theorem 5.4.8 allows to apply the fixed point theorem: BDfl' in
\[
(C \xrightarrow{\text{state}} C^O) \xrightarrow{\text{tot,cont}} (C \xrightarrow{\text{state}} C^O)
\]
has exactly one least fixed point ($\kappa = \text{card}(C)$)
\[
\mu\text{BDfl'} = \bigcup_{i \in \text{Nat}_0} \text{BDfl'}^i(\text{inh}_\perp) = \text{BDfl'}^\kappa(\text{inh}_\perp)
\]
which is
\[
\bigcup_{\text{inh}_{\perp} \prec_S \text{inh}} \text{inh} = \text{inh}_{\perp}^{\text{max}}.
\]

Theorem 5.4.16. If the given Java-program is Well-Formed then the function $\text{Bind}_{inh_B0}^{\perp}$ is satisfying all six rules of IPET. The rules are the same as in Theorem 5.3.2, only $inh_B0$ , bind and $\text{bind}_{inh_B0}$ are to be replaced by $inh_B0$, Bind and $\text{Bind}_{inh_B0}$ respectively.

Proof. The verification of the rules I and II is the same as earlier in the proof of the theorem 5.3.2.

III. (ET-SimpEncl)
Consider a user declared class $P$. From the first premise: $\text{Bind}_{inh_B0}(D \text{ in } P) = T$ we have that there exist the word $w_0$ - the least word such that $w_0(P).D = T$. From the definition of Bind
\[
\text{Bind}_{inh_B0}(\text{ext}(P.C).D \text{ in } P) = \text{Bind}_{inh_B0}(D \text{ in } \text{Bind}_{inh_B0}(\text{ext}(P.C) \text{ in } P)).
\]
Since $inh_B0$ satisfies the property $I_1$ we can simplify the right-hand side of the equation to $\text{Bind}_{inh_B0}(D \text{ in } inh_B0(P.C))$.
The third premise reads: $\text{Bind}_{inh_B0}(\text{ext}(P.C).D \text{ in } P)$ is undefined. From it we have that for every word $w''$ of the form $w'' \prec inh$ the value of the expression $(w'' \prec inh)(P.C).D$ is undefined. From the second premise we have: the value of $\lambda(P.C).D$ is undefined. Now consider $\text{Bind}_{inh_B0}(D \text{ in } P.C)$. Remark that the equality $(w_0 \prec decl)(P.C).D = T$ holds. Making use of the observations based on the second and third premises we conclude that the word $w_0 \prec decl$ is the least word $w$ such that the expression $w(P.C).D$ is defined. Hence, $\text{Bind}_{inh_B0}(D \text{ in } P.C) = T$. If $P.C = \text{Object}$ then $P = \text{Root}$ and $P.D = T$. The conclusion is holding also due to definition of $\text{Bind}_{inh_B0}$. If $P = \text{Root}$ then the conclusion is holding trivially.

IV. (ET-SimpSup)
The equality $inh_B0(P.C) = \text{Bind}_{inh_B0}(\text{ext}(P.C) \text{ in } P)$ is valid if $P.C$ is a user declared class because condition $I_{B1}$ is holding. Due to definition of $\text{Bind}_{inh_B0}$ we have $T = \text{Bind}_{inh_B0}(\text{ext}(P).D \text{ in } P) = \text{Bind}_{inh_B0}(D \text{ in } inh_B0(P))$ and $T$ is $\mu w(\text{inh}_B0(P)).D$. Because $P.C.D$ is undefined $P$ is not repeated on the path $\mu w \text{ inh}_B0$ from $P$ to $\text{decl}(T)$. So $\mu w \text{ inh}_B0$ is the least path, denoted $\mu w$, without repeated classes such that $\mu w(P).D = T = \text{Bind}_{inh_B0}(D \text{ in } P)$. If $P = \text{Object}$ or $\text{Root}$ then the conclusion is holding trivially.
V. (ET-Long)
The conclusion is holding due to definition of $\text{Bind}_{inhB_0}$.

