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Review of the PhD thesis of Karol Węgrzycki "Provably optimal dynamic programming"

The thesis is focussed on dynamic programming and computational problems for which fastest known algorithms are based on dynamic programming techniques. Presented results include algorithms finding optimal or close to optimal solutions as well as reductions showing computational equivalence of a class of studied problems. They fall into the recently very active line of research on fine grained complexity. The common theme of most of the work is the question of existence of sub-quadratic or sub-cubic algorithms for a class on natural optimization problems. The presentation of the obtained results is divided into two parts: algorithms and equivalences of problems. Each part contains two chapters, each chapter corresponds to a single publication.

Chapter 3 of the thesis contains results based on employing fast decomposition of matrices into Frobenius normal form. This can be seen as an extension of a line of work that utilized fast algorithms for matrix multiplication to find solutions to combinatorial problems in sub-cubic time. New algorithms for finding short cycles, counting cycles of fixed length, and constructing distance oracles in directed graphs are presented.

In Chapter 4 Polynomial Time Approximation Scheme (PTAS) for special cases of the Knapsack problem are being studied. The particular focus is on the dependence of the running time on the precision parameter ϵ . The main result here is the first such algorithm for the Partition problem with sub-quadratic dependence of the running time on $1/\epsilon$. Additionally a randomized algorithm for the more general Subset Sum problem slightly violating the sum constraint also with sub-quadratic dependence on $1/\epsilon$ is presented.

Chapter 5 of the thesis is dedicated to the class of problems that are shown computationally equivalent to computing Min-Plus Convolution. The most outstanding consequence is that it shows that it is unlikely that there would exist substantially sub-quadratic algorithm for approximately solving the Knapsack problem. Interestingly it contrasts with the existence of sub-quadratic algorithm for Subset Sum which is an important special case. Interestingly, testing super-additivity (up to ϵ precision) of a sequence of numbers is also shown to fall into the same category and hence probably does not have a sub-quadratic algorithm.

Chapter 6 contains results that show computational equivalence of finding exact solutions to one group of problems and finding approximate solutions to another group. More precisely it is exact computation of Min-Max product being show equivalent to approximate solution to Min-Plus product. As a consequence the approximate computation of All Pairs Shortest Paths is shown to correspond to exact computation of so-called All Pairs Bottleneck Path. Such an

equivalence of computational tasks of different type appears to be a very elegant and somewhat unexpected result.

I'd like to complement the thesis for being composed of such a collection of consistent high quality results. They all were published at top CS conferences: STACS, SODA, ICALP, and STOC respectively. As the set of coauthors changes substantially between different papers, it appears that the author of this thesis played an important role in the presented line of research.

I must emphasize that the editorial quality of the thesis is in my opinion very good. The candidate paid attention to provide exhaustive introductory part defining and discussing the studied settings and concepts. I was particularly pleased with the fragments that discussed the strength of the obtained results by clearly contrasting them with the state of the art knowledge for the closely related computational problems and also with the cover story presented in the Foreword.

Summary

It is a very good thesis. The presented results fulfill and exceed the required by law standards for a PhD disertation. I propose to accept the thesis and allow for the further steps of the PhD procedure. I also propose that the PhD committee considers awarding the thesis a distinction (pol. wyróżnienie).

Janoslaw Beprka