Final report on the doctoral thesis: “Secant varieties, Waring rank and generalizations from algebraic geometry viewpoint” by Maciej Galazka

November 1, 2023

In the revised version of the dissertation, the candidate has addressed all my the questions and suggestions in a completely satisfactory manner. I think that the presentation of the results is very good.

I confirm my previous opinion about the quality of the thesis: it deals with relevant problems in Algebraic Geometry and presents new and very interesting results.

In conclusion, **I consider the revised thesis to be, without any doubt, sufficient for the award of a PhD.**

Sincerely yours,

Maria Chiara Brambilla M Chiara Brambilla

UNIWERSYTET WARSZAWSKI
BIURO RAD NAUKOWYCH
2023 -11- 01
WPŁYNĘŁO
L.dz................Podpis............

Report on the doctoral thesis: "Secant varieties, Waring rank and generalizations from algebraic geometry viewpoint" by Maciej Galazka

July 25, 2023

The dissertation of the candidate concerns the theory of secant varieties, which is an essential topic in algebraic geometry and has connections with many areas of mathematics, in particular with the theory of tensors. The thesis collects some interesting results obtained by the candidate, some of them already published (see [42,43,44] of the bibliography).

The main results presented can be summarized as follows:

- *Explicit computation of X-rank, in case of a toric surface X.* A bound for the $\nu_{d,e}(\mathbb{P}^1 \times \mathbb{P}^1)$-rank for monomials (Theorem 1.6) is proved by toric methods. Other analogous computations are presented in the case of the Hirzebruch surface and some weighted projective planes. See Section 5.

- *Finding equations of secant varieties.* Determinantal methods (in particular the vector bundle method) are confirmed tools for the description of secant varieties. The candidate studies extensions of such methods to the problem of finding equations of cactus secant varieties. See Theorems 1.13, 1.14 and 1.15 and Section 3.

- *Geometric description of cactus variety.* Starting from results on the Hilbert scheme the candidate is able to describe explicitly the cactus variety (in particular $\kappa_{14}(\nu_d(\mathbb{P}^n))$). This study gives information on the difference between secant and cactus varieties. Also a result on the Grassmann secant variety is provided. See Section 4 and 6.

Recommendation.

I think this thesis satisfies the standards of academic work. Anyway I would like to ask the author for some minor corrections.

Comments and questions.
Here is the list of corrections and remarks for the candidate:

- page 8, after Remark 1.3: I think it is worth to remind here that Problem 1.2 can be formulated also for the real field and this is the case more relevant for the applications. Which of your results can be extended and studied in the real field?
- page 8, Definition 1.4: you should also assume X irreducible and reduced
- page 8, line -7 and line -3: "the" X-rank
- page 9, line 11: "the" rank
- page 9, line 16: you could mention that in [2] a conjecture is proposed and is still open.
- page 9, line -10: "the" rank (please check this along all the dissertation)
- page 9, line -7: you should write $r_{\nu_d(\mathbb{P}V)}$ instead of $r_{\mathbb{P}V}$
- page 10, line 8: when you introduce the rank with respect to Segre-Veronese you could mention that it is called "partially symmetric rank" (and point out the case of the Veronese is the symmetric rank, which coincides with the Waring rank)
- page 10 line 6: you should write $r_{\nu_{d,e}(\mathbb{P}^1\times\mathbb{P}^1)}$ instead of $r_{\mathbb{P}^1\times\mathbb{P}^1}$ The same for every occurrence in this page, in particular in the statement of Theorem 1.6.
- page 13 line 10: delete a "the"
- page 13 line 16: when you introduce the cactus rank you should mention that it was fisrt introduced in the book of Iarrobino-Kanev as "scheme length"
- page 14 line 1: you should also assume "non-degenerate". Please check also in the other statements of the introduction.
- page 14 line 12: "Proposition"
- page 14 line 16 "there are"
- page 17 line 15: delete "is"
- page 19 line -2: I would recall the definition of the class group and some more preliminary results about the Cox ring

- page 34, line 10 you should call it "dual" vector space
- page 45: in the Terracini lemma you should assume X irreducible and reduced
- page 45 line 2: at "a" point q
- page 45 line 12: delete "a"
- page 45: line -10 and following: it seems that the sentence is not finished.
- page 47 line 13: of "a"
- page 55, line -9: delete "is"
- page 56 line 1: "necessarily"
- page 56 line 16: "affine"
- page 60 line 2: "multi"
- page 65 line 12: apolarity "and" we
- page 69 line -1: explain in which sense you take the "closure", the same in theorem 4.2
- page 71 line 13: delete "the"
- page 76, line 11: "defined"
- page 82, line 1: "dimensional"
- page 86 line-11: a bracket is missing
- Section 5: do you think that the techniques applied here can be extended to other toric varieties?
- page 92, line -15: delete ","
- page 92, line -12: has "the" desired
- page 110, line -11: you should be more clear: "other component with respect to the secant"

25/7/2023

Maria Chiara Bubbles

UNIWERSYTET WARSZAWSKI
BIURO RAD NAUKOWYCH
2023 -08- 16
WPŁYNĘŁO
L.dz. 1579 ....Podpis............