VI. (ET.LongSup)
The equality $inhB_0(T) = Bind_{inhB_0}(ext(T) \text{ in } P')$ is valid if $T$ is a user declared class because condition $I_{B_1}$ is holding. Due to definition of $Bind_{inhB_0}$ we have $U = \mu w (inhB_0(T)).C$ and $U = Bind_{inhB_0}(ext(T).C \text{ in } P') = Bind_{inhB_0}(C \text{ in } inhB_0(T))$. Because $T.C$ is undefined $T$ is not repeated on the path $\mu w inhB_0$ from $T$ to decl($U$). So $\mu w inhB_0$ is the least path, denoted $\mu \tilde{w}$, without repeated classes such that $\mu \tilde{w}(T).C = U = Bind_{inhB_0}(C \text{ in } T) = Bind_{inhB_0}(X.C \text{ in } P)$.

If $P = \text{Object}$ or $P = \text{Root}$ then the conclusion is holding trivially. 

We may observe that the model $Bind_{inhB_0}$ may be calculated by an algorithm. Hence we have two different algorithms to construct the models of IPET calculus. Now we see that the statement “a straightforward algorithm ...[11]p.34“ is not justified at all. First of all the authors of [11] gave no description of the algorithm. Second, there exist at least two algorithms. The cited statement does not answer to the question which of algorithms is the proper one?

5.5 The dilemma with IPET’s rule III. (ET-SimpEncl)

The dilemma with IPET does not end itself on that IPET allows at least two different binding functions $bind_{inh_0}$ resp. $Bind_{inhB_0}$ which satisfy all six rules of IPET and which yield two different notions of well-formedness of (the structure of) a Java-program. If there were a clear criterion for the calculus how to elect the distinguished inheritance function resp. binding function everything would be fine. Let us remark that, in case all premises and conclusions are positive logical formulas then the intersection of any two satisfying functions is satisfying as well. Moreover, the distinguished function can be gained in a constructive manner by successive top-down applications of the rules. We have already seen that rule III. (ET-SimpEncl) has a premise which is a negative, and a metatheoretic formula such that clear application of the rule is a great problem. It is even so that there is a program, namely Example 5.4.4, which is well-formed w.r.t. $bind_{inh_0}$ and $Bind_{inhB_0}$, but the intersection function $bind_{inh_0} \cap Bind_{inhB_0}$ does not satisfy rule III. (ET-SimpEncl), especially is different from $bind_{inh_0}$ which Java Language Specification JLS [7] has prescribed to be the appropriate binding function.

Lemma 5.5.1. The intersection of the two models of IPET calculus need not to be a model of IPET.

Proof. The proof of the lemma consists in exhibiting a counter-example program of the previous section.

Example 1 continued:

We have

- $inh_0(B) = inh_{B_0}(B) = A$
- $inh_0(B\$D) = inh_{B_0}(B\$D) = A\$C$
- $inh_0(B\$D\$F) = B\$E$
- $inh_{B_0}(B\$D\$F) = A\$E \neq B\$E$

Let us calculate the binding functions:

- $bind_{inh_0}(A \text{ in } \text{Root}) = Bind_{inhB_0}(A \text{ in } \text{Root}) = A$
- $bind_{inh_0}(C \text{ in } B) = Bind_{inhB_0}(C \text{ in } B) = A\$C$

Now $bind_{inh_0}(E \text{ in } B\$D) = B\$E$
5.6 What was left open by Igarashi and Pierce

because path $decl$ from $B$ to $E$ is lexicographically less than path
$inh^{\sim}decl$ from $B$ to $A$.

But $\text{Bind}_{\text{inh}_{B_0}}(E \text{ in } B$ $D) = A$ $E \neq B$ $E$!

because path $decl^{\sim}inh$ from $B$ to $A$ is lexicographically less than
path $decl$ from $B$ to $B$.

Let us define the intersection relation

$Int \overset{\text{df}}{=} \text{bind}_{\text{inh}_{B_0}} \cap \text{Bind}_{\text{inh}_{B_0}}$.

We shall prove that $Int$ relation does not satisfy rule III (ET-SimpEncl). From the facts
gathered till now one deduce (using the rule III) that the value of $Int(E \text{ in } B$ $D)$ should be $B$ $E$.

