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Prof. Anna Gambin
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Dear Professor Gambin,

I am writing to you - in response to your e-mail of November 11,2018 - to present my report on the doctoral thesis, written by Mr Michał Strzelecki under the supervision of Prof. Radosław Adamczak, entitled *Functional and transport inequalities and their applications to concentration of measure*.

The PhD dissertation by Michał Strzelecki deals with functional inequalities related to the concentration of measure phenomenon, namely some variants of the well-known Poincaré inequality, logarithmic-Sobolev inequality and transport-entropy inequalities. The use of these functional inequalities to describe the (concentration) properties of the studied measures, has been pushed forward by pioneers such as Michel Talagrand, Vitali D. Milman, Katalin Marton, Michel Ledoux, Sergey Bobkov in the nineties. Many authors have afterwards studied the interplay of these different notions. In the same times, concentration of measure properties have become a fondamental tool in various branches of mathematics, among them high dimensional probability, geometric analysis, statistics, machine learning, computer science.

After a brief chapter in which the functional inequalities are introduced and the well known results are given with precise references, Michał Strzelecki presents the scope of his thesis divided into two parts.

The first part (Chapter 2) concerns some new results about connections between the *Latała-Oleszkiewicz inequality* and the *modified log-Sobolev inequality* by Gentil-Guillin-Miclo [31]. Namely if an alsolutely continuous probability measure satisfies *Latała-Oleszkiewicz inequality* or equivalently F_q*-Sobolev inequality*, it also satisfies a *modified log-Sobolev inequality*. As an interesting byproduct Michał Strzelecki obtains that the concentration properties of a probability measure satisfying *Latała-Oleszkiewicz inequality* are stronger than what was known until now. At the end of the chapter, he also presents an example of measure satisfying the *modified log-Sobolev inequality* and also the *Latała-Oleszkiewicz inequality* without satisfying a so-called *weighted log-Sobolev inequality*, to show that their result can not be derived from Wang's paper [83] involving *weighted log-Sobolev inequality*. This work in progress is a recent collaboration with Franck Barthe in spring 2018 while Michał Strzelecki was visiting the Institut of Mathematics in Toulouse, France. These results are still not published and show the ability of Michał Strzelecki to enter a new subject. With this work, done at the end of its doctoral studies, Michał Strzelecki also enlarges his knowledge to prove functional inequalities by using Hardy-type conditions, capacity criterion or by using several known connections with many other functional inequalities, such as *F_q-Sobolev inequalities* or *super Poincaré inequalities*.

The second part (Chapter 3-7) is the heart of the thesis and deals with *convex* concentration properties related to the *convex* Poincaré inequality, *convex* log-Sobolev inequalities and the theory of weak transport-entropy inequalities recently introduced by Gozlan-Roberto-Samson-Tetali [15], inspired by seminal papers by Bernard Maurey [54] and Michel Talagrand [77,78]. Michał Strzelecki presents the results of three papers from different successful collaborations, published in famous international journals. Two papers are collaborations with young researchers, one with Marta Strzelecka and Tomasz Tkocz published in the journal ALEA, and another one with Yan Shu published in the Annals of Institut Henry Poincaré. The third paper with his supervisor Radosław Adamczak is accepted in the Bernoulli journal.

After an overview in Chapter 3 of the known results, Chapter 4 is devoted to answer some open problems about characterizations of probability measures satisfying *convex* (modified) log-Sobolev inequalities on the real line. With Yan Shu, Michał Strzelecki substantially improves seminal characterisations given by Gozlan-Roberto-Samson-Tetali [15] in terms of weak transport-entropy inequalities or in terms of regularity of some transport maps. A consequence of their results is that if a probability measure μ satisfies a *convex* log-Sobolev inequality then a dimension-free deviation's inequality holds for any convex function on the product space $(\mathbb{R}^n, \mu^n)$ both above and under its mean. The deviations above the mean are simple consequences of the so-called Herbst's argument, however up to now, the approach by Michał Strzelecki is the only one to derive deviations under the mean from *convex* log-Sobolev inequalities. Another consequence of their result is that a probability measure μ satisfies T_2-Talagrand's transport-entropy inequality if and only if it satisfies a Poincaré inequality together with a *convex* log-Sobolev inequality. Chapter 4 ends with few interesting open questions that emerge from their research.

