

Madrid, 21 June 2023

Review on the PhD thesis “Graph Width Parameters. Dependencies, Algorithms and Decompositions” submitted by Wojciech Nadara

The submitted PhD thesis is primarily focused on the structural and algorithmic aspects of width and depth parameters of graphs. This line of research holds significant importance in theoretical computer science and has been mainstream for over three decades. Among the width and depth parameters of graphs, tree-width clearly stands out. This parameter that measure the tree-like structure of a graph is of substantial importance both in structural and algorithmic graph theory. Indeed, it plays a prominent role in the graph minor theory. Courcelle’s Theorem, the seminal result in the area, states that every graph property expressible in monadic second order logic can be tested in linear time for graphs with bounded tree-width.

The submitted thesis deals primarily with tree-depth and path-width, which are more restrictive graph width parameters. Both of these parameters are very important in algorithm design, which is briefly outlined in the introductory Chapter 1 of the thesis. However, the focus of thesis lies in exploring their structural properties and developing algorithms for computing decompositions with small tree-depth and path-width, which are equally significant questions from the algorithm design point of view.

The submitted thesis consists of seven chapters. The first two chapters serve as an introduction to the subject and the results of the thesis. Chapter 1 provides a brief overview of the thesis topic, presenting both a general introduction and an overview of the results presented in the subsequent chapters. I find it confusing that some results are presented in separate sections while some not. Chapter 2 focuses on introducing the notation that will be used throughout the thesis.

The remaining five chapters are each based on a paper coauthored by Wojciech Nadara, with different groups of collaborators; in case of Chapters 4 and 6, only a part of the paper is included. While some chapters contain concluding sections, it would be beneficial if the thesis also included a brief concluding chapter to tie together the main contributions and present directions for further research.

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I will next give a brief overview of the results presented in the thesis. Chapter 3 explores the interplay between tree-width and tree-depth. It is established that graphs with large tree-depth contain one of three natural obstructions: have large tree-width, a subdivision of a deep full binary tree, or a long path. Additionally, a polynomial-time approximation algorithm for tree-depth is designed with an approximation factor being a function of tree-width. Chapter 4 provides an upper bound on the number of vertices of minimal obstructions for tree-depth, and Chapter 5 presents a fixed parameter algorithm for computing tree-depth-decompositions of depth d with the hidden constant exponential in $O(d^2)$.

Chapter 6 addresses path-width and presents a result analogous to that in Chapter 3 for tree-depth. Specifically, it proves that graphs with large path-width either have large tree-width or contain a subdivision of a deep full binary tree. This result proves the conjecture of Kawarabayashi and Rossman (with the best possible exponent). Chapter 7 substantially differs in topic from the previous four chapters, focusing on the maximum average degree of a graph. It addresses the problem of identifying a well-behaved set whose removal decreases the maximum average degree. In my view, the thesis would retain its strength without this chapter and its removal would lead to a higher level of topic coherence.

The quality of the results presented in the thesis definitely meets the expected standards of a PhD thesis. However, there are areas where the presentation of the results appears rushed, and additional effort could be made to enhance the coherence of the thesis. One aspect that could benefit from improvement is the unification of notation used across different papers that the thesis is based on. For example, the terms 'complete binary tree' and 'full binary tree' are used in different parts of the thesis, depending which paper the particular part is based on. Additionally, on page 52, the term 'paper' is used to refer to a chapter.

Both Chapters 1 and 2 would benefit from providing more context for the presented notions and new results. In particular, the general introduction to the research area in Chapter 1 is unexpectedly brief, and expanding it would provide a better context for the presented results. In addition, including figures to illustrate key concepts in Chapter 2 would enhance the exposition. Despite the expressed points of criticism in regard to the presentation, I find the submitted thesis to fulfill the requirements expected from a PhD thesis in terms of quality of result presentation.

In summary, the submitted thesis concerns important topics in theoretical computer

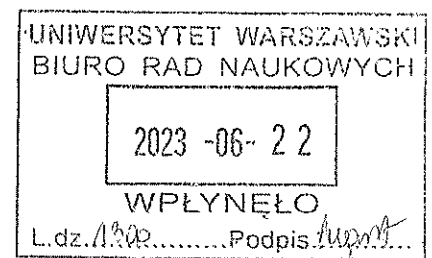
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science and makes progress in advancing the understanding of width and depth graph parameters. The quality of the results presented in the thesis meets the expected standards of a PhD thesis, and the author's ability to conduct research of international quality is clearly demonstrated. While there are areas concerning the presentation of the results that could benefit from further improvement, the presentation of the results in the thesis overall meets the requirements expected from a PhD thesis, both in terms of the style and the rigor. Without any hesitation, I deem the thesis as sufficient to grant a PhD.

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