Report on the PhD thesis of

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"Singularities of harmonic and biharmonic maps into compact manifolds"

1. Background

This thesis belongs to the field of mathematical analysis, more precisely, the field of geometric analysis and partial differential equations. The main goal is to study regularity and singularities of harmonic and biharmonic maps into manifolds. Since by explicit examples it is known that singularities may occur, partial regularity (regularity outside an exceptional set) is in general the best result one can expect. Therefore, a natural question is the analysis of the singularities and the exceptional set, like estimates for the size or the dimension. Although the regularity theory for harmonic and biharmonic maps is a classical field of research, there are still many open problems.

2. Content of the thesis

The thesis consists of an introduction, two research results on harmonic and biharmonic maps and an appendix, where some technical lemmas are proved in detail. The fourteen pages long introduction provides a very nice overview of the background and explains the main results. It becomes apparent that Ms Mazowiecka has a broad overview of the whole field of harmonic maps. One of the two research results is in collaboration with the PhD supervisor Paweł Strzelecki and already published in *Adv. Calc. Var.* (reference [39] in the thesis).

2.1. **Lavrentiev gap phenomenon for harmonic maps.** This chapter is concerned with the study of harmonic maps into spheres with singularities and their stability. The main result shows that singularities are unstable in the sense that for any given boundary datum φ and any $M \in \mathbb{N}$ and $p < 2$ there exists another boundary datum $\widetilde{\varphi}$ which on the one hand is close with respect to the $W^{1,p}$-norm and on the other hand admits a unique minimizer with at least M singularities. In particular this result implies that the Lavrentiev gap phenomenon appears for the modified boundary datum $\widetilde{\varphi}$. For boundary data with zero degree this result is contained in publication [39]. The result is sharp, in the sense that it is wrong for $p = 2$. The proof uses a very nice construction which is based on a result by Almgren and Lieb. A particularly nice feature is that the arguments from the paper by Almgren and Lieb are carried out in detail so that other researchers can profit from this in the future.

In her thesis Ms Mazowiecka also treats the case of nonzero degree. Here, the construction from before is not possible anymore since it uses the existence of two antipodal points with $\varphi(q) = \varphi(-q)$. For this reason Ms Mazowiecka has to develop a different construction for the case of nonzero degree.

2.2. **Conditional boundary regularity.** In the last chapter Ms Mazowiecka studies the boundary regularity for minimizing biharmonic maps. She proves that for dimensions ≥ 5 every minimizing biharmonic map is smooth on a full neighborhood of the boundary provided it satisfies a boundary monotonicity inequality. The boundary monotonicity inequality is not included in the thesis. It has recently been proved by S. Altundas as part of his PhD thesis. Therefore, in combination both thesis yield a very nice result.

The proof of the boundary regularity result is based on the analysis of limits of rescaled maps. In the boundary situation several extra difficulties have to be handled, like the treatment of extra terms and reflection arguments. One of the main contributions here is a compactness result at the boundary. The method of proof and the difficulties appearing at the boundary are described in a very nice and honest way. I particularly like that Ms Mazowiecka does not try to hide the difficulties she had to face when proving the result.

3. Evaluation

The main results obtained in this thesis are new and give new insight into the theory of harmonic and biharmonic maps. Existing techniques are combined and modified in an innovative way and new methods are developed in order to overcome the difficulties. To give some impression of the variety of the used methods, we mention that amongst others compactness results, monotonicity formula, installing singularities, co-area formula and defect measures are used in the arguments. These methods are extensively used in contemporary research.

This is a thesis of high quality in a field of current interest. The objectives and results are clearly stated. The only drawback is that the boundary regularity is obtained under the condition of a boundary monotonicity formula. I evaluate as good research practise that this condition is clearly stated. The overall presentation of the thesis is excellent. The author demonstrates a very good knowledge in her field of research. This work clearly shows that Ms Mazowiecka is able to do independent research at an international level. This is manifested by the fact that one of the results is already published in a high level international journal and two results are obtained without the collaboration of the superviser. Additionally Ms Mazowiecka published two articles in collaboration with K. Kazaniecki, M. Lasica and A. Kałamajska not contained in her thesis. Ms Mazowiecka has also given talks on her results at international conferences. I particularly enjoyed her enthusiastic lecture style during her talk at the ÖMG-DMV Congress 2017 in Salzburg.

In conclusion, I recommend without any doubts the PhD thesis to be accepted at the University of Warsaw.

Salzburg, January 30, 2018

(Prof. Dr. Verena Bögelein)