Whereas Chapter 4 mainly concerns probability measures on the real line, Chapter 5 is devoted to the characterisation of measures satisfying the *convex* Poincaré inequality in higher dimension. One difficulty is that in higher dimension, no equivalent Hardy-type conditions or regularity conditions on transport maps are known, even for the classical Poincaré inequality. As a main result, Michał Strzelecki and its supervisor prove that a measure on $\mathbb{R}^n$ satisfies a convex Poincaré inequality if and only if it satisfies a weak transport-entropy inequality with a particular quadratic-linear cost. The sketch of the proof follows the one of the same kind of result obtained for the Poincaré inequality by Bobkov-Gentil-Ledoux [18]. However Michał Strzelecki manages to circumvent the difficulties resulting from the lack of symmetry due to the *convex* restriction and to simplify some elements of the proof by Bobkov-Gentil-Ledoux [18]. As explained at the end of Chapter 5, in their result, a constant for one of the implications is not dimension-free as expected, pointing out new open questions connected to their proof.

Chapter 6 deals with refined concentration properties that can be derived from weak transport-entropy inequalities or Poincaré inequality for non Lipschitz-functions in the same way as some recent results by Bobkov-Nayar-Tetali [21]. Michał Strzelecki points out that by the use of Orlicz norm, the known deviation results for a convex function deriving from a weak transport-entropy inequality, can be improved as regards to the hypotheses on the gradient of this convex function. This relevant observation will be very useful for other concentration results induced from weak transport-entropy inequalities.

The last chapter concerns one of the remarkable results of the thesis in the study of probability measures satisfying *optimal* weak transport-entropy inequalities on the real line, or equivalently *optimal convex* infimum-convolution inequalities. This terminology has been introduced by Latała-Wojtaszczyk for infimum-convolution inequalities, observing that the optimal cost is given by the Legendre transform of the cumulant-generating function of the underlying measure. In collaboration with Marta Strzelecka and Tomasz Tkocz, and following Latała-Wojtaszczyk's line of research, Michał Strzelecki obtains that product of symmetric measures with log-concave tails satisfy an *optimal convex* infimum-convolution inequality with a universal constant. The proof uses the characterisation of weak transport-entropy inequalities by Gozlan-Roberto-Samson-Tetali [15] studied in Chapter 4. His results complement the one of Latała-Wojtaszczyk for product of log-concave measures. As a main application, comparaison inequalities for weak and strong moments, reached by Latała-Wojtaszczyk for random vectors with independent log-concave coordinates, are extended to vectors whose independent coordinates have log-concave tails. Moreover, to complete his work, Michał Strzelecki partially answers a question by Latała-Strzelecka about comparaison moment's inequalities with constant one at the first strong moment, by constructing a counter-example to discuss the log-concave tails assumption. The level of complexity of the proofs and of this example needs a demanding work. It shows the mathematical skills of Michał Strzelecki in the field.

As a conclusion, Michał Strzelecki has worked on a very interesting topic which includes functional and transport inequalities, and concentration phenomena. As his PhD dissertation shows, he has acquired an extensive experience in this field of research and his work solves or reveals open questions. All along the document, none of the technical mathematical difficulties are avoided but are very clearly addressed. Beside his thesis supervisor, he already successfully collaborates with young or experienced researchers. The published papers by Michał Strzelecki are of high level and new collaborative papers are in progress. Therefore I deem the doctoral thesis by Michał Strzelecki as widely sufficient to grant a PhD, and because of the high quality of his work, with an honorary distinction.

Sincerely,

Paul-Marie Samson