Report on the thesis submitted by Jedrzej Kołodziejski

The thesis entitled “Bisimulation-Invariant Logics : Beyond Finite (and Infinite)” authored by Jedrzej Kołodziejski described contributions in two distinct topics related to bisimulation invariant logics : first, on the model theory of modal logic (ML), and in particular the notion of bisimulation categoricity, that is, the property of a set of formula having a unique model up to bisimulation, and second, countdown μ-ML, an extension of the modal μ-calculus (μ-ML) that adds the capacity to talk about boundedness with novel ordinal fixpoint operators.

I will first discuss my appreciation of these contribution and the different parts of the thesis in the order of the manuscript, and then conclude with a global evaluation. Some minor suggestions toward improving the manuscript are included for the author’s benefit.

The first chapter is an introduction to the contributions of the thesis. It begins with a gentle presentation of bisimulation, leading to a discussion of bisimulation invariant logics, specifically modal logic (ML) and its fixpoint extension μ-ML. With these main elements in place it then proceeds with a high level description of the contributions of the thesis. I found this introduction to be well written and pedagogical.

The second chapter mostly introduces notations and definitions, as well as recalls the fundamental results relating automata, games and μ-ML. In addition, it also develops tools for reasoning compositionally about games. Indeed, while games are a well-established tool in logic and automata theory, making formal arguments about games can often be laborious, even for standard techniques. To facilitate proofs in Chapter 4, here are introduced notions of partial games, that can be combined in a compositional way, as well as gamemulations, which are bisimulation-like relations specific to games that preserve winning conditions and allow for strategy translations. I find this technical development to be well-motivated, and indeed it does seem to simplify the presentation of Chapter 4. This part of the thesis demonstrates a keen technical mastery of the subject of games, as well as an appreciation of a common challenge within this topic, and a certain flair necessary for building a toolkit that can circumvent these.

Chapter 3 concerns the Model Theory of ML, and constitutes the technical meat of the thesis. More specifically, it focuses on the notion of bisimulation categoricity, that is, having a unique model up to bisimulation. This is a very natural notion which remarkably has not been previously studied, a gap that this chapter steps in to fill. It characterises bisimulation categoricity for ML with the existence of an image-finite model. It then considers the bisimulation categoricity for restricted classes of models, namely two-way or bidirectional models, in which two accessibility relations are each other’s converses, and transitive monomodal models. In the former case, categoricity can be characterised by having a model with finite in and out degree, while in the latter case, if the set of propositional variables is finite, then categoricity coincides with having a finite model. The final part of the chapter then focuses on ordinal models, that is, monomodal models with a well-founded linear order as their sole accessibility relation. On these models, bisimulation categoricity coincides with having a finite model. It then studies comptactness of ML over ordinal models, and shows that ML is compact over ordinal models if and only if the set of atomic propositions is finite. Finally, it proves a ”short model property” for ML over ordinal models.

The notion of bisimulation categoricity is natural in the context of ML and bisimulation invariant logics in general, and it is somewhat surprising that the questions studied here have not been tackled previously. The results are interesting and paint a first picture of the landscape of bisimulation categoricity for ML over difference types of models. The various phenomena that arise are clearly illustrated with detailed examples. In particular I found the discussion in 3.1.3.

very well explained and illustrated, with accessible, well-explained examples. Both the results presented and the discussion of their limits show the author's deep technical mastery of the subject and ability to effectively present the subtleties of the various phenomena that arise.

I also enjoyed the focus on ordinal models, which is well-justified by the different proof techniques required, as well as the more in-depth study of this model theory, with the section on compactness.

Chapter 4 is largely independent of Chapter 3, although both directions of research can be seen as novel angles on the classic bisimulation invariant logics, ML and μ-ML. μ-ML enjoys a powerfool toolkit based on automata representations and parity games, which capture the semantics of the logic. In this chapter, this trifecta of logic, automata and games is extended to capture notions of *boundedness*, via countdown μ-ML, countdown games, and countdown automata. It also introduces a syntactic variant of the automata in which the counters are independent of the fixpoint hierarchy, but are organised on a stack to capture the hierarchy among them. The author proves that the correspondence between logics, games and automata remains robust with these countdown operators, with the caveat that scalar and vectorial countdown mu calculus do not have the same expressive power.

In section 4.8 the author studies the hierarchy obtained from increasing the number of countdown operators, which is shown to be infinite. Finally, it studies decidability questions, and shows that the finite model checking problem is decidable, as is the satisfiability problem for the fragment in which only μ-operators are ordinal countdown operators, and the so-called Büchi fragement, with only two ranks, over infinite words. The general satisfiability problem is left open, although conjectured to be decidable.

Although the thesis does not show decidability of this logic (which seems to be quite an ambitious problem), the logic and automata models that it builds are natural extensions of μ-ML, and, as the thesis argues, the fact that they preserve the relationship between logic, games and automata implies a certain robustness to these models. I found the observations on the differences that arise in the countdown version of the logic, namely a difference in expressivity between vectorial and scalar forms of the logic, as well as the lack of positional strategies, quite insightful. Even though this chapter does not provide as many results as the previous one, it lays down valuable groundwork for the study of these formalisms.

The final Chapter concludes with some open problems and directions for future work. Since both parts of the thesis handle novel notions, the landscape of interesting questions on these is quite vast. For the model theory of ML, the most striking is that of a metatheorem that would unite some of the techniques and insights developed in the theses. On the countdown μ-ML side, the question of decidability of satisfiability, of course, stands at the center of this landscape, but there are also several questions of expressivity (is the countdown operator hierarchy infinite over infinite words, for example ?) and comparison with other models that seem interesting.

