Title: *Stochastic games and their complexities*
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The thesis of Marcin Przybyłko is concentrated on analysis of branching games, that is stochastic two-player zero-sum turn-based infinite-duration games of imperfect information. The author studies classical questions such as determinacy and computational complexity. In addition, an application of stochastic games to a practical problem of modeling ecosystem of a fruit farm is included at the end of the thesis.

The thesis is based on three papers published in proceedings of international conferences, one in MFCS (rank A according to Core 2018) and two in Gandalf. Marcin Przybyłko is also a co-author of another MFCS paper that seems to be strongly related to the thesis.

The thesis is divided into 10 chapters. After a short, and relatively well written, introduction the author presents definitions of basic notions such as trees, topology and measure, games on graphs, tree automata and monadic logic. The presentation is rather technical with almost no illustrative examples but is standard with respect to most papers I have seen in this area.

Chapter 3 presents the main model, branching games. The definitions are formally correct and clean. The author illustrates the main concepts using an example presented in Figure 3.1. I have to say that I would have appreciated a more detailed illustration, especially of behavioral strategies. A much better explanation is given later in Example 3.2.2 which helped me to truly understand the fundamental concepts. The author also shows that branching games subsume various standard types of games.

Chapter 4 describes problems of interest that are pretty standard: Identify nicely behaved subclasses of branching games (w.r.t. determinacy) and analyze decidability and complexity of brachning games. The studied computational problems are also standard: Compute the value of the game or, at least, decide whether the value is above a threshold. As branching games are not determined in general, another meaningful problem is to decide determinacy.

Chapter 5 is concerned with regular pure branching games that is games with no stochastic elements. First, single player games with regular winning conditions are studied. Here the author analyzes complexity of computing winning strategies. The proofs seem to be based on well known principles and not too technically demanding. Then two player games are studied. Here the author proves that determinacy is decidable in doubly-exponential time. Moreover, the author provides tight complexity bounds on some instances of the so called simple threshold problem for the value. This chapter also contains a technique of dealternation which, in essence, removes the alternation from the alternating tree automaton specifying the winning set. This technique itself is not completely new and surprising but in this setting can be seen as one of the contributions of the thesis.

Chapter 6 extends the previous results to stochastic branching games. The author proves that stochastic branching games are determined with open (closed) winning sets (however, they are not determined for regular winning sets in general). Further, the simple threshold problem is proven to be undecidable even for single player stochastic branching games. This result is based on reduction from a word problem of very simple non-deterministic automata. The construction is not difficult but it is always hard to estimate how much effort went into finding it. The main technical part of this section is concerned with a derandomization construction which eliminates the “Nature” player from the game. The construction builds on top of results of Matteo Mio but brings, in my opinion, new non-trivial insights. Thus can be considered as another contribution of the thesis.

A very short chapter 7 deals with determinacy and complexity of branching games with winning objectives described by so called game automata via a relatively simple reduction to meta-parity games. Chapter 8 is concerned with a slightly unrelated problem of computing the uniform measure of regular sets of infinite trees. The sets are specified by formulae of various fragments of monadic second-order logic. The author identifies several fragments (such as first-order formulae without descendant relation) defining regular sets whose uniform measure is rational and computable. The results apply some relatively standard techniques and do not seem to be too demanding. Maybe I am missing something but it is hard for me to see the direct connection of these results to the rest of the work. I understand that Section 8.1 tries to describe such a connection but still I would like to see a more detailed explanation of the relationship. Namely, could we use results of chapter 8 to solve the games?

Chapter 9 is concerned with a so called plantation game which is a model of an ecosystem of a fruit farm. Unfortunately, the formal framework used in this chapter is not exactly the one developed in the rest of the thesis. Also, as the author admits, the chapter makes an impression of a work in progress. The presentation looks rushed and definitely not sufficiently polished. The author omits some definitions (e.g. most of the semantics of the game) which is not usual in this type of publication. Even though I appreciate applied flavor of this chapter, it is really the weakest part of the thesis.

To sum up, the main contribution of the thesis consists of several technical observations. I have not found any serious errors in reasoning or in presented results. On the other hand, even though some techniques presented in the thesis might be useful in further research (e.g. the derandomization), none of the presented results seems to be particularly surprising or difficult. As indicated above, presentation might have been better in some places but it is acceptable.

Despite of my objections, I found **the thesis as sufficient to grant a PhD**.

Brno February 3, 2019 Tomáš Brázdil