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To whom it may concern:

Karol Wegrzycki's dissertation *Provably Optimal Dynamic Programming,* advances our understanding of fundamental polynomial-time solvable problems, and casts light on the relationship between fundamental problems via fine-grained reductions. The volume and character of these results is more than sufficient to award Karol Wegrzycki a Ph.D.

Chapter 3 presents applications of the Frobenius normal form to graph algorithms, yielding simplified algorithms for several fundamental graph problems, e.g., preprocessing graphs in $O(n^{\omega})$ time to support distance queries in $O(n)$ time, and counting and detecting cycles passing through any vertex.

Chapter 4 addresses a class of NP-hard problems that are unusual in that they admit fully polynomial time approximation schemes (FPTAS), such as (Unbounded) Knapsack, Subset Sum, and the simplest of them all, Partition. It was known that $(1+\epsilon)$-approximation schemes for these problems existed with running times of the form $O(n+\epsilon^{-C})$ for various constants C. This dissertation presents sophisticated algorithms to solve "weak" $(1+\epsilon)$-approximations for Subset Sum and $(1+\epsilon)$-approximation for Partition in $\tilde{O}\left(n+\epsilon^{-\frac{5}{3}}\right)$, breaking a longstanding ϵ^2 barrier for FPTASs of NP-hard problems.

The area of fine-grained complexity has found lower bounds for problems in *P*, conditioned on plausible hardness hypotheses. Each of these hypotheses (SETH, 3SUM, APSP, OV, etc.) rules over a "fiefdom" of problems, and these fiefdoms have been related in various ways, but never united under a single hypothesis (*à la P=NP*). Chapter 5 of Wegrzycki's thesis makes a major contribution to this theory by building an entirely new fiefdom of problems equivalent in hardness to (min,+)-convolution. It was known that (min,+)-convolution is related to APSP and 3SUM, but developing a class around (min,+)-convolution is new and interesting. It contains interesting problems like Tree Sparsity and Necklace Convolution.

Chapter 6 addresses a major open problem in fine-grained complexity. There are numerous problems equivalent to APSP and, by hypothesis, require $n^{3-o(1)}$ time, and many other problems that are equivalent to matrix multiplication, and require n^{ω} time. However, there are a large set of problems that are intermediate in complexity, such as directed APSP, (min,max)-product, domination product, and others. Problems in this class are varied, but they all have running times that collapse to $O(n^{2.5})$ in the event that $\omega=2$. Chapter 6 makes the first connection that

I'm aware of between this class and existing ones. Wegrzycki shows that strongly polynomial $(1+\epsilon)$-approximate APSP is equivalent to (exact) (min,max)-product. In particular, there is an $\tilde{O}(n^{\frac{3+\omega}{2}}/\epsilon)$ algorithm for approximate APSP (matching Duan and Pettie's $O(n^{\frac{3+\omega}{2}})$ algorithm for (min,max)-product), and improving either algorithm automatically improves the other. This chapter introduces a number of interesting techniques to approximate (min,+)-style problems with (min,max)-type formulas. Chapter 6 is at the forefront of research in fine-grained complexity. Understanding, at a fine-grained level, the relationship between exact and approximate versions of polynomial-time solvable problems is not well understood. Wegrzycki's results are an important contribution to this area.

To summarize, Wegrzycki's dissertation is excellent and makes substantial contributions to theoretical computer science. This would be among the better dissertations at any of the universities that I have been affiliated with in the last 20 years, and clearly merits a Ph.D.

Sincerely,

S Pettie

Prof. Seth Pettie