"A weakly compressible, diffuse-interface model for two-phase flows"

A novel mathematical model of nearly incompressible interfacial flows is presented. Unlike the traditional modelling where zero Mach number is imposed, our approach assumes the speed of sound to be finite yet much higher than the convective velocity scale. The two-phase nature of the considered systems is accounted for using the one-fluid formulation; in particular, the diffuse-interface approach is applied. Therefore, smooth solutions of the governing equations are expected in the considered range of low Mach numbers. The resulting governing equations are of mixed parabolic-hyperbolic (or purely parabolic) type instead of the parabolic-elliptic type of the incompressible Navier-Stokes equations. Although being favourable from the numerical and algorithmical standpoint, such modelling brings some issues when fluid-fluid interfaces are considered. These stem from weak but yet present compressible effects: the density is no longer constant in the bulk of the phases. We discuss these problems in details and propose successful solutions.

Our main motivation to undertake the topic is a recent development of the computing devices architecture, especially the availability of powerful multithreaded shared memory systems, like GPUs and multicore CPUs. The ellipticity-free mathematical models are better suited for the efficient parallelisation of the computational process. The reason is that the possible use of the fully explicit in time and local in space numerical schemes allows one to avoid solving algebraic systems. Therefore we briefly present an easily parallelisable, universal numerical algorithm to solve the proposed model; it has proven to be successful in a wide variety of flow cases.

Our research was published in the following papers:
1. A. Kajzer and J. Pozorski, 2020, A weakly compressible, diffuse interface model for two-phase flows, *Flow, Turbulence and Combustion*, **105**, 299-333
2. A. Kajzer and J. Pozorski, 2022, A weakly compressible, diffuse interface model for two-phase flows: numerical development and validation, *Computers and Mathematics with Applications,* **106**, 74-91