The thesis is overall well-written, with particular attention paid to pedagogical examples that illustrate interesting subtleties. The proofs are detailed and the results seem sound. The author shows a reasonable appreciation of the scientific context of his work. Some minor suggestions for improving the presentation of the thesis are included below for the author's benefit. These do not detract from the merits of the thesis.

A particularity of this thesis is that it delves deep into these two distinct sets of tools and practices. On one hand, the work on bisimulation categoricity falls into classic model theoretic questions, and in this context, the thesis uses FO-axiomatizations, saturation techniques, and topological arguments. The work on countdown μ-calculus on the other hand requires working with completely separate set of tools, namely parity games, reasoning about positional and nearly positional strategies, fixpoint theory and automata. The author shows an impressive mastery of both of these subjects and the tools needed to work on them. His contributions and choices of topics show a deep appreciation of classic topics and foundational questions on both the model theory side and the logic/automata/games side. Yet, from these classics, the thesis manages to uncover novel ground, with the study of bisimulation categoricity on one hand, and countdown μ-calculus on the other. In this way, this thesis successfully marries both a healthy respect for classic topics and a mathematical curiosity that enables the author to bring forth new ideas and angles of study to already well-trodden grounds.

I appreciate the author's contribution on the countdown μ-calculus for setting down robust definitions that extend the logic/games/automata toolkit to questions of boundedness, which seem to be attracting more and more attention. I expect these to be useful in the future, in particular in their potential relation to B-automata and related models. That being said, to my mind it is the work on the model theory of ML that is the most significant contribution. Indeed, the questions studied are both original yet quite fundamental, and the answers provided seem thorough, with a variety of techniques used to show them.

In this thesis Jedrzej Kołodziejski has successfully demonstrated his ability to pursue research in the field of model theory, logic, games and automata. My recommentation is that the candidate should proceed to a public defence of his thesis.

12.03.2024 in Marseille, France

Karoliina Lehtinen

Minor suggestions for the author

Chapter 3 Overall, there are several points at which you could add valuable discussions that help place the work in its context. For example, the choices of focus in several places could have been justified or motivated in more detail : for instance why are the cases of bidirectional and transitive models particularly interesting ? Are there cases that proves out to be difficult to handle ? What were the difficulties and what are the conjectures ? I would have also appreciated a discussion of how the features of the classes studied affect the proofs/their difficulty, and how the situation is likely to change for other cases not considered here. For example is the fact that the transitive models are monomodal important, or just convenient ? Can this restriction be lifted ? That being said, the above comment does not apply to the choice of focusing on ordinal models, which I find well-motivated, both by the different proof techniques that arise, as well as the more in-depth study of this model theory, including compactness.

Similar questions arise in Chapter 4, where you chose to study the hierarchy obtained from adding ranks, rather than the alternation hierarchy, which is typically more important in the μ-calculus. A brief discussion of such choices would help the reader follow your intuition of why these are the right questions to study.

Overall, the proofs are well-written and convincing. However, at times I found them hard to follow, and believe that they could be signposted better. For instance, the proofs of Theorems 3.1.4, 5.1.5 and 3.1.6. come quite a bit later after some discussions and are the location and structure of the proof is not explained. For example, when on page 52, referring to "the implication (3) $\implies$ (1)", it is unclear which theorem (or all of them) this sentence is referring to. Also consider whether the discussion of classes of models vs altered semantics is optimally placed between the theorems and their proofs, or whether it should come later.

Minor : on page 55, I could not find the definition of ω-saturation. On page 60 Figure 3.1.19 a bracket is overlapping with ω

I would have enjoyed a discussion at the end of chapter 3 for some perspective on the results there. For example, you compare your results with the Ryll-Nardzewski Theorem (page 45) on maximally consistent FO-models. Can you comment on how the proof strategies differ in the FO context ? Are there notable similarities in the arguments ? What are the key differences which require different techniques ?

Chapter 4 On guardedness. Prop 4.5.2. While I believe the statement to be true, the proof is not detailed enough to convince me of its correctness. In particular, the complexity/size increase of the operation is suspicious to me. Indeed, to the best of my knowledge the existence of a polynomial transformation of μ-formulas into guarded form is open (see for instance [BFL15]), and non-trivial. Indeed, replacing unguarded occurrences with $\top$ or $\bot$ is not a correct transformation.

Regarding the conjecture on the decidability of the countdown logic, could you comment on reasons to believe in decidability ? Can you say anything about decidability when the ordinals are restricted to omega ?

example 4.3.3. p 87 The example showing that there are no positional strategies has infinite branching. Under the assumption of finite branching, are there still no positional strategies ?

The caption of figure 4.1. is confusing red and blue.

Thm 4.6.2. The argument is quite involved, and dense. An overview would help.

I would have liked to see a discussion of the implications of this difference in expressivity. Which version is "better", that is, is there any reason to think the scalar version might be algorithmically easier to handle ? How significant is this difference ?

In general, please include all assumptions and conditions in the theorem statements so that they i) are more correct ii) can be cited. For example, the k bound on the stack height is not included in the definition of a countdown automaton so the condition should be stated in Thm 4.7.4. Similarly, the "mild assumptions" should be stated in Thm 4.8.1. Similarly the statement of Prop 4.9.7 is false since you need to specify that the game is winning for Eve.

Références

[BFL15] Florian Bruse, Oliver Friedmann et Martin Lange. “On guarded transformation in the modal μ-calculus”. In : *Logic Journal of the IGPL* 23.2 (2015), p. 194-216.