# Evaluation of the Thesis of Marcin Przybytko 

## Summary

The thesis considers two issues. On the theoretical side, it studies problems related to the analysis of a certain type of games called branching games. These are a type of two-player turn-based games, a computational model that is used in reasoning about systems, logic, and finite-model theory. On the practical side, the thesis studies how these types of techniques could be used in the context of agricultural planning.

The main part of the thesis considers two-player stochastic games with branching positions. The particular type of games considered in the thesis were suggested recently by Matteo Mio in his PhD thesis. Mio used these games to show a connection between game theory and a certain type of temporal logic. However, many questions regarding these games remained open and some of them are answered in this thesis.

One of the most expressive types of games considered in this area of computer science is called Blackwell games. These are rich games that incorporate many modelling options regarding options for the players (such as probabilistic choices, knowledge regarding the arena and other players, and simultaneity of decision making). In a landmark result, Donald Martin showed in 1998 that these games are determined for very expressive goals for the players. That is, even for very complicated goals for the players, it is possible for a player to disclose their strategy to the other player and this does not give the other player an advantage.

While Blackwell games supply the foundations for many of the studies of two-player games, Mio showed that branching games are more general than them. This means that the issue of determinacy is open for these types of games and does not follow from Martin's landmark result. It was clear from Mio's work that the most general types of games do not have these good features.

In the thesis, the boundaries of determinacy and algorithmic computation of values for branching games are beginning to be charted. In order to do that the thesis studied various restricted settings of branching games. Starting from restricting the types of options that are available to the players and continuing with studying the complexity of the goals that the players need to achieve. The thesis is rich with results about different settings where the games are determined and what algorithmic results can be achieved in some cases. It also includes negative results showing what cannot be done and which cases are too complicated to be algorithmically solved. In particular, the results about the interaction between the complexity of the options available to the players and the complexity of the goals given to the players are surprising and very original (called "derandomization" and "dealternation" in the thesis).

The study of different winning conditions for these games raises another interesting question on the boundary of measure theory and computer science. This is the issue of measure of sets of trees that are defined by logical sentences. The thesis shows how to compute the measure for several types of restricted sentences.

Finally, the thesis handles a practical question of how models similar to those studied in the theoretical part could be used for agricultural planning. The thesis includes a case study based on real data from a fruit farm in New Caledonia. In the case study, existing historical data is analyzed and formed into a model and then analysis of the model creates a method of prediction and advice as to how to apply treatments in the farm. Though not using the elaborate game models that are handled in the theoretical part of the thesis, this part shows the power of these models and, at the same time, the diversity of techniques applied by the candidate.

## Evaluation

The topic of two-player games is a widely studied topic. Branching games are a relatively recent addition and are exciting as they break some of the boundaries of determinacy and decidability that usually characterize these kinds of games (with full observability). The thesis combines knowledge in games, automata theory, topology, and measure theory. It contributes to the theoretical foundations of this field of study and extends the theory of branching games alongside related issues. The practical part shows additional techniques that are mastered by the candidate. Overall, the thesis shows that the candidate mastered techniques and knowledge from very diverse fields and makes new contributions to the field.

As further evidence to the quality of the work, the thesis is based on results that have been published in the following conferences: the symposium on Games, Automata, Logics and Formal Verification (GandALF) 2014 and 2018, and the symposium on Mathematical Foundation of Computer Science (MACS) 2016.

Overall, the thesis is very well written. Deep technical material is clearly explained. The contributions are properly put in the context of the literature with good discussions.

Therefore, I find that the thesis is sufficient to grant a PhD and recommend the award of a PhD degree to Marcin Przybyłko.

I am including a list of minor comments about the write-up. I would suggest to the candidate to try to use these comments to improve the write-up for posterity's sake.

