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Review of doctoral dissertation of mgr Rafał Martynek

The following is my evaluation of the dissertation "*Estimates of suprema of stochastic processes with application of the chaining method*" submitted by mgr Rafał Martynek. The review is divided into three parts: I. An overview and evaluation of the results; II. Suggestions to the author concerning the manuscript; III. Some typos. Citations will follow bibliography numbers of the dissertation.

I. An overview and evaluation of the results

Mgr Martynek's research is in the area of probability theory. More specifically, he is developing mathematical tools to investigate suprema of stochastic processes over general index sets. Such problems arise in diverse branches of probability theory and stochastic processes, high dimensional statistics, and computer science, just to mention a few. The key idea to this problem is to relate expected supremum of the process to the geometry of its index set. The roots of this idea can be traced to V.N. Sudakov's talk at the International Congress of Mathematicians, Moscow 1966. Following this idea, the upper bound for the expected value of the supremum of Gaussian processes was later obtained in terms of the metric entropy condition for the index set. However, there was a gap between the upper and lower bound. M. Talagrand [28], with a substantial contribution by X. Fernique, introduced a new concept of majorizing measures and using them obtained a complete characterization of boundedness for all Gaussian processes. This approach has propagated since then to investigate other stochastic processes of importance, which included Bernoulli, empirical, selector, and infinitely divisible processes. However, it occurred that majorizing measures were not well suited to handle such processes and they were replaced by general partitioning schemes.

In 2014 the influential monograph [32] by M. Talagrand was published. It included this new approach of partitioning schemes and geometry related γ_α numbers to handle expected suprema.

The book included a number of open research problems, among them the *Bernoulli Conjecture* - solved by W. Bednorz and R. Latała [2] and three other major conjectures solved only recently by W. Bednorz and R. Martynek [4].

M. Talagrand [33], in a preprint of the new edition of his 2014 book, writes:

"Even more importantly, this approach, when combined with the Latała-Bednorz theorem allowed (after a crucial contribution by Witold Bednorz and Rafał Martynek) a positive solution of three of the main conjectures of the old edition, which are presented respectively in Theorems 5.8.3, 9.11.1 and 10.4.5 (of the new edition)."

Martynek's dissertation consists of five chapters which are packed with results. The main result of the dissertation, Theorem 28, combined with Theorem 19, gives complete geometric characterization of the boundedness of infinitely divisible processes. This is a huge progress. Previously, only Theorem 20 was available for the lower bound, with a horrible restrictive $H(C_0, \delta)$ condition. This condition was not applicable in many cases of interest.

Chapter 1 provides some introductory material, definitions, and basic tools. In Chapter 2 the problem posed by K. Oleszkiewicz is considered. It concerns the comparability of weak and strong moments of Bernoulli series in a Banach space. Some of the Bernoulli Conjecture techniques are adopted in this setting, giving a partial answer to the problem. Chapter 3 presents a neat piece of mathematics concerning Bernoulli processes with monotone coefficients. Chapter 4 contains the above mentioned Theorem 28 (the Decomposition Theorem for infinitely divisible processes). Chapter 5 has two fundamental results: Theorem 33 (The fundamental theorem of empirical processes) and Theorem 34 (Generalized Bernoulli Conjecture).

Results of Chapters 2 and 3 have already been published in papers [5] and [3], respectively. Results of Chapters 4 and 5 are available in [4] as a preprint at arXiv:2008.09876v2. Papers [3] and [5] are beautiful contributions to mathematics but paper [4] is superb, which makes the case for the following.

This is an outstanding dissertation. I am very much impressed by its scientific level and the quality of results. I have cited above the opinion of Michel Talagrand, a member of the French Academy of Sciences, who was also impressed. I think the results on decompositions will have lasting value. In my opinion, this dissertation fully deserves an award.

II. Suggestions to the author concerning the manuscript

To improve the readability of this manuscript, the referee suggests to consider the following.

General remark. The thesis manuscript contains 34 theorems but it is not immediately clear which ones are due to mgr Martynek and which ones to other authors. It would be helpful to the reader if theorems are named by the authors and bibliography numbers. For example, Theorem 28 (Bednorz and Martynek [4]), Theorem 20 (Talagrand [29]). In the cases where the theorems are taken from books, the source with page number should be given; for example, Theorem 6 ([32], p. 108).

P. 1, 2nd paragraph: The history of the study of suprema of Gaussian processes is very instructive and should be mentioned here in a greater extent. The survey article by R.M. Dudley, one of the main players in the earlier stages of this theory, entitled "*V.N. Sudakov's Work on Expected Suprema of Gaussian Processes*", in Progress in Probability vol. 71, pp. 37- 43, Birkhäuser, 2016, can be very helpful in this task.

P. 2 line 1: It is better to mention that $\|G_t\|_p \leq L\sqrt{p}\|G_t\|_2$ follows from Stirling's approximation of gamma functions than giving a reference to an unsolved exercise in a book.

P. 4 lines 14 - 15: There is an inconsistency between the roles of T_1 and T_2 in these lines and in Theorem 1.

P. 51 line 14: This formulation is slightly confusing. j_n are integer-valued functions on $\mathcal{A}_n$ (tags of sets in partitions), not just single numbers. Should they satisfy (1.8), i.e., $j_{n-1}(B) + 1 \geq j_n(A)$ as well? Maybe one can write $r \geq 4$, $u > 0$ instead of "a number $r \geq 4$ ... where $u > 0$ is a parameter"?

III. Some typos

P. 1 line 16-: centered; line 13-: classical;

P. 1 line 7-: it should be 2^{-1} instead of 2;

P. 1 line 1-: correct the expression accordingly, also L should be in the last term;

P. 2 line 14 and further occurrences: Majorizing;

P. 9 line 4: $u = t$?

P.9 line 1-: Expectation is in the wrong place;

P.9 line 2-: Expectation is missing.

Sincerely yours,

J. Rosiński

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