However from the definition of $Int$ we have

$B = decl(B$ $D)$

$B$ $D$. $E$ is undefined

$Int(E \text{ in } B) = B$ $E$

$Int(C.E \text{ in } B)$ is undefined

because

$\text{bind}_{\text{inh}_{B_0}}(C.E \text{ in } B)$

$= \text{attrbind}_{\text{inh}_{B_0}}(E \text{ in } \text{bind}_{\text{inh}_{B_0}}(C \text{ in } B))$

$= \text{attrbind}_{\text{inh}_{B_0}}(E \text{ in } A$ $C)$ is undefined

and

$\text{Bind}_{\text{inh}_{B_0}}(C.E \text{ in } B)$

$= \text{Bind}_{\text{inh}_{B_0}}(E \text{ in } \text{Bind}_{\text{inh}_{B_0}}(C \text{ in } B))$

$= \text{AttrBind}_{\text{inh}_{B_0}}(E \text{ in } A$ $C)$

$= A$ $E$.

But

$\text{bind}_{\text{inh}_{B_0}}(E \text{ in } B$ $D) = B$ $E$

$\text{Bind}_{\text{inh}_{B_0}}(E \text{ in } B$ $D) = A$ $E$

hence

$Int(E \text{ in } B$ $D)$ is undefined.

Hence $Int$ is not a model of IPET.

In the previous sections we have unexpectedly seen two different models of IPET calculus. Therefore the IPET can not be treated as a definition of function of binding. Now, the immediate instinct to enrich the IPET with a (metatheoretic!) clause: "choose the least of all IPET's models" leads to nowhere.

5.6 What was left open by Igarashi and Pierce

The identification of declarative occurrence $T$ of a class that is bound to an applicative occurrence of a (class) type $X$ within a class $P$ is basic for the understanding how a program works. The paper [11] offers the IPET calculus for deducing the values of the function $\text{bind}(X \text{ in } P) = T$, in original paper it is written $P \vdash X \Rightarrow T$. It turned out the formal system of IPET has many models, hence, the system does not define the binding function.

The discussion of this chapter shows how important is to state a few questions known already in metamathematics:

1. (determinacy or consistency) It is obvious that a formal system may allow to prove a sentence on many alternative ways. However, a sound system may not allow to deduce mutually negating answers. In this case the question should be: is it true

\[ \text{is it true} \]
that for every class \( P \) and for every type \( X \) if IPET calculus allows to deduce two triplets \( P \vdash X \Rightarrow T \) and \( P \vdash X \Rightarrow U \) then \( T = U \)? We should be sure that the relation \( P \vdash X \Rightarrow T \) is a function, that binds an applicative occurrence of the type \( X \) inside the class \( P \) to the declaration \( T \) of a class.

2. (categoricity or completeness) How many models has a proposed formal system? In our case the question is are there different functions \( \text{bind} \) that are models of the IPET calculus? The positive answer tells us that something important escaped our attention.

3. (repairing an incomplete system) If there are several models, one should try to repair the formal specification by adding either more axioms and inference rules (this way, we believe, is the correct one) or by adding some metatheoretic rule like, for example, among all possible models choose the least one. Or better, among all possible models choose the one calculated by certain algorithm.

These questions were not addressed in the paper [11].

Still there are open questions (to be answered in forthcoming papers):

- is it possible to repair the IPET calculus and present a formal system which will enable to deduce the values of the function \( \text{bind}_{\text{inh}} \)?

- how many models the calculus IPET admits? The eventual answer may interest the designers of programming languages.
Chapter 6

Static and runtime binding

In this chapter we shall discuss the problem of determining the meaning of identifiers during a compilation and later an execution of a program. In the preceding two chapters we discussed the problem of identifying the direct superclass of a given class. We gave the algorithm for computing function \texttt{inh}. This algorithm uses another function \texttt{bind}. The latter algorithm finds application during the phase of compilation which checks whether a source code is well formed program. This phase of compilation is usually called “static semantic analysis”. The application of function \texttt{bind} during compilation allows to simplify the access to variables during execution of program as it will be seen from the theorem 6.1.6.

6.1 Binding of variables

Variables are referenced in method bodies. Variables declared inside a method are considered to be local. All other references to variable identifiers should be bound in the environment of the method. This environment consist in the class which declares the method and all classes textually enclosing the declaring class. Without loss of generality we assume that all variables are of class types. We assume that for every class \( K \) there exist a function \( .v \) such that \( K.v \) returns the class \( M \) being the type of the variable \( v \) declared within the class \( K \) or remains undefined if the class does not declare a field variable named \( v \). Since no class should declare two different variables with the same name \( K.v \) is really the function.

\textbf{Definition 6.1.1.} An applied occurrence of a variable identifier \( v \) within the body of a method \( M \) will be called nonlocal iff the variable identifier \( v \) is not declared within the method \( M \) neither as a local variable nor as a formal parameter.

The way of searching the environment of nonlocal variable \( v \) applied occurrence in method \( M \) body is isomorphic to a class identifier binding when they occurs within classes (cf. definition 2.1.2).

\textbf{Definition 6.1.2.} Let \( K \) be a class. An applied occurrence of a variable identifier \( v \) in the class \( K \) is bound to a class \( L \) such that \( L \) is the type of the applied variable \( v \) occurrence.

\[ \text{bind}(v \text{ in } K) \triangleq (\text{inh'\,decl'}(K)).v \]

where the pair \( (j, i) \), \( j \geq 0 \), \( i \geq 0 \), is the least pair in the lexicographic order such that the class \( (\text{inh'\,decl'}(K)).v \) is defined.
Definition 6.1.3. The nonlocal applied occurrence of a variable identifier \( v \) with the body of a method \( M \) is bound to its type \( L \) iff its bound to the same type as applied variable occurrence in class \( K \) where \( K \) is the class declaring the method \( M \).

During runtime applied occurrences of variable identifiers should be bound to the corresponding locations in store. Each class together with its declared variable fields is the pattern used to allocate location for every variable declared in this class. During runtime any given class may be used many times to reserve new locations to declared field variables. All locations for field variables of a given class should be reserved at once and we call such collection of locations a data field of the class. As we will see in the next section data fields forms during the run time a structure:

\[
\mathcal{DF} = \langle D\text{fields}, Decl, Inh \rangle
\]

homomorphic to the compile time structure

\[
\mathcal{CL} = \langle Classes, decl, inh \rangle
\]

with the homomorphism \( h : D\text{fields} \rightarrow Classes \) defined as follows

\[ h(\kappa) = K \] iff the data field \( \kappa \) was reserved for locations of class \( K \) field variables.

Since field variables have unique names within each class we can assume that for every data field \( \kappa \) there exist function \( .v \) such that \( \kappa.v \) returns the location of variable field \( v \) within data field \( \kappa \) iff such location exists.

Definition 6.1.4. Let \( \kappa \) be a data field of an arbitrary class. An applied occurrence of a variable identifier \( v \) in the data field \( \kappa \) is bound to a location \( \lambda \) such that \( \lambda \) is the value of the function \( \text{bind} \) computed in the following equality.

\[
\text{bind}(v \text{ in } \kappa) \overset{\text{df}}{=} (\text{Inh}^{j} \text{Decl}^{i} \kappa).v
\]

where the pair \((j, i), j \geq 0, i \geq 0,\) is the least pair in the lexicographic order such that the class \((\text{Inh}^{j} \text{Decl}^{i} \kappa)).v\) is defined.

Definition 6.1.5. The nonlocal applied occurrence of a variable identifier \( v \) while invocation \( \varphi \) of the method \( M \) is bound to its the location \( \lambda \) iff its bound to the same location as applied variable occurrence in class data field \( \kappa \) where \( \kappa \) data field of the classs defined as the value of the expression “this” within the invocation \( \varphi \).

Theorem 6.1.6. Let \( \kappa \) be a data field of class \( K \), \( v \) be applied variable identifier occurring in \( \kappa \) and in \( K \) and \( L \) be a class designated as type of \( v \) by the formula

\[
(\text{inh}^{i}\text{stat} \text{decl}^{j}\text{stat}(K)).v = L
\]

from the definition 6.1.2 and \( \lambda \) be a location designated as the location of \( v \) by the formula

\[
(\text{Inh}^{i}\text{dyn} \text{Decl}^{j}\text{dyn}(\kappa)).v = \lambda
\]

from the definition 6.1.4

then \( j_{\text{stat}} = j_{\text{dyn}} \) and \( i_{\text{stat}} = i_{\text{dyn}} \)

6.2 Instantiation of classes

6.2.1 Static typing of instantiation expression

Class instantiation is done by the expression \texttt{new}. The result of class instantiation is producing a class instance. A class instance consists of data fields for the instantiated class
and data fields for all its superclasses. The expression `new` may be invoked in one of two contexts

- `e.new C(...)`
- `new X(...)` as top level expression

In both cases the first step is to determine the declaration of the class to be instantiated. Static semantics analysis requires the declaration of the instantiated class while in runtime we are obliged to define the values of functions decl and Inh as well for data field of the instantiated class as for data fields of all its superclasses, which are to be reserved simultaneously.

Let we start with determination of the class to be instantiated. If the `new` expression is in the first form then the expression `e` should be well typed to some class `K`. Class identifier `C` is bound to the class `L` iff

\[ L = (inh^i(K)).C \]

for some `i` and `i` the least number \( \geq 0 \) such that the right hand side of the equality is defined.

For the second form we must first establish

\[ L = bind(X \text{ in } K) \]

where class `K` is the owner of the method instantiating class. Then we must check whether for some `i` and `j`

\[ decl(L) = inh^i decl^j(K) \]

**Lemma 6.2.1.** Let `new X(...)` be an class instantiation expression within a class `K`, and let `length(X)=1` i.e. `X=C` where `C` is a class identifier. Then the type of the expression `new X(...)` is equal `bind(C \text{ in } K)`

The observation stated in the lemma above does not generalize. It is possible that if the type of the expression `new X(...)` in `K` is `L` then `L \neq bind(name(L) \text{ in } K)` as it can be seen from the following

**Example 6.2.2.** Class instantiation.

```java
class B1{
    class A {}
    class B2 {
        class A {}
        void f() {
            new B1.A();
            new A();
        }
    }
}
```


### 6.2.2 How to generate class instance using class instance generator

**Definition 6.2.3.** Let \( \kappa \) be a data field of class `K`. This data field may be used as class instance generator for any direct inner class of `K`. 
The aim of a class instance generator $\kappa$ is to instantiate a direct inner class. Let it be the class $L$ with the $\text{name}(L) = C$. Each class instance consists of data field of an instantiated class and data fields of its all superclasses. Each of the data fields should have defined its $\text{Decl}$ and $\text{Inh}$ functions properly. So, the first task of class $L$ generator is to generate an instance for direct superclass of the class $L$. Since it is done we obtain instance of superclass consisting of data field of all superclasses of $L$, each data field with functions $\text{Decl}$ and $\text{Inh}$ already defined. Then we reserve new locations for data field $\lambda$ of class $L$ and define $\text{Inh}(\lambda) = \eta$ and $\text{Decl}(\lambda) = \kappa$ where $\eta$ is data field of the direct superclass of the class $L$.

Hence it is only left to define how class instance generator instantiates the direct superclass of a class to be instantiated. In other words the only problem is how to find a direct superclass $L$ generator when the class $L$ generator is known as $\kappa$. We remain that it means that $\text{Decl}(L) = h(\kappa)$. In order to deal with this problem we have class $L$ constructor of the form.

$\text{C}(\ldots)\{\ \text{e.super(...)}\}$ or 
$\text{C}(\ldots)\{\ \text{super(...)}\}$

where it is assumed that $\text{name}(L) = C$ and $\text{inh}(L) = N$

In order to better understand the second form its worth to remain that the expression super should instantiate the superclass which is already known from the earlier stage of static semantics analysis as $N$. Let we treat the super() expression as a fictious class generator new $N()$, in a very specific form where the class identifier is changed by the class itself. Now we check if a formula

$\text{decl}(N) = \text{inh}^i\text{decl}^j(K) \quad (\ast)$

holds for any $i, j \geq 0$. This test may be performed during static semantics analysis and for well typed program at least one such formula should hold. However if we have more candidates we have a problem how to define class instance generator for $N$. In fact we choose, perhaps a little bit arbitrary, to set class $N$ instance generator as data field $\tau$ of the class $T = h(\tau) = \text{decl}(N)$ where $\tau = \text{Inh}^i\text{Decl}^j(\kappa)$, such that and $j_0$ is the least integer such that the formula $(\ast)$ holds. Its worth to notice that for such $j_0$ $i_0$ is defined uniquely since there is no cycle in the graph of the inheritance function $\text{inh}(L)$

If the $L$ constructor is in the first form then we assume, as in the previous case that the direct superclass of $L$ is $N$ and $\text{decl}(N) = T$. Then it is a static semantics analysis responsibility that the static type $E$ of $e$ is the subtype of $T$. During run time we must evaluate the expression $e$ before class $N$ is instantiated. The expression $e$ is evaluated in
the dynamic context of $\kappa$ instance of class $K$ which is the class instance generator of class $L$. Let $\eta$ designates the value of $e$. We assume the dynamic run time type correctness which means that the type of $h(\eta)$ is a subtype of $E$. In other words $Inh^i(h(\eta)) = inh^i(E) = T$ for some $j \geq i$. So $Inh^j(\eta) = \tau$ where $h(\tau) = T$. Then $\tau$ is the instance generator of superclass $N$.

6.2.3 How to find the proper class instance generator from the place of class instantiation.

Let we assume that the class instantiation expression new $X(...)$. occurs at the time of the method $M$ execution and at this time the value of the expression this within method $M$ is $\nu$. Let $M$ be a method of a class $K$. Then $K = h(Inh^k(\nu))$ for some $k$. Then assume that static analysis has defined the class to be instantiated as $L$ and $j$ is the smallest integer such that $decl(L) = inh^i decl^j(K)$. Note that for such $j$ the number $i$ is defined uniquely while the existence of such $j$ is guaranteed by static semantics analysis. Then the $L$ class instance generator is $\tau = Inh^i Decl^j Inh^k(\nu)$

Now it remains to define class instance generator for the expression of the form $e$.new $C(...)$. Let $K$ be a static type of the expression $e$ and $C$ be bound to class $L$ by static semantics analysis. It means that $decl(L) = inh^i(K)$ for an uniquely defined $i$. Then let $h(\nu) = E$ where $\nu$ is run time value of the expression $e$. So $K = inh^j(E)$ for some $j$ due to run time type correctness. Then the class $L$ instance generator is $\tau = Inh^i Inh^j(\nu)$
Chapter 7

Conclusions

The Java programmers and the compilers of Java must resolve the problem of which of possibly many classes B is extended by the class A extends B{...}. This problem is not trivial because Java admits inner classes. The problem becomes even more complicated, when we recall more general form of class declaration

\[
\text{class } A \text{ extends } B.C.D \{ \ldots \}.
\]

In the specification of Java language [7] one may find conditions which must be satisfied by the proper solution. This specification however doesn’t help to find a proper solution. It also doesn’t guarantee nor that a solution exists neither that the solution is unique.

The problem, treated informally in JSL[7] was was given an algebraic formulation in Chapter 2. Chapter 3 brings a non deterministic algorithm and the proof of its correctness. Next chapter presents a deterministic algorithm together with its analysis. Chapter 5 discusses earlier works on this problem in various Object Oriented languages. A special attention was given to the work of Igarashi & Pierce [11]. This paper contains a system of axioms and inference rules which were invented in order to give reduction semantics of the subset of Java. We proved that this work has a flaw.

There are at least two possible applications of the results of this dissertation. First, the algorithm determining the extended classes may and should be a part of any Java compiler. Second, the theoretical work made in the context of the problem may constitute a good foundation for the future research of the semantics of Java programming language. Chapter 6 may be treated as a first step of such research. The research which may lead to, among the others, an extension of the Eclipse environment and an axiomatic definition of the Java programming language.
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