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## Detailed Comments and Minor Issues

1. Page 18 Fact 2.2.1 and 2.2.4 are repeated.
2. P20 L-12 the explanation of $G=\langle B, W>$ explain first what is $B$ and then what is $W$. At the end of the paragraph you should also give some technical details about what is W.
3. P21 first sentence: Say also something about ownership.
4. P21 for reachability games and parity games: you give an independent definition but don't explain this very well. I would change the order of the paragraphs in both subsections to have first the explanation of how these are graph games and then give an independent and slightly different definition with a specialized notation. But I'm not sure that you are using the specialized notation elsewhere so perhaps drop this?
5. P22 Sec 2.3.5 L1 introduces $\rightarrow$ introduce
6. P23 Finite automata - don't you want to restrict the behavior after reading \flat to match the structure of trees?
7. P23 L-1: You mention winning strategy but this was never explained before
8. P24 Automata on words: Your definition allows the trees to have multiple branches that are labeled by non-blank symbols. Wouldn't it make more sense to include a part that forces all the rest of the tree to be blanks?
9. P30 Definition of Branching games: reorder the explanations to match the order of the elements in the tuple.
10. P38 the equation for val_ $\mathrm{G}^{\wedge}\{\mathrm{BE}\}$ third line is missing $a+$
11. P40 Gale-Steward games - your explanation is somewhat simplistic. You have to translate arbitrary branching of the set $S$ to the binary branching of the $\{\mathrm{E}, \mathrm{A}\}$ branching game.
12. P44 Theorem 4.3.3 +31 algorithms like strategy $\rightarrow$ algorithms like the strategy
+31 exponential in time $\rightarrow$ exponential time
+41 or fix-point $\rightarrow$ or the fix-point
13. P44 Theorem 4.3.4 non-deterministic automaton $\rightarrow$ non-deterministic word automataon.
14. P48 last line: What about the case that A is alternating?
15. P49 line 2: What about a deterministic or alternating automaton?
16. P49 line 2: You are referring to $[54,15,17]$ for the details. It would be nicer to localize the exact results. E.g., the emptinesss of nondeterministic tree automaton is in UP\cap co-UP [Thomas].
17. P50 just before sec 5.3: concludes the proof of the lower bounds. The EXPTIME upper bounds are missing!
18. P55 Fig 5.2: top left label should be \phi \wedge $\backslash \mathrm{psi}$
19. P58 Proof of Theorem 5.3.4: As before, upper bounds are neglected. The case of E is simpler as it just goes back to Theorem 5.3.2. But the case of A requires going back into Lemma 5.3.1.
20. P61 Example 6.1.3 be an arena as depicted $\rightarrow$ be the arena depicted Let words $(\mathrm{t}) \rightarrow$ For a tree t , let words $(\mathrm{t})$
21. P66 The undecidability of .... refinement.

I don't see why this is clear. The automata that [20] work with have either 1 or 2 transitions per letter. So, in general, your simple automaton falls into this category. But why cloning states? And how to clone states in a way that does not change the proportion of accepting/rejecting runs?
22. P67 explanation of (b) Probabilistic automata were never mentioned before
23. P68 L-7 dealternation $\rightarrow$ derandomization
24. P69 last paragraph. I confused the winning set with the set of plays and found this very hard to follow. I think that you should improve this paragraph.
25. P70 expressive power: complex $\rightarrow$ irrational
26. P71 simple branching games: I think that a reminder what are simple branching games is in place.
27. P72 paragraph 3: verify the guess $\rightarrow$ compute the probability $r$ (and 1-r in the second appearance)
28. P72 last line: $\mathrm{f} \_\mathrm{N} \#$ leta ${ }^{\wedge *}$ _ $\left\{\mathrm{B}^{\prime}\right\}$ chooses:

This is not really a strategy. The value r (or 1-r) is the integral over all the choices of nature in the gadget G_r. It is equivalent to choosing the left child with probability val^E_\{G_r\}.
29. P73 Figure 6.5: Please use the same convention as in Figure 6.4. That is, include the figure of $\mathrm{z}<-\mathrm{x}->\mathrm{y}$ and add the dashed lines to $\mathrm{g}, \mathrm{y}$, and z
30. P74 L-11: with the Nature's $\rightarrow$ with Nature's
31. P78 the inequalities numbered 7 and 8 are actually equalities.
32. P80 just before Theorem 7.1.1: In Theorem 3.3.5 you cite 33 and not 34
33. P84 Problem 8.1.2: exists $\rightarrow$ exist
34. P84 Lemma 8.1.3: perhaps use different names for $B_{-}\{t, u\}$ where $t$ is finite and where $t$ is infinite?
In 3. It should be $t^{\prime} \backslash$ triangle $u$
35. P85 L3: This is where you are using $B_{-}\{t, u\}$ for infinite tree $t$ and finite trees $t \_i$ and $t \_\{i+1\}$. This is quite confusing notation.
36. P85 You start with an introduction to Kolmogorov's zero-one law, say that this is not the case but then: the first three cases in example 8.1.4 are still zero-one. Wouldn't it make more sense to have these examples before the statement attached to Lemma 8.1.3 and then have the $4^{\text {th }}$ item by itself refuting the zero-one-ness?
37. P85 l-3 stay for $\rightarrow$ stand for $\quad$ at room $\rightarrow$ at the root
38. P87 definition of root formula: wouldn't it make sense to define r-root formula? Mentioning specifically the parameter $r$, which otherwise is used but not really bound.
39. P88 Moreover, for every tree .... \varphi_i(x):

I found this very unclear until turning the page to Figure 8.1. You should refer to the figure in the text. It is very helpful but would be even more helpful if you explain it in the text.
40. P91 L3: at depth $r+1$ or more
41. P92 after the computation of $\backslash m u^{\wedge *\left(B \_t\right) . ~ S t a t e ~ t h a t ~ i t ~ i s ~ c l e a r l y ~ r a t i o n a l . ~}$
42. P94 the definition of edges in G_\pi

Before reading the proof the decision to include $\mathrm{x} \backslash \mathrm{subsetneq} \mathrm{y}$ and not $\mathrm{y} \backslash$ subsetneq x as well as lepsilon( x ) look quite arbitrary. I think that some explanation regarding that here would be useful.
43. P95 Figure 8.2. Again, a useful figure but you neither refer to it in the text nor explain it.
44. P96 Graph of Firm sub-patterns is and edge $\rightarrow$ is an edge
45. P96 just before lemma 8.4.4: decides of the $\rightarrow$ decides the
46. P97 last three lines of the proof of Lemma 8.4.4

It is not clear to me why this is an iff.
$t \backslash$ models $q$ implies $t \backslash$ models $p$ is clear as $p$ is a subpattern of $q$. But $I$ do not understand why t models p implies t models q
47. P97 Proof of Theorem 8.4.1: State the rationality as well.
48. P99 L-9: considered a work $\rightarrow$ considered as a work
49. P100 colleges $\rightarrow$ colleagues
50. P101 definition of POMDP

Shouldn't Q,E, and P form part of the tuple?
You are using P and O for the observations in this definition paragraph.
51. P101 L-15 O\times $\mathrm{E} \rightarrow \mathrm{E} \backslash$ times P
52. P101 L-6 Missing * on $\mathrm{M}(\mathrm{w})$ and $\left\langle\right.$ tau_i>_ $\{i=0\}^{\wedge}$ ^infty $\backslash \mathrm{pi}^{\wedge} 0=\backslash \mathrm{pi}^{*}$-> you haven’t defined $\backslash \mathrm{pi}$. Should probably be $\backslash \mathrm{pi}$

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\tau^*_i=B(b_i) should be e_i
\i^*_i=\ldots \tau^*_{i-1} should be \tau^*_{i-1}(o)
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53. P102 L-6 Env:N\times H

You should introduce N and H before using them.

