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Report on the Thesis:

Analytic properties of operators on the non-reflexive spaces of smooth functions

submitted by

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to the Faculty of Mathematics, Informatics and Mechanics at Warsaw University

1 The Thesis of Krystian Kazaniecki

1.1 The Content of the Thesis

The thesis presents the central results obtained by Krystian Kazaniecki, as graduate student under the supervision of Prof. Michal Wojciechowski.

1. Chapter 1 reviews the results and discusses the background and the origin of the problems addressed in later chapters.

2. Chapter 2 on the Fourier analytic treatment of Ornstein non inequlities and the closely related chapter 3 on Fourier multipliers acting on the homogeneous Sobolev space $\dot{W}_1^1$, are based on two separate publications in *Analysis and PDE* and *Ann. Inst. Fourier (Grenoble)* respectively.

3. Chapter 4 presents the L^p estimates for transfer operators documented in the ArXive article 150.05994.

4. Finally Chapter 5 is devoted to the recent spectacular result on the trace space of $W_1^1(\Omega_K)$ where Ω_K is the Jordan domain bounded by the v. Koch curve. This result, obtained jointly with his adviser Prof. M. Wojciechowski, is documented in the thesis for the first time. It -once again- reveals the astonishing power of the *spanning tree method* invented by Prof. M. Wojciechowski in connection with the bounded approximation property of Sobolev spaces.

Thus Krystian Kazaniecki produced an excellent mathematical thesis, presenting with high clarity, his own recent achievements as well as the field in which he obtained his impressive results.

We now turn to describing in more detail the content of the five chapters forming Krystian Kazaniecki's thesis.

Chapter 1. In the first chapter the author provides a conceptual overview of his work emphasizing the origin and history of the problems solved in his thesis. Of particular interest is the thorough discussion of the Ornstein-non-inequlities and the presentation of the competing research conducted by Kirchheim and Kristensen.

Chapter 2. This chapter is based on the paper "Anisotropic Ornstein non-inequalities" (ONI) which appeared 2017, in the fiercely competitive recent journal **Analysis and PDE**, published jointly with D. Stolyarov and his advisor M. Wojciechowski.

The setting for anisotropic ONI is specified by fixing an affine hyperplane Λ and differential operators T_j with Newton diagram contained in Λ. In that case T_j is called Λ homogeneous. Under the additional constant parity assumption the main result of chapter 2 (Theorem 2.2) asserts that for any choice of Λ-homogeneous differential operators T_j, the a priori estimates

$$\|T_1 f\|_1 \leq C \sum_{j=1}^{\ell} \|T_j f\|_1, \qquad f \in C_0(\mathbb{R}),$$

implies that $T_1 \in span\{T_j : 2 \leq j \leq \ell.\}$.

The method for proving Theorem 2.2 is quite *ingenious and merits intensive further study.* The authors set up a variational problem, (eqn. (2.4) and eqn. (2.5)), use the ONI hypothesis to show that generalized quasi convexity condition holds true (Theorem 2.6.), deduce from it the generalized rank-one convexity condition (Theorem 2.9.) and transform it into a separately convex function, homogeneous of order one. Thereafter it is shown that the latter are non negative (Theorem 2.14.), from which it is finally deduced that $T_1 \in span\{T_j : 2 \leq j \leq \ell\}$.

The chapter closes by discussing ONI for martingale transform operators, thereby pointing out fresh, interesting connections between the Calculus of Variation and Probability.

Chapter 3. The original ONI provide the motivation for the problems solved in chapter 3. Indeed, as noted by S. Poornima (1982) , ONI implies that there exists a bounded Fourier multiplier operator T_m on $\dot{W}_1^1(\mathbb{R}^d)$ which is not given by the Fourier transform of a finite measure—nevertheless the main result of chapter 3 (Theorem 3.1.) asserts that a Fourier multiplier operator T_m on the homogeneous Sobolev spaces $\dot{W}_1^1(\mathbb{R}^d)$ is bounded if and only if the multiplier m itself is a bounded and continuous function.

This chapter is based on the paper "Fourier multipliers on homogeneous Sobolev spaces $\dot{W}_1^1(\mathbb{R}^d)$" which appeared 2016 in the classical first rate journal **Ann. Inst. Fourier (Grenoble)**. The authors (K. Kazaniecki and M. Wojciechowski) obtain therein a definitive answer to one of the *first and natural questions* on Fourier multipliers acting on Sobolev spaces.

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The method of proof, presented in chapter 3, proceeds by contra-position and assumes, that there exists a bounded Fourier multiplier whose symbol m fails to be continuous at 0, (at all other points the continuity of m is deduced from general facts and considerations). Two distinct cases are isolated by which a multiplier may fail to be continuous. For Case I, matters are reduced to multipliers of homogeneity zero, and the theorem of S. Poornima. (See Lemma 3.4.) For Case II the proof proceeds by constructively exhibiting normalized testing functions h_s (Riesz products) for which

$$\|T_m(h_s)\| > Cs, \quad s \in \mathbb{N}$$

contradicting continuity. (See pp 29-36). Masterful command of the Riesz product technique is displayed throughout the proof of Theorem 3.1 in chapter 3 of K. Kazanieckie's thesis.

Chapter 4. This chapter is devoted to transfer-operators acting L^p spaces over the infinite torus product $\mathbb{T}^{\mathbb{N}}$. The underlying article entiteled "On the equivalence between the sets of trigonometric polynomials" is documented as ArXive 1502.05994. Prior to the paper by K. Kazaniecki and M. Wojciechowski, only for the case $p = \infty$ there existed a proof (by Y. Meyer) that transfer-operators are bounded in L^p—and for the cases $p < \infty$ the literature did not contain a valid proof.

This is now rectified by the paper of K. Kazaniecki and M. Wojciechowski who demonstrate(!) that Y. Meyer's condition on the transfer parameters imply that the transfer-operator satisfies upper and lower L^p estimates. Moreover, by way of very sharp examples, the authors show that transfer-operators fail to have the interpolating property: Hence the passage from $p = \infty$ to $p < \infty$ poses a delicate problem, which was solved by K. Kazaniecki and M. Wojciechowski.

Chapter 5. This chapter presents a deep study of the trace operator acting on the Sobolev space $W_1^1(\Omega)$. First for Lipschitz domains Ω the author presents a very lucid proof of the fact that the trace operator does not possess a linear right inverse, (Peetre theorem for Lipschitz domains). A real surprise is given by Theorem 5.2 –the main result of chapter 5: It asserts that for the domain bounded by the v. Koch curve a right continuous inverse exists and in fact an explicit construction is given in the last chapter of Kazaniecki's thesis. Moreover a key point in the proof identifies the isomorphic type of the trace space to be ℓ^1.

1.2 Assessment and Recommendation

Through a series of impressive results, and their excellent presentation in his thesis, Krystian Kazaniecki demonstrates that he is a problem oriented mathematician in full command of most advanced techniques developed in the fields of geometric -, harmonic -, and functional analysis, and that he has the ability to solve problems left open by leading experts in the area.

In my opinion Krystian Kazaniecki produced a first rate thesis, exceeding the requirements for **PhD with distinction (summa cum laude)**.

$